Preliminary results from an agent-based adaptation of friendship games

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This paper presents agent-based model (ABM) equivalents of friendship-based games and compares the results with the theoretical models. A friendship game is a network game in which a player's immediate neighbors on the network are considered friends. Two friendship-based game models are examined: strategic complements and strategic substitutes. Strategic complements represent decisions for which it is preferable to do what one's friends are doing, such as adopting a common software product. Strategic substitutes represent decisions for which it is preferable to let one friend act alone, such as the local provision of a public good. The game theory models predict the rate of change of preferences and specific equilibrium outcomes over specific time scales for each model. The ABM equivalents provide a means to examine the motivations for behaviors of specific individuals in these models beyond closed-form payoff functions. The ABM results are found to be sensitive to the topology of the random network, and alternate topologies are examined in this respect. (JEL Q32)

Keywords: Agent-Based Modeling; Agent-Based Computation Economics; Game Theory; Network Games; Friendship Games.

Lamberson (2011)presents a network game model of the influence that friends - defined as immediate neighbors on a network - have on individual preferences and the effect this has on overall equilibrium in the long run. Friendship games are applicable to problems for which peer choice is important. Examples include the adoption of standards or common

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tools, such as word processing software. Additionally, friendship games find use in problems of free-riding, such as the provision of a public good, such as a community web page or a web server.

1 Network games

Galeotti et al. (2010) present the theoretical basis for, and some examples of, network games. In these games, the players are distributed on a random network and the payoffs are functions of the expressed preferences of the immediate neighbors on the network. For a model of strategic substitutes, the payoff is such that, if at least one neighbor is paying the cost, none of the other neighbors has an incentive to also pay it. This is a free-rider model, similar to the local provision of a public good. For a model of strategic complements, the payoff is highest for the choice that is supported by a majority of neighbors. This is similar to a network externality, where adopting the most common word processing software, for example, maximizes the ability to share documents with neighbors.

Lamberson (2011) adopts the term *friend* for these network neighbors, reflecting the fact that adjacent nodes in a social network can be quite distant geographically. Lamberson extends the network game models to include *clusters*: the extent to which a players friends are friends amongst themselves. Clustering is shown to increase the equilibrium provision of a public good - perhaps inefficiently - in a strategic substitutes model. In a strategic complements model, however, clustering can improve the diffusion of new ideas or technologies.

1.1 The strategic substitutes models

Suppose there are two strategies, x and y. If an agent has k friends, then, at any given instance, there are k_x of them playing strategy x, and k_y of them playing strategy y. For the strategic substitutes models, the payoff for playing strategy x is

$$\pi_x(k_x) = f(k_x) - c_x \tag{1}$$

and the payoff for playing strategy y is

$$\pi_y(k_x) = f(1-k_x) - c_y \tag{2}$$

where f is a non-decreasing function andc_x and c_y are the costs of play x and y, respectively.

The provision of a public good is a strategic substitute: an agent needn't provide it unless none of its friends do. The decision of friends has a negative affect in that an agent tends to take the opposite choice of any friend that favors a strategy.

The strategic substitutes model presented in Lamberson (2011) is simply this: play strategy x if fewer than four neighbors (in a random network of degree 10) are playing x. That is, costs are zero and

$$f(k_x) = \begin{cases} 1 & k_x < 4 \\ 0 & \text{otherwise} \end{cases}$$

1.2 The strategic complements model

For the strategic substitutes models, the payoff for playing strategy x is

$$\pi_x(k_x) = 1 - c_x \tag{3}$$

where $0 < c_x < 1$ and the payoff for playing strategy y is

$$\pi_y(k_x) = \begin{cases} 1 & k_x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

The adoption of a standard is a strategic complement: an agent chooses what most of its friends choose. The decision of friends has a positive affect in that an agent tends to take the same choice as friends that favor a strategy.

The strategic complements model presented in Lamberson (2011) is simply this: play strategy x if four or more neighbors (in a random network of degree 10) are playing x. That is

$$\pi_x(k_x) = \begin{cases} 0 & k_x < 4 \\ 1 & \text{otherwise} \end{cases}$$

2 Approximating a random network

The Lamberson (2011) models feature 1000 players on a Bernoulli random network with an edge probability of 0.10. That is, for a network connecting all players to all players, there is a probability of one in ten that a given connection will actually be there. The number of other players to which a player is connected is that player's *degree*. The average degree for this random network is approximately 10. That is, players have, on average, ten friends.

In the ABM context, there are three ways in which a random network might develop. These are referred to as the *regular*, *minimum*, and *Gilbert* network models. The following are descriptions of these network models.

2.1 The regular random network model

One way to form a random network is for each agent to make two friends, but only with other agents that don't already have two friends. This results in a random regular network where the nodes have a uniform degree of two, ensuring that the average degree is two. This is a *2-regular* random network. It is a high connectivity network: no agents will end up completely disconnected from the network.

2.2 The minimum random network model

Alternatively, each agent can make two friends irrespective of how many friends those agents already have. Agents that made friends early are increasingly like to have more than two friends as later agents make friends. This results in a minimum degree of two and an average degree of nearly four. Here again, no agent is disconnected. The network described here is a *minimum-2* random network.

2.3 The Gilbert random network model

If all friendship pairs are equally probable with probability p then a Gilbert random network is formed. The mean degree is np, where n is the number of agents. The degree of the agents is distributed binomially. For a mean degree of two, this will be referred to as a *2-random* network. The algorithm used in this paper for a Gilbert random network is presented in the discussion.

2.4 The ABM platform

This paper uses the NetLogo[1] agent-based modeling platform. As a typical NetLogo model, the friendship games would update all agents in each time step, advancing time steps until there are no further changes. In order to make a direct comparison with the theoretical models, however, it is necessary to model just a single agent's update at each time step. This is done in the NetLogo models by updating a single, randomly selected agent at each time step. This random sampling means that, for a network with 1000 nodes, in the first 1000 time steps, some agents may not be updated at all, and others may be updated more than once. In this sense it differs from the theoretical models, though this difference is probably not significant.

2.5 The 10-random network models

The theoretical models (Lamberson, 2011) use a 10-random graph with 1000 nodes, and a tie probablity of 0.01 (the probability that a specific edge exists). For a strategic substitute,

[1] See http://ccl.northwestern.edu/netlogo/ [accessed 12 June 2011]

the payoff for playing x is positive if fewer than four neighbors are playing x, and the payoff is zero otherwise:

$$\pi_x^{substitute} = \begin{cases} 1 & k_x < 4 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

For strategic complements, the payoff for playing x is positive if more than four neighbors are playing x, and the payoff is zero otherwise:

$$\pi_x^{complement} = \begin{cases} 1 & k_x > 4 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

A plot of the ABM results for strategic substitutes is shown in Figure 1. The thin black lines are from the ABM results and the thicker blue lines are from Lamberson (2011). The ABM equilibrium drifts upward toward 0.42, while the theoretical drifts slightly above 0.40.

A plot of the ABM results for strategic complements is shown in Figure 2. The thin black lines are from the ABM results and the thicker blue lines are from Lamberson (2011). The equilibrium outcomes are the same at starting states near zero and near 1.0, but the dividing starting state is somewhat different. For example, for a starting state with 0.25 playing x, the theoretical equilibrium state is zero, but for the ABM the equilibrium state is 1.0.

One possible source of the differences between the ABM and the theoretical curves may be the underlying random network or, specifically, the algorithm for forming the random network. Figure 3 shows the distribution of node degrees for the network. Superimposed is the expected binomial distribution

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where k is degree, n is the number of nodes, and p is the tie probability (the probability that a specifc edge exists). The coefficient of correlation between the 10-random graph sample distribution and the binomial population distribution is 0.88. Nodes with degree near 10 are over-represented, while those with degree between 3 and 6 are particularly under-represented.

2.6 Comparison of 2-random networks

For degrees less than 10, the payoffs (4) and (5) cannot be used directly, so the more general forms (1), (2), and (3) are used in the ABMs. For the strategic substitutes model, the additional constraint $c_x + c_y = 1$ is applied.

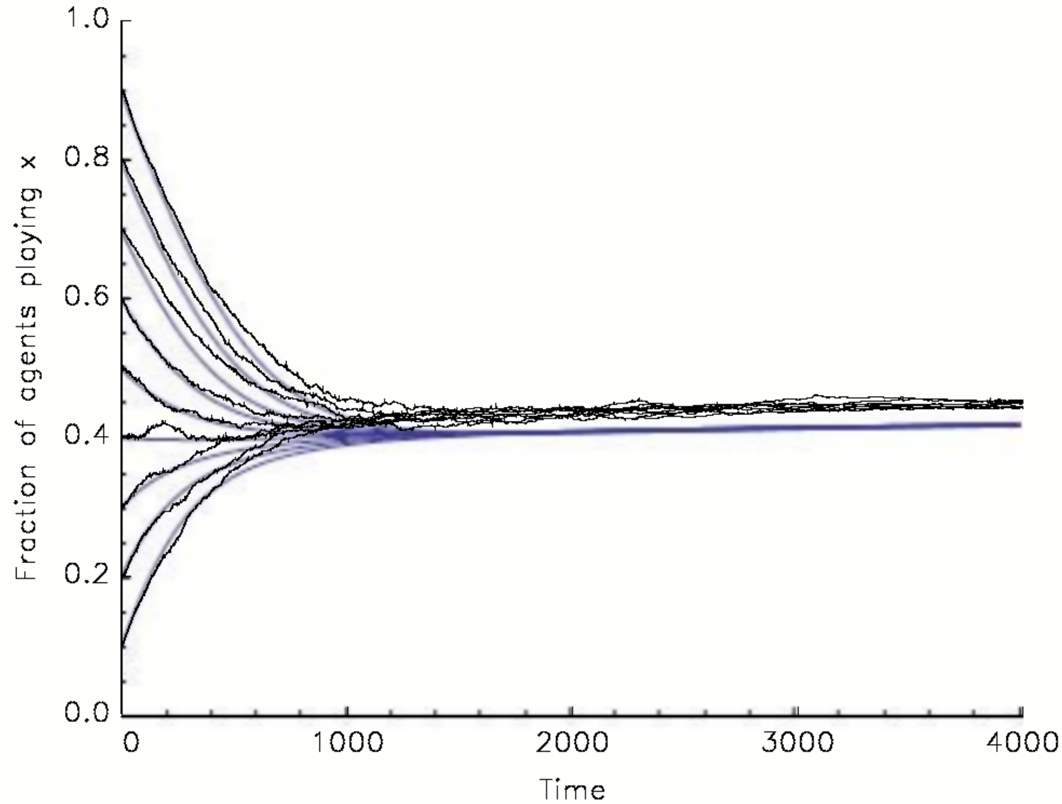

Figure 1: Strategic substitutes, $n = 1000$, $p = 0.01$. ABM 10-random results overlaying the theoretical curves from Lamberson (2011).

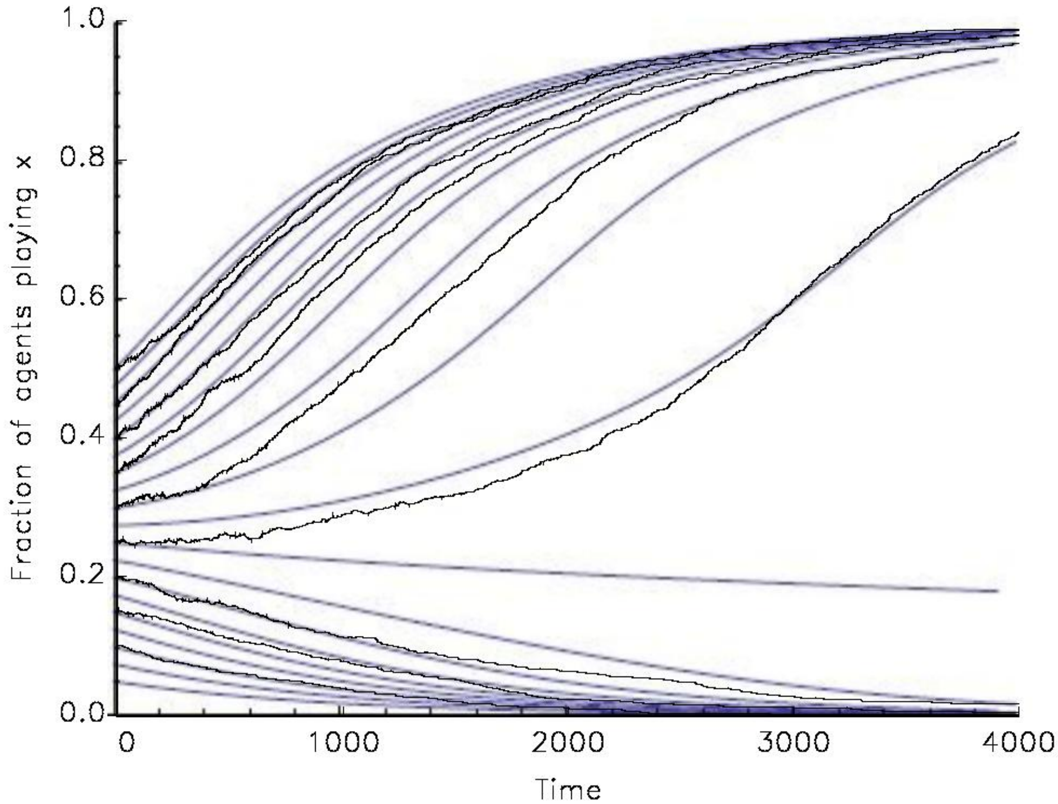

Figure 2: Strategic complements, $n = 1000$, $p = 0.01$. ABM 10-random results overlaying the theoretical curves from Lamberson (2011).

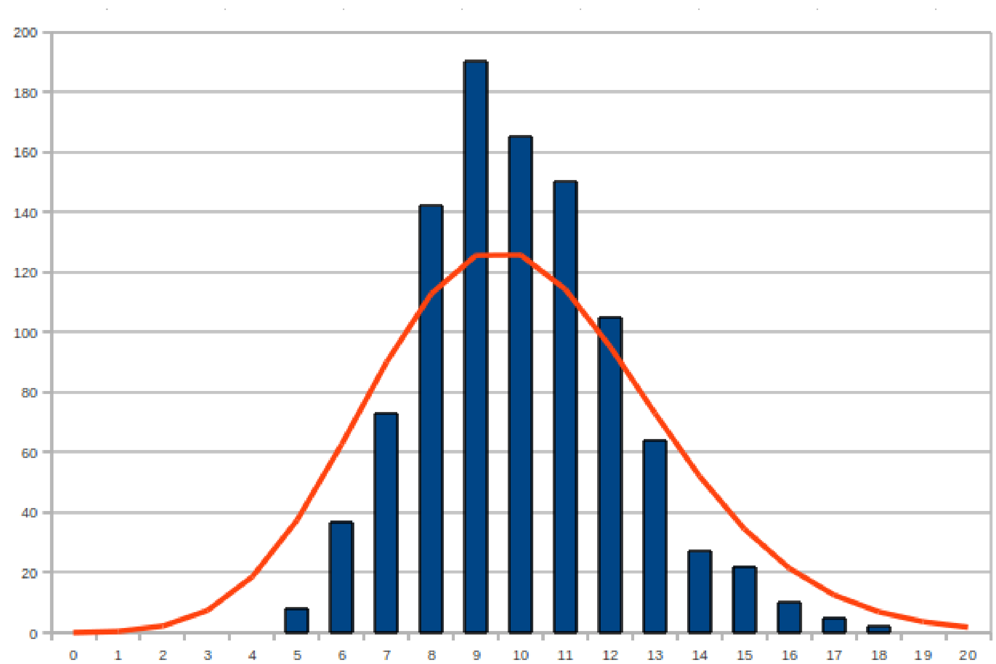

Figure 3: Distribution of node degree (blue bars) and the corresponding binomial distribution.

For the strategic substitute models, the ABM with a 2-regular network model reproduces the theoretical results remarkably well. This is illustrated in Figure 4, which shows the ABM results (thin dark lines) with cost ratio 50:50 together with the results from Lamberson (2011, Fig. 2). Note that, although each ABM curve converges more slowly than the theoretical, the overall convergence is very similar.

Figure 5 presents the same results for a minimum-2 network. Like the preceding graph, each curve falls below the theoretical initially, but it is more pronounced in this model. As a result, the curves converge below the theoretical, unlike the preceding, but eventually stabilize at the theoretical equilibrium after some time. This is markedly different from the 10-random model (Figure 1), for which the equilibrium tended above the theoretical.

Figure 6 shows the equivalent of Figure 4 for cost ratios $c_x : c_y$ of 25:75, 50:50, and 75:25. Specific cost appears to have no affect on either the dynamics or the equilibrium.

For the strategic complements model, the lowest degree to yield stable results is four. As seen in Figure 7, however, the trends for intermediate starting distributions is not consistent with theory.

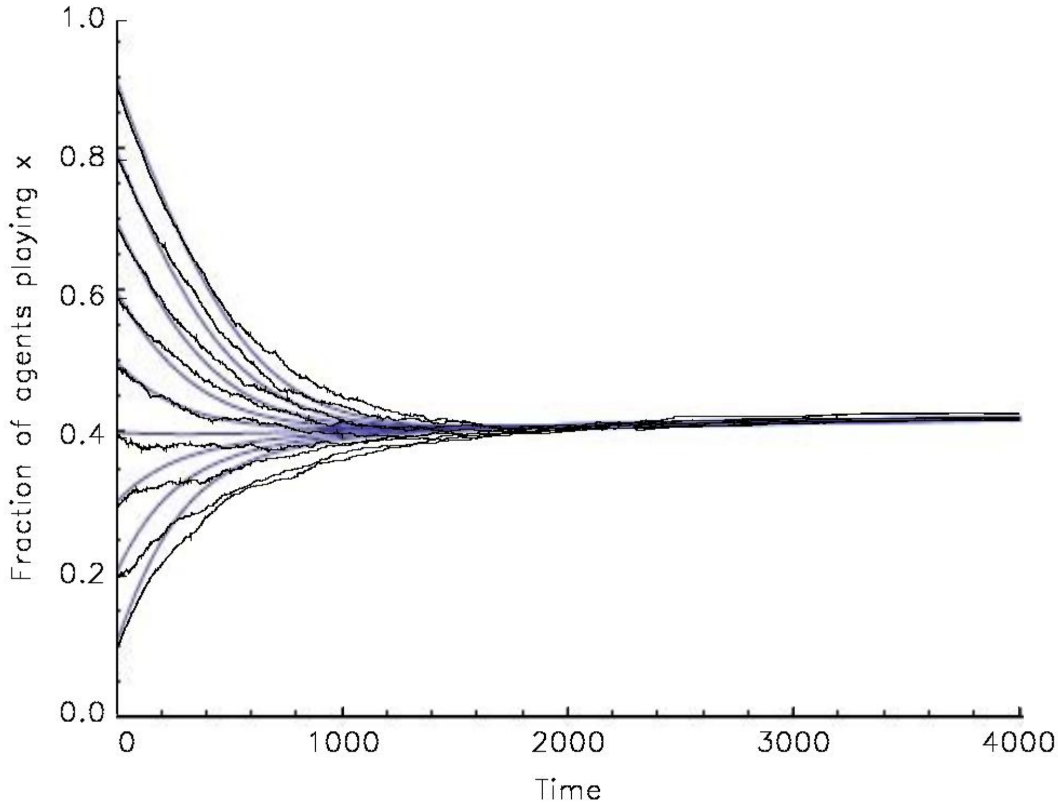

Figure 4: Degree two, substitutes, cost ratio x:y = 50:50, 2-regular network overlaid with Lamberson (2011, Fig. 2).

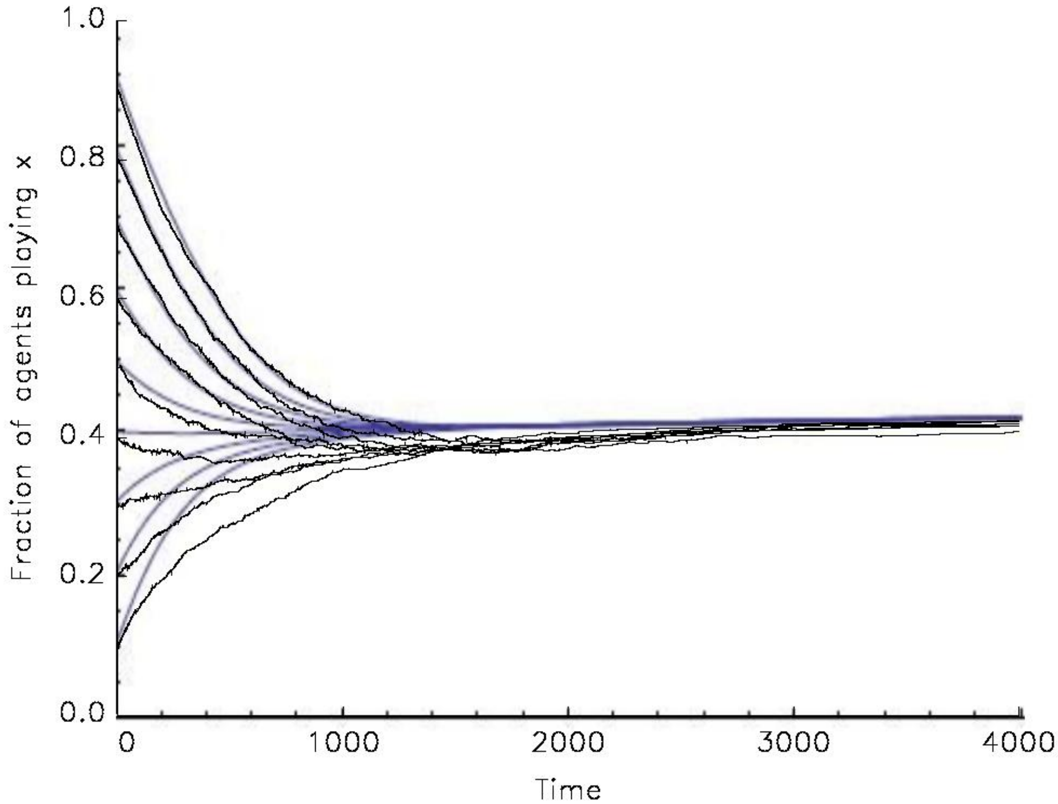

Figure 5: Degree two, substitutes, cost ratio x:y = 50:50, minimum-2 network overlaid with Lamberson (2011, Fig. 2).

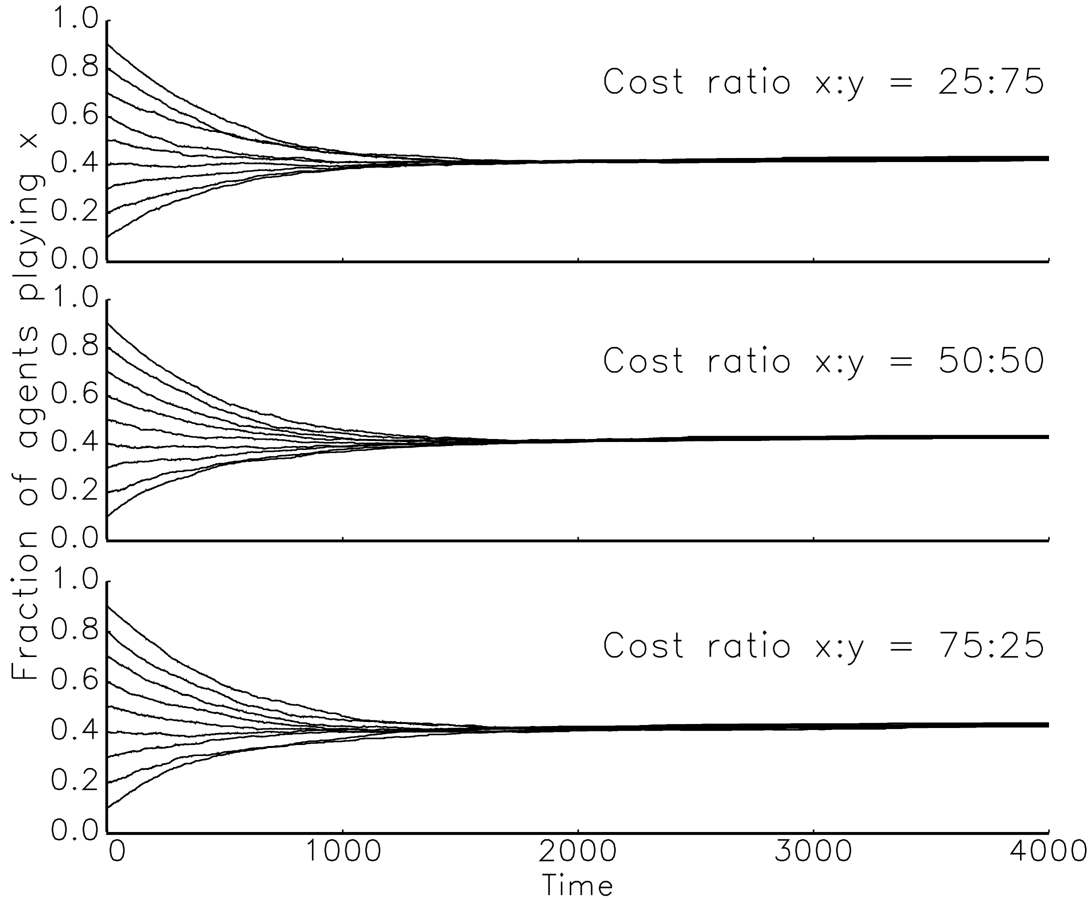

Figure 6: Degree two, substitutes, cost ratios compared.

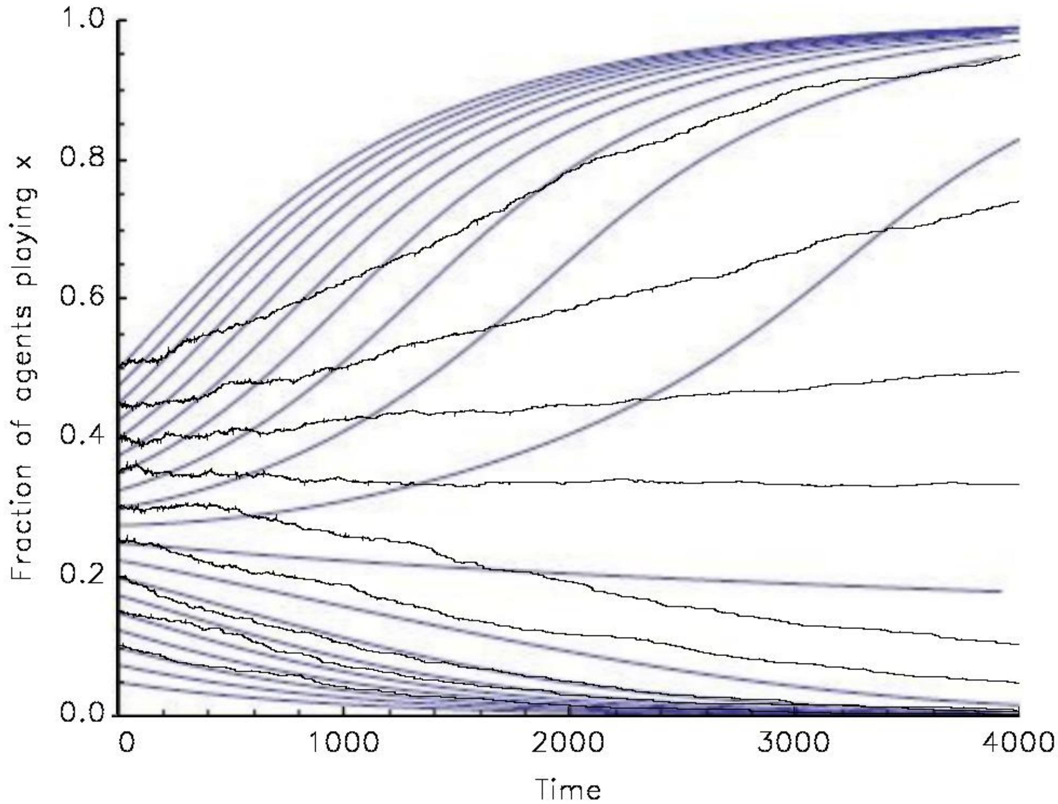

Figure 7: Degree four, complements, cost of x = 0.50, minimum-4 network overlaid with Lamberson (2011, Fig. 2).

Algorithm 1 Formation of an n-random network.

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p = tie probability
N = the number of nodes (agents) in the network
nSample = pN
For each agent α:
    select nSample/2 other agents
    for each agent β of these :
        if link exists between α and β
            attempt a link between α and another randomly selected agent
            until a new link is made
        else
            make a link between α and β
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3 Discussion

These preliminary results suggest that the outcomes are sensitive to the network topology or, more precisely, to the distribution associated with the random topology. The algorithm for forming the random-n network is shown as Algorithm 1.

Selecting random sets of agents of size $nSample$ makes this algorithm much faster than selecting random edges. The outer loop over all agents ensures that every agent attempts at least $pN/2$ links for every agent. This results in an expected degree of pN, but the distribution is narrower than binomial, as seen in Figure 3. Further study is expected to reveal if this error in the distribution of degree is the cause of the discrepency between the ABM and theoretical results.

4 Further research

The next task is to establish an algorithm for producing a true random network. If this is shown to reliably reproduce the theoretical result, research can proceed to examine more complex payoffs, including extended form games. If a truly random network doesn not yield results within a few percent of theoretical (in terms of the coefficient of correlation) then the source of the discrepency must be found.

Once correspondence with theory is established, other network structures can be examined, such as a power-law distributed network such as the Internet. Also of interest is the sensitivity to degree and to the number of nodes. Finding that lower degree networks produce qualitatively the same results will make it possible to explore the stochastic space of these network models. That is, given a distribution of a behavior, under what circumstances are the outcomes unchanged, reversed, or unstable. For example, if there is small probability that an agent's preferences are not transitive, at what incidence of intransitivity does the outcome fluctuate perpetually? Other behaviors of interest include trend-setting, resistance to change, contrariness, bargain switching, scams and fraud. Free-riding problems of interest include voting, recycling, immunization, charitable giving, and so on. To what extent are the outcomes affected by mobility and changing social structures, such as the emergence of cyber neighborhoods?

5 Conclusion

Friendship games are a tool for examining the affect of social decision-making and the impact of social connectedness. Agent-based models of these games may be a rich laboratory in which to explore nonlinear behaviors under these circumstances. In this preliminary study, it is found that, for models of strategic substitutes, even very small circles of friends produce results consistent with theoretical models. For strategic complements, however, the circle of friends must be greater than two for stable results, and greater than four to achieve results consistent with theory. Ongoing research will find better algorithms for constructing random networks, and will establish the minimum degree for an agent-based model that is consistent with theory. Once consistency is established, agent-based modeling can be used to investigate decision spaces that are difficult or impossible to model with network games.

References

References

A. Galeotti, S. Goyal, M.O. Jackson, F. VEGA-REDONDO, and L. Yariv. Network games. *Review of Economic Studies*, 77(1):218–244, 2010.

PJ Lamberson. Friendship-based Games. 37th Annual Conference of the Eastern Economics Association, 2011.