MATHEMATICAL REPRESENTATION OF REAL SYSTEMS: TWO MODELLING ENVIRONMENTS INVOLVING DIFFERENT LEARNING STRATEGIES

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Abstract: Many research studies, based on the representational nature of human knowledge and describing how people construct their knowledge about the world, focus on strategies of modelling, i.e.,: a facilitating process for the construction of adequate mental models that will help the understanding of physical models. We report examples where learning of physics follows the same steps than the learning of a new language: as it happens in the case of a new language, the learning of semantic precedes the learning of syntax, that is, mathematical representations. Teaching strategies are implemented that take into account that the students already have the basic tools to generate mental models, which are the same they use to interpret the world: to make analogies, idealizations and abstractions. The reported examples use two different kinds of modelling environments: the first one belonging to the category of the so-called "aggregate" modelling engines (STELLA) and the second one belonging to the category of "object-based" modelling languages (NET LOGO).

Introduction

Models are a central topic in discussion about contemporary science education with debates focused on including a modelling perspective in science curricula and on pragmatic strategies for designing classroom activities that enable students to learn about science as a modelling endeavour. At the core of the debates are questions connected with the different ways in which the term model is used, and in particular:

1. the role of models and modelling procedures in the process of knowledge construction;
2. the relationship between Physics Models and Mathematical Models.

Cognitive scientists have identified "mental models" as fundamental tools of human thought used to structure our experience and thereby make it meaningful. Researches on cognitive processes (1, 2) and epistemological analysis of evolution of physics (3) have shown that a modelling approach can constitute a good frame of reference in order to design teaching-learning situations both faitful to the discipline and relevant for the learner. Moreover, some pedagogical strategies have been developed focusing on transfer of knowledge and competences (4, 5) and allowing pupils to better understand many content areas, since they enable them to see similarities and differences among apparently different phenomena (6).

However, many pedagogical questions, concerning methods and strategies making the modelling approach teachable and/or learnable, are to be deepened.

It is widely accepted that the pedagogical use of Information Technologies (such as Microcomputer Based Laboratory, multimedia, simulation programs etc.), in mathematics and physics may help pupils to overcome some difficulties dealing with learning of scientific concepts and procedures.

In this paper, we present some pedagogical strategies and tools giving pupils the possibility to formalise a problem, not necessarily dealing with physics, following well defined steps tailored to identify relevant elements of a given observation, define the associated variables, predict relationships among them and check predictions using tools scaffolding the process of personal knowledge construction. Furthermore, we show how this strategy allows to overcome the well-known difficulty that students encounter in the understanding of some mathematical formalism, such as differential equations, and their real meaning. The reported examples are aimed to students in 16-18 year range and have been experimented in courses for pre-service teacher preparation.

Difficulties in mathematical learning

Physics teaching usually use the differential calculus in an operational and mechanical way: very soon, even in simple situations (7), students do not understand what is beeing done and why it is done. Most high school pupils as well as many undergraduate students have a inadequate comprehension of the mathematical symbology and, what is surely worse, they do not know when and why differential calculus becomes necessary and/or the strategy that it offers in order to solve problems.

A typical case is the understanding of the time derivative symbol, *d/dt*, seen as an operator that, applied on a generic function *y(t)*, gives its rate of change. In this sense *d/dt* could be seen as the "rate of change" operator. The application of the operator *d/dt* to the variable position is very often well understood by the majority of students: probably, because the concept of speed as an indicator of how much quickly a body is moving belongs to the customary experience. On the contrary, the same cannot be said for other physical quantities. So, if we apply *d/dt*, for instance, to the temperature of a cooling body, very seldom students are able to grasp the meaning of the symbol, that is rate of variation of temperature. This simple example partially explains why the formalism of differential equations, although considered the starting point in the *mathematisation* of physical situations, results to be badly understood.

In this context modelling activities could help students to develop competencies in translating different descriptions of real world into each other. In fact, modelling may be considered as a translation procedure from verbal description of real world to other forms of representation, such as the mathematical or iconic ones. The use of "simulation environments ", whose components allow to easily visualize physical objects and processes, makes possible the construction of operative thinking forms.

“Aggregate” modelling engines: STELLA

STELLA is a simulation environment developed to generally represent the process of “thinking” (8); its main advantage is to eliminate the need to manipulate symbols and to make complex mathematics understandable and easily manageable. STELLA represents the dynamic changing of a variable by a stock that can fill or drain through incoming or outcoming flows. Its main graphic interface makes available the basic elements assigned to the model building. These are: *Stocks* - containing amount of the variable changing in time, *Flows* -representing the action to fill or drain stocks, *Converters* -additional objects used to complete the logic sentence, *Connectors* -used to link objects together and define their relationships.

In natural language a mental model is generally associated to a verbal description; STELLA allows the users to translate the verbal model of the system under scrutiny into a symbolic scheme, by representing carefully all the elements of the idea describing its evolution. The program automatically generates a code describing the scheme built by users and simulates the time evolution of the system representing it through graphs, tables etc. Modelling becomes a “translation” from verbal descriptions to iconic representations; mathematical equations are, consequently “translated” from the specific iconic language of STELLA.

Some examples with STELLA

In order to introduce the concept of “*rate*” (the amount of change of some quantity during a time interval divided by the length of the time interval) we usually begin by studying what happens to the volume of water in a container in the situation represented in fig. 1.

We can recognise (see fig.2) the *stock* used to represent the volume of water and the *flows* representing the action to fill or drain stocks at defined constant rates R_{in} and R_{out}.

The system can be verbally described by a statement like: *The volume of water is dependent both by the outflow and the inflow of water.* In order to quantify the relationships we can say some more, that is : *Its rate of change is given by the algebraic sum of the two rates.*

The iconic model of the process and the results for different values of the ratio *Inflow-rate/Outflow-rate* are represented in fig. 2.

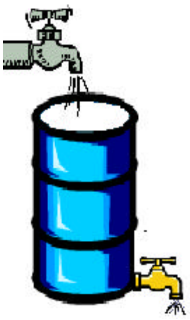


Figure 1

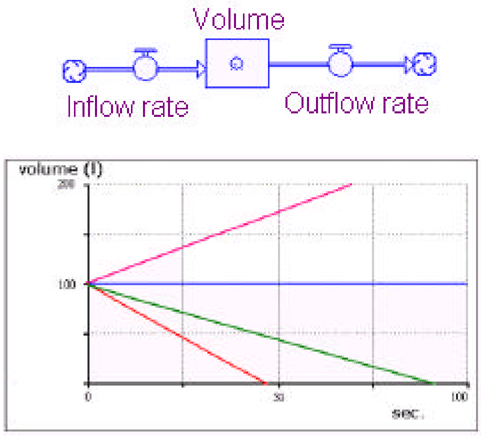


Figure 2

The translation of the iconic model to the mathematical formal model is straight:

$$\frac{dV}{dt} = R_{in} - R_{out}$$

1. Decay of pollution: the lake purification

The next point is the introduction of the concept of variable rate. We approach the problem by considering a lake, of constant volume, V_L, contaminated by some kind of substance. If the lake is well mixed, we can assume that the contaminant concentration is uniform. Assuming that at time $t=0$ we start to decontaminate the lake by making flow in clean water at rate R and allowing that at the same rate R water containing the polluter flows out, in order to model the process of the lake purification and calculate the time dependence of the contaminant volume, $V_C(t)$, we can begin with a simple verbal description of the phenomenon. A possible one can be the following: *The volume of the contaminant must decrease, due to both the outflow of polluted water and the inflow of clean water. The clean water volume consequently increases until it reaches the (constant) volume of the lake.* Now, how can we try to schematise the relevant variables and all the logic links between them influencing the process of lake purification?

A possible STELLA iconic representation of the lake purification is shown in figure 3.

The figure shows that the model is built by using two stocks, three flows, two converters and some connectors.

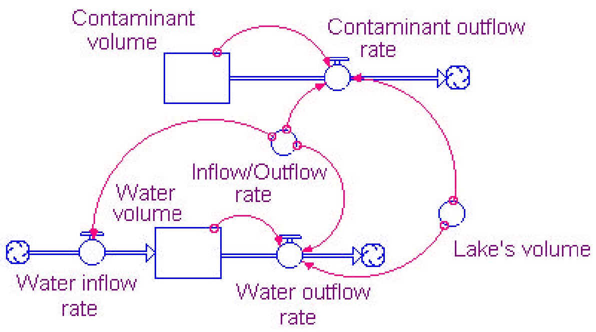


Figure 3

The relevant variable, the contaminant volume, V_C, is represented by the first stock and the other stock does represent the volume of clean water in the lake, V_W.
One of the converters indicates the lake volume, V_L, equal to $V_C + V_W$ and constant by definition. The other converter represents the rate of inflow of clean water into the lake and, at the same time, the rate of outflow of polluted water from the lake, equal because of the constant lake's volume.

The flow marked "Contaminant outflow rate" represents the rate of decrease of the contaminant volume. It must obviously depend from the lake's volume, V_L, the contaminant volume, V_C, itself and from the Inflow/Outflow rate and is linked to them by connectors. The two flows connected to the water stock indicate the two ways the clean water volume has to vary; the "Water inflow rate" flow represents the constant inflow of clean water into the lake and is linked (or, better, equal) to the "Inflow/Outflow rate" converter; the "Water outflow rate" flow, on the other hand, must depend from the Inflow/Outflow rate, the lake's volume and the clean water volume. Note that the links between The "Contaminant volume" stock and the "Contaminant outflow rate" flow and the "Water volume" stock and the "Water outflow rate" flow actually give two feedback loops, indicating that these flows must take into account the instantaneous value of the variable represented by the stocks. As a consequence the two rates "Contaminant outflow rate" and "Water outflow rate" are not constant, but depending from the instant values of the volumes.

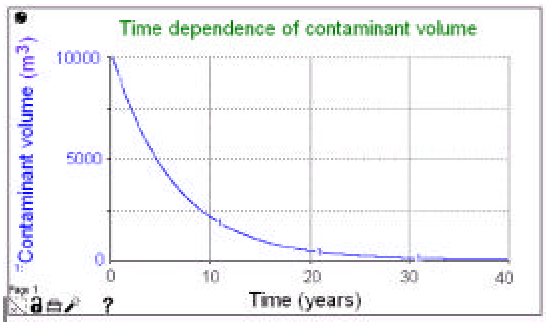
The equations lying behind the model are:

$$\frac{d}{dt}[V_C(t)] = -\frac{R}{V_L}V_C(t)$$

$$\frac{d}{dt}[V_W(t)] = R - \frac{R}{V_L}V_W(t)$$

Their solutions should exhibit exponential time dependences. In figure 4 are reported the results of the STELLA simulation, with $V_L = 10^8$ m^3, $V_C(0) = 10^4$ m^3, $R = 0{,}5$ m^3/s.

It is worth noting that the first equation contains all the necessary information to solve the problem if we make a more adequate choice of the variable describing the evolution of phenomenon: i.e. the concentration of the contaminant in the lake, $c(t)$, (the mass of contaminant per cubic meter of water). The equation behind the model is very similar to the one for the contaminant volume:



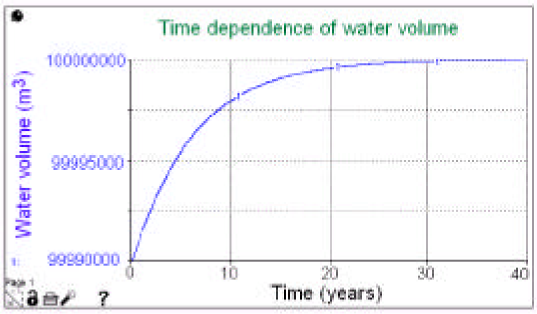


Figure 4

$$\frac{d}{dt}[V \cdot c(t)] = -Rc(t)$$

In this case the STELLA model is simpler, mainly because we need not to take into account the water volume time dependence (see figure 5). The solution shown in figure 6 is obtained with $V_L = 10^8$ m^3, $c(0) =$ 0.1gr/m^3 and $R = 0{,}5$ m^3/s

2. Cooling a substance in an environment at constant temperature

We now analyse and model a classical physical problem showing behaviours similar to those analysed in the previous paragraph.

We consider a container with a given mass of water at temperature T_W set in an environment at a lower. constant temperature T_E. The process of water-cooling is easily observed as well as the fact that the cooling

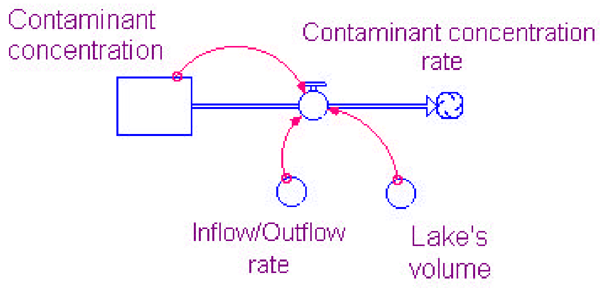


Figure 5

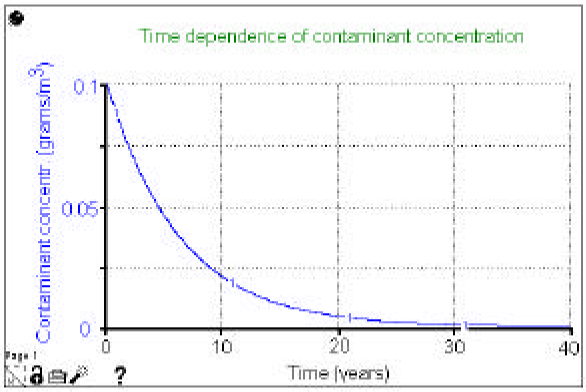
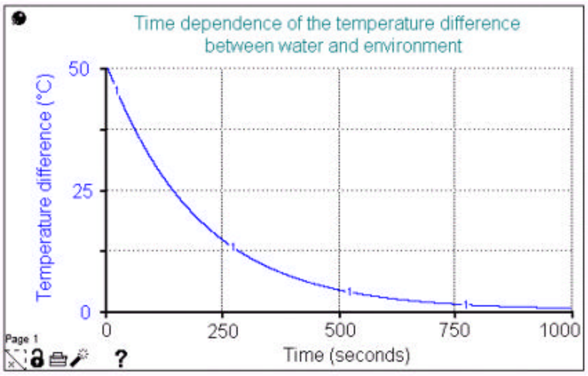


Figure 6



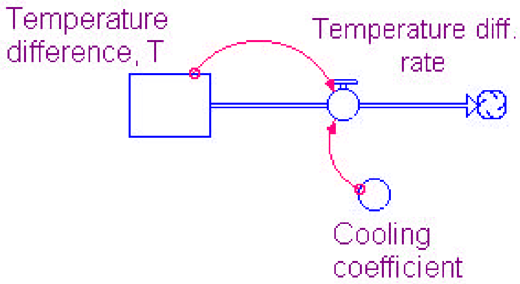


Figure 7

rate is not constant. The appropriate variable to describe the phenomenon is the temperature difference between water and the environment, $T(t)= T_W(t)-T_E$. It decrease as a consequence of the heat flow from the system to the environment. If we call K the cooling coefficient, depending from the physical parameters of the system (liquid and container)[1], we can hypothesize a dependence of the cooling rate dT/dt from the instantaneous value of T.

The STELLA iconic representation of water cooling model is very similar to the lake's purification model and consequently to its results (see figure 7) .

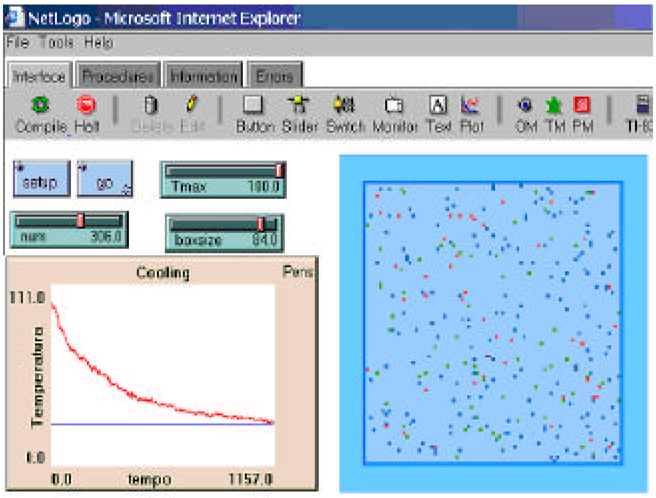
The NetLogo modelling environment

NetLogo (9) is a programmable modelling environment for simulating natural and social phenomena. It is particularly well suited for modelling complex systems developing over time. Modellers can give instructions to hundreds or thousands of independent **"agents"** all operating in parallel. This makes possible to explore the connection between the micro-level behaviour of individuals and the macro-level patterns that emerge from the interaction of many individuals. The agents can represent animals, cells, trees,….., that is individual elements interacting and consequently also molecules of a substance cooling in an environment at low temperature (see figure 8).

NetLogo lets students open simulations and "play" with them, exploring their behaviour under various conditions. It is also an "authoring tool" which enables students, teachers and curriculum developers to modify models and/or create their own models. NetLogo is simple enough that students and teachers can easily run simulations or even build their own. And, it is advanced enough to serve as a powerful tool for researchers in many fields.

Many real systems are usually so complex that a description and interpretation at macroscopic level requires mathematical competencies usually not mastered by high school pupils. Their analysis at level of their constituents can, some times, simplify understanding by making pupils enable to construct causal explanations of a wide range of phenomena, and provide them with a framework that is useful across a wide range of disciplines.

[1] From the Newton's cooling law, the cooling coefficient, K, is equal to hS/C, were h is the "external conductivity coefficient", S is the thermal contact surface between water and the environment and C is the thermal capacity of the sample of water.



The figure shows the NetLogo interface representing a given number of gas molecules, at a given temperature, contained in a box in thermal contact with the environment at low temperature. The cooling of the gas is represented using different colours for molecules with different energies.

A graphical display of the gas temperature as a function of time is also represented. Data fittings of the cooling curve can be performed and its mathematical characteristics pointed out.

Figure 8

Conclusion

The didactic approach, here described, has been tested in courses for pre-service teacher preparation, in order to make prospective teachers aware of the power of informatics tools in facilitating the construction of mental models adequate to help the understanding of physical models as well as the formalism of differential equations. Several parts of the modelling approach have also been tested in some high school classrooms.

The STELLA approach offers real advantages in helping users to construct models and to eliminate the need to manipulate symbols, making complex mathematics more understandable. Student teachers, graduated in mathematics or physics, at the beginning encountered some difficulties in switching the symbolic perspective: they have studied differential equations in their degree courses and were familiar with their formalism. They began to appreciate the new representational formalism when they met more complex systems to model: they understood the power of the iconic formalism and showed to appreciate its pedagogical impact.

Some teachers, introduced to STELLA during a in-service teacher training course, recognized the advantage of using it in the teaching of science, but showed the need to have a well performed didactic guide introducing to the language. An introductory guide is in preparation.

NETLOGO uses a different approach that allows users to model systems directly at the level of their individual constituent elements. Using it, students can learn to think about actions and interactions of individual objects and to describe complex system properties as the result of individual actions.

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