

THE NATURE AND FUTURE OF CLASSROOM CONNECTIVITY: THE DIALECTICS OF MATHEMATICS IN THE SOCIAL SPACE

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New theoretical, methodological, and design frameworks for engaging classroom learning are provoked and supported by the highly interactive and group-centered capabilities of a new generation of classroom-based networks. This discussion group situates networked learning relative to a dialectic of (1) seeing mathematical and scientific structures as fully situated in socio-cultural contexts and (2) seeing mathematics as a way of structuring of our understanding of and design for group-situated teaching and learning. Acknowledging (1), significant classroom examples are then used to illustrate the reciprocal process (2) of mathematics structuring the social sphere (MS3). The mathematically informed ideas of *space-creating play* and *dynamic structure* are then used to update our ideas of generative teaching and learning and to situate these classroom examples. Then, returning again to the dialectic, this current work is critiqued from a socio-cultural perspective (1). *Participation* and *agency* are highlighted in this critique. The session closes with a discussion of future possibilities for classroom connectivity.

The highly interactive and group-centered capabilities of a new generation of classroom-based networks are helping both to support and to provoke the development of new theoretical, methodological and design frameworks for engaging classroom learning. This discussion group situates networked teaching and learning relative to a dialectic of (1) seeing mathematical and scientific structures as fully situated in socio-cultural contexts and (2) seeing mathematics as a way of structuring our understanding of and design for group-situated teaching and learning. The idea is that if mathematical and scientific structures are seen to fully participate in the social plane, then not only are they structured by the social plane (i.e., (1)) but they also structure social activity (i.e., (2)), including learning and teaching. Due to the group-focused interactivity and data collection capabilities of next-generation networking, we now have a new tool to explore the dynamics of—and design for—classroom learning. A number of projects have responded to the challenge of learning in a network space by using mathematical/scientific ideas to organize and analyze classroom activity. Some of these recent projects focus on student learning and one is focused on teacher understanding. All of these projects have begun to use mathematics itself to organize their classroom-based work. This use of domain-related “big ideas” to organize and analyze group learning is what is meant by mathematics structuring the social sphere (MS3). The mathematically informed ideas of *space-creating play* and *dynamic structure* are then used to update our ideas of generative teaching and learning and to further situate the previous classroom examples. Our argument is that to take full advantage of these notions, and of the new classroom tools, researchers and educators must acknowledge explicitly the dialectic that exists between the domains of mathematics or science as structuring agents and the structuring functions of the social, cultural, historical milieu in which classroom learning and teaching in the domains exist. Returning again to the dialectic, this current work is then critiqued from a socio-cultural perspective focusing on ideas of participation and agency. Consistent with this dialectic framework, an overall notion of what is called generative teaching and learning is clarified in a way that both draws on previous work and uses the mutually constitutive relations captured by the dialectic for extending the prior analyses.

What's New about Next Generation Classroom Networks

This paper is not about technology *per se*, but rather about a specific instance of the interaction and co-evolution of design, technological affordance, and cognitive theory (Dewey, 1938; Hickman, 1990). Networks and ideas of generative teaching and learning can be and have been discussed without specific reference to technology (Wittrock, 1978, 1991; Learning Technology Center [LTC], 1992). The idea is that next generation networking can better support or “resonate” with generative practices and also, in iterating back to theory, allow us to further develop our ideas of what generativity is. To start this conversation, we may need to understand first how these new network designs are functionally evolved, and thus distinct, from the kinds of networking that has been with us for a long time. In this section the question becomes: What might be said to be “new” about next-generation network functionality in classrooms?

With a few important exceptions (among them, *CSILE*, associated with Scardamalia (1993); Scardamalia and Bereiter (1991); Scardamalia, Bereiter and Lamon (1994); and Scardamalia, Bereiter, McLean, Swallow and

Woodruff (1989); collaboratory notebook associated with Edelson, Pea and Gomez (1996); *ClassTalk* as discussed by Abrahamson (1998); Mestre (1996), or Hake (1998), or some of the projects discussed herein), networking in the classroom has served primarily in two ways: (i) As a portal to sources of information or interactivity having their “centers of balance” outside the classroom (e.g. visiting the *CNN* web site or filling out a web-based questionnaire), or (ii) To implement Computer Assisted Instruction (CAI) or tutoring environments. Two top-level features characterize these uses of networking in classrooms. First, the experience is fundamentally individual – most of the activity could be carried out at home or in a local library as little or no use is made of the social space of the classroom. Second, the knowledge and/or the trajectory of learning is owned by someone who is not a member of the classroom (e.g., a distant “expert” for web content or the computer programmers for CAI/tutoring environments). Especially for low socio-economic status (SES) students who have had their experience of school-based technology skewed heavily toward the use of CAI environments, a fundamental message of mathematics and science reform—that the classroom should be a community of inquiry characterized by joint ownership and construction of understanding—is undermined. Traditional uses of classroom networks place the locus of knowledge making outside the classroom (even as the individual experience is often described as highly “customizable”).

The next-generation network functionality, provoking a new round of theorization and design, does mark a significant break with this tradition in terms of pedagogical vision and technical capability. To begin with, these systems are typically designed “from the ground up” with the classroom in mind. Rather than constrain the learning experience to be narrowly individualistic, this technology supports socially situated interaction and investigation. Moreover, the learning trajectories and the processes of knowledge construction are owned by the group itself. Software and hardware work together in supporting this “group-oriented” design. A significant number of these networks are about to become widely available and are poised to become a major presence in classroom learning. The implementations may vary but at the top level the design features of these systems are remarkably similar and typically include: individual devices or “nodes” that are relatively unobtrusive; the network supports a range of topologies for real-time or near-real time interaction (peer to peer, peer to group including whole-class, or group to group); the network is “wireless” and hence very flexible and portable; there is a core set of meaningful functionality in each device (e.g., at least that of a graphing calculator); there is a mixture of public and private display spaces (e.g., the public space can be a computer projection system as with participatory simulations (Wilensky & Stroup, 1999) or a calculator Viewscreen™ with the SimCalc materials (Kaput, Roschelle, Tatar, & Hegedus, 2002) and the private space can be the students’ own individual displays on a calculator, Palm Pilot™ or laptop computer); the network experience is “author-able” in that it allows teachers or others to create new activities; the network can readily support the exchange of a range of kinds of both group and individual artifacts/data-types including text, strings, numeric values, ordered pairs, lists, matrices, individual and whole class graphs, images and, in some cases, sounds or video; and the network enables new (as well as old) forms of gesture (e.g., turning to the left with an arrow key on a calculator).

An interesting facet of working with these kinds of networks is how traditional research methodologies and theories, based on some notion of simply “scaling up” to the group methods and models based on individual cognition, are overwhelmed. This kind of challenge has always been present in analyzing classroom learning, especially as we move toward a reform-based ideal of classrooms being communities of inquiry (Brown & Campione, 1996). The theoretical and methodological disconnect seems even more pronounced for researchers attempting to make sense of what happens in these highly interactive networked spaces. All the projects discussed in this paper are struggling to situate and organize what they are doing in terms of activity design, learning theory, and research methodology. The good news is that new patterns of analysis and design are beginning to emerge. One of these new frameworks is the use of mathematical ideas not just as “content” to be learned but also as an interpretive framework for learning analysis and activity design. This is what will be discussed as mathematics structuring the social sphere (MS3). And then, in turn, the mathematically informed ideas of space-creating play and dynamic structure are used to clarify and update our understandings of generative teaching and learning. Here we can begin see how technological affordance can loop back to change the shape of cognitive theory and activity design. Returning to the dialectic, we then use insights from a socio-cultural perspective to critique and extend the ideas of participation and agency related to generative teaching and learning.

Mathematics Structuring the Social Sphere: Examples From Networked Classrooms

During a recent session at the Annual Meeting of the American Educational Research Association, a group of investigators presented results from working in classrooms with this kind of next generation classroom network (Kaput, Roschelle, Tatar & Hegedus, 2002; Mack, 2002; ;Stroup, 2002a, 2002b; Wilensky;). A striking features in common to these presentations was the way in which ideas from mathematics proper were either starting to become, or already were, central to the organization of their work. This engagement with mathematics went well beyond thinking of it as content to be learned by individual students. These presentations are discussed briefly in this paper

as part of helping us understand what it might mean to think of mathematics as a socially structuring tool in learning research and design. These examples illustrate how thinking of mathematics as an active participatory agent in social activity, might be considered a reasonable “next step” in our ever-evolving understanding of the nature of mathematical and scientific activity itself. That is, once we relinquish the assumption of the truly solitary mathematical or scientific researcher – as either a practical possibility or theoretical ideal – we are then free to fully engage how mathematics is a socially structured form of activity (part (1) of the dialectic). With this freedom also comes the possibility of investigating how mathematics is, or can be, socially structuring (part (2) of the dialectic). Each of the research efforts discussed in this section is responding to the challenges of creating learning activities in next generation networks. Each illustrates the directions being taken in trying to say something meaningful about the learning and teaching that takes place in relation to these activities.

Parametric space

Moving their work from developing software for learning calculus ideas with individual computing devices to now situating the use of this kind of functionality in a network space, Jim Kaput, Jeremy Roschelle, Deborah Tatar and Stephen Hegedus from the SimCalc Project have been pushed to think anew about the design of the activities themselves (2002). Prior to the availability of classroom connectivity, they had presented students or small groups of students with individual learning tasks related, for example, to giving in advance a *velocity* graph that controls the motion of one animated character and then asking them to author a *position* graph for a second character such that its motion would now match that of the given character (thereby using the simulation and graphs to obtain the integral of the original velocity function). Given the network, they have begun to use the idea of a parametric space to organize the network-supported activity.

For example, in the curricular context of linear functions in $y = mx + b$ (slope-intercept) form, to help understand the roles of the “m” and “b,” each group of students is assigned its own value of “b” (the y-intercept), which controls the starting point of the group’s “mascot” in a “race.” In this race they are asked to finish in a tie with all the other groups’ mascots at a given time (six seconds) and position (twelve meters). They then must determine the velocity (and hence slope) that accomplishes this task for their given initial conditions. In submitting their solutions to a public display space the resulting aggregation of graphs has properties that none of the individual graphs have: A “star” of lines all meeting at the same place is created. When simulated characters associated with each of these lines are animated, the characters all come together at (6, 12) and, if continued for an additional two seconds, they then spread apart again in a kind of mathematical dance. In this way, the SimCalc Project has begun to use the mathematical idea of parametric space as both the “content” and the cognitive organizer for the network-supported learning activity to raise the level of organization of mathematical objects as well as the focus of mathematical attention in the classroom. For further specific examples, see Kaput & Hegedus (these proceedings).

Proof

In a related way, Andre Mack (2002) has begun significant network-based research related to how teachers’ notions of mathematical proof structure and organize their real-time decision making about where to “go” with the classroom learning, especially as related to moments of mathematical uncertainty (e.g., when students ask questions that the teachers (and/or the students) are uncertain about). Just as moments of uncertainty in mathematics proper provoke a need to construct proofs, some idea of how valid reasoning “works” in mathematics animates teachers’ organization of learning in their own classrooms. Teachers’ notions of aesthetics for mathematical proof serve as an instructional resource for making sense of and validating (or disproving) student-generated mathematics claims. A wide range of understandings of mathematical proof is represented in Mack’s investigations. What is important here is the way in which the mathematical idea of proof is used to organize his analysis of classroom learning. Much of this work moves well beyond traditional understandings of pedagogical content knowledge (Shulman, 1987) centered on the interaction of pedagogy (typically understood as *how* to teach) and content (understood as *what* gets taught), the content – proof in this case – *becomes* or is *enacted* as the pedagogy in moving to the group as the unit of learning analysis.

The network-supported activities bringing about significant instances of uncertainty in the classroom include participatory simulations (discussed next) where, for example, each student uses the arrow keys on a networked calculator to move an individual point around on his/her screen. At the same time, this point and all the others from the class move on a computer screen projected at the front of the class. The teacher asks the students to move according to a rule like “move until your y-value is two times your x value.” When a line forms, the points are then aggregated and sent back to the students with the challenge using the calculator to find equivalent functions that go through these points. The student functions can then be collected and displayed (Wilensky & Stroup, 1999). Mack has found (2002) that often, in the course of this activity, students will submit functions where the teacher is unsure of their equivalence. The teacher then has to make decisions about how to proceed based on his/her notions of how

proof works in mathematics. The trajectory of the group learning experience is structured by the teacher's ideas of how mathematics reasoning is carried out.

Complexity

In a similar way, content becomes pedagogy for the Participatory Simulations Project (Wilensky & Stroup, 1999). Participatory simulations are activities where learners act out the roles of individual system elements and then observe how the behavior of the system as a whole can emerge from their individual behaviors. These emergent results then become the focus of in-class discussion and analysis. Using network technology with a public display space, students can, for example, become agents in a population where a disease is introduced and be part of the system when the disease spreads. Or they can control a stoplight in a simulated city's traffic grid and work toward improving the traffic flow. Not only is dynamic systems modeling the content being introduced into the curriculum but also the learning itself is organized in terms of the classroom *becoming* the dynamic system. By assuming roles in a system, mathematical ideas like emergence, feedback, and complexity are literally embodied by the network-supported learning activity. The traffic control or "gridlock" activity helps to illustrate both how students already have ideas about how complex systems may work and how – in using this embodied learning activity – students develop significant insights and conjectures related to emergent phenomena like traffic.

When introduced to the gridlock activity, students are presented with the following scenario: The mayor of the City of Gridlock is unhappy with the traffic congestion in town and she has commissioned the class to improve the situation (Wilensky & Stroup, in review). The goal of the activity is for the students to find ways of optimizing traffic flow for the simulated city. The Gridlock activity has been run in a variety of settings, from middle school science classrooms, to secondary social science classrooms, undergraduate and graduate education classes, and at research conferences. The responses reported below are from one of these instances of running the activity in a class made up of twenty-four seventh graders taking science at a relatively low income, ethnically/racially diverse school located near Austin, Texas.

After the goal of the activity was introduced students are asked what they know about traffic flow. In response to the teacher asking, "What are some of the things that you guys listed that would be indications that traffic would be good or bad to you?," the students presented ideas that covered more than two chalk boards. Ideas ranged from "slow drivers" and "barrier walls too close for cars" to "too many cars" and "special events" like "crashes." The sense is that learners can articulate a wide range of factors that can impact complex phenomena like traffic. Some of these responses are behaviors of individual drivers and others are related to the structure of the roadways or the context for the behavior (e.g., lights too long). Taken collectively, these lists suggest that learners do have an initial appreciation of how agent behavior in an environment can have consequences for the emergent features of a complex system. For traffic this insight comes from their first-person experience. Part of the purpose of participatory simulations is to leverage the learners' first-person perspective in a way that can lead to more robust, incisive and powerful understandings of complex phenomena.

Next the students engage in the participatory simulation using the network. After a number of rounds of "play" starting with getting familiar with controlling the lights and then moving to improving the traffic flow, the teacher stopped the class and asked:

Teacher: Has anybody started to think of some ideas like what are you doing at your stop light that you think is working really well? Who has some ideas of why they think maybe their stoplight is working better than somebody else's stoplight. Anybody have any ideas? Anything that you're trying to do yet? Yeah (pointing to student).

Student 1: Letting one go and then the other go.

Teacher: Okay, so you're letting one go ... are you letting it go for a certain amount of time?

Student 1: Nah, just go this and that way.

Teacher: So you're just doing one then enter, two then enter. ...

Student 2: Pick a space between the line of cars to turn the light red, so that way, so then all the cars will be stuck together.

Teacher: So get the cars staggered and spaced out and pick a spot between ... yes, sir (pointing to a third student).

Student 3: Have it like every place where I saw the cars go like this (gesturing downward) and once they're done have them all go like that (gesturing to the side).

Teacher: So have the down ones green (gesturing downward with fingers extended) and then all the side-to-side ones green (similar gesturing to the side).

These are just some of the strategies students generate. The first strategy is simply alternating the lights at a regular interval. The second strategy looks for open spaces to shoot for in turning the light red in that direction. Other sections of classes at this school articulated what we call a "traffic cop" strategy where you simply look locally to

see in which direction there are the most cars at your intersection, and then let that direction go (i.e., like what a traffic cop might do at a busy intersection). Some students discussed the possibility of developing “smart cars.” With smart cars there might not be a need for lights, *if* a way could be found to “coordinate” with cars coming from the “other” direction. As is true for all the groups we have worked with, the Austin area students came up with a phase-related strategy for “synch-ing” the lights. The actual classroom exchange related to this strategy highlights some of the pedagogical challenges related to engaging and extending the developing strategies of students. These issues help point to possible future directions for developing the HubNet functionality and are under development.

Self-Organization

In a closely related line of research based on using participatory simulations (Wilensky & Stroup, 1999), Andy Hurford is looking at the learning in a network through a “self-organizing, critical systems” (SOCS) lens (1998). SOCS characterizes both the behavior of the simulated system and the learning that occurs in relation to *living in* that system. Bak and Chen (1991) provide an accessible real-life example of a SOCS and identify some general attributes of these systems. SOCS are: 1) sensitive to initial conditions; 2) scale invariant; 3) self-organizing; and 4) evolve in a way that may be characterized as a one-over-f distribution (Bak, & Chen, 1991, p. 48). SOCS are said to be “critical” because they alternate between relatively stable and relatively chaotic phases (Peterson, 2000). An application of SOCS theory to a learning system where learning was defined in terms of reorganization of individuals’ conceptual structures (Hurford, 1998) argued that the first SOCS attribute, sensitivity to initial conditions, might be compared with the importance of prior knowledge in learning. That is, what one learns depends upon what the student knows— learning is sensitive to initial conditions. Conceptual reorganizations may occur in many sizes from small “ahas” to grand gestalt moments (Strike & Posner, 1992; Demastes, Good, & Peebles, 1996; Kuhn, 1996) and large reorganizations “look” just like small reorganizations except for their scale. Self-organization, “a process in which pattern at the global level of a system emerges solely from numerous interactions among the lower-level components of the system... executed using only local information, without reference to the global pattern” (Camazine, Denoubourg, Franks, Sneyd, Theraulaz, & Bonabeau, 2001, p. 8), is what happens as the learner interacts with the subject matter. In this analysis it is the learner’s conceptual structures, evolving according to the learner’s own sets of rules and in the individual’s unique context that are self-organizing. The final attribute of a SOCS as discussed here is the notion of a one-over-f distribution, and here we can only assert that conceptual change may actually occur as a “continuous spectrum of learning events” (Hurford, 1998, p. 21) that may be characterized by the distribution function.

Although the above example is based on an investigation of individual learning, SOCS are scale-invariant, and as such, the model contends that learning should be self-similar across several orders of magnitude. Hence it seems reasonable to assume that learning in an individual and learning in small groups, classrooms, or school systems should share significant similarities. Beyond this attribute of self-similarity the fundamental assumption of this research is that learning can be profitably viewed as a SOCS, and all self-organizing critical systems share attributes as described above. The challenge now is to “watch” for these SOCS attributes in classrooms and to use that information to promote student learning. At least initially the justification for attempting to look at learning through a SOCS lens is based on a plausibility argument: In contrast to viewing a classroom as the simple linear summation of the individual learning units, it seems at least plausible that 1) the classroom is better understood as a complex dynamic system and 2) that a SOCS perspective might stand to “fit” as an account of this complexity. This “plausibility” argument is similar to that advanced at an early stage of theory development in other areas of cognitive science (Riesbeck & Schank, 1989; Kolodner, 1993). In now moving beyond the individual, SOCS rubrics and real-time computational tools are being developed for use with network-supported participatory simulations. The network functionality can carry out simultaneous analyses of student activity, such that it should be possible to look for SOCS signatures at various learning scales *while students are engaged* in network-mediated participatory simulations. Next generation connectivity can support new, real-time, analyses of student learning.

MS3 Overview

What all these projects have in common is a sense in which mathematics is more than content. In the social space of network-supported interactivity, mathematics is seen as structuring the classroom activity and the related learning. A way of situating this development is to say a “next step” may follow from accepting that mathematical and scientific learning is fully socially situated. The insight that learning is socially situated has – certainly in the last twenty years – brought to the table of learning analyses perspectives and approaches from disciplines as diverse as anthropology, sociology, ethnic studies, and critical theory. Methodologically, case studies and ethnography have become commonplace both in analyzing scientific activity itself, and in making sense of interactions in math and science classrooms. The inclusion of social analyses has heralded a new era of not just greater subtlety in the ways we look at learning, but also in what it means to learn. Learning has come to be understood as a form of participation in activities and processes much larger than individual comprehension. In a related way, the design of

classroom activity is being increasingly situated relative to socially-derived frameworks and analyses. Many learning researchers now embrace the idea that cognitive structures or big ideas exist in, and as a result of, social activity.

What may follow as a next step in this analysis, then, is this: If mathematical ideas participate fully in the social space, they are not just *organized* by the social space they are also *organizing* of this social space. Just as the move to situating mathematics and science in the “larger” social plane brought new theoretical, methodological, and design perspectives to the table of understanding learning, so too the move to seeing mathematics as structuring of social space may invite to the table perspectives from, among others, formal mathematics, complexity theory, mathematical biology and computer science. Extending this MS3 idea to previously existing ideas of generative teaching and learning allows us to add new facets to activity design and learning theory, especially as they come to be supported by next generation networks.

Updating Generative Teaching and Learning from an MS3 Perspective

We begin to move from that part of the dialectic focused on using mathematics to structure the social space (2) to use mathematically informed ideas of space-creating play and dynamic structure as a way to move back to situating mathematics in the social sphere (1). In a sense, this analysis of generativity begins to situate these various projects relative to the dialectic of mathematics in a social space. To be sure, models of generative teaching and learning have been around for a while. Some aspects of space-creating activity and dynamic structure are nascent in previous analyses of generativity, but this paper is pushing to make these ideas more visible and more explicitly about designing for highly interactive group spaces.

Generative teaching as discussed by Wittrock is “a model of the teaching of comprehension and the learning of the types of relations that learners must construct between stored knowledge, memories of experience, and new information for comprehension to occur” (1991, p. 170). What Wittrock means by the learners’ active construction of new “relations” is close to what we might call constructivist teaching pedagogy. Consequently, generative learning in his framework involves students’ ability to create artifacts that embody their constructed understandings. In a closely related way researchers from the Learning Technology Center at Vanderbilt emphasize aspects of creating “shared environments that permit sustained exploration by students and teachers” in a manner that mirrors the kinds of problems, opportunities, and tools engaged by experts (1992, p. 78). Teaching involves “anchoring or situating instruction in meaningful, problem-solving contexts that allow one to simulate in the classroom some of the advantages of apprenticeship learning” (1992, p. 78). Extending these practices to the new functional capabilities associated with next generation networking may push us to update these existing ideas of generative teaching and learning. Although each of these previous theories is generative at the level of the individual learner (or even at the level of a small group) there is not enough of a picture of how to structure the cross individual or cross small-group learning. Using mathematically structured ideas to organize classroom learning helps us to augment these understandings of generative teaching and learning in ways that can be well-supported by next generation network capabilities. Moreover this update is intended to include a wider range of activities and allow for greater precision in saying what it means for learning and teaching to be considered “generative.” Using an MS3 stance the ideas of *space-creating play* and *dynamic structure* are highlighted.

Space-Creating Play

In a technical sense mathematical “space” refers to, “An arbitrary collection of homogeneous objects (events, states, functions, figures, values of variables, etc.) between which there are relationships similar to the usual spatial relations...” (Union of International Associations, 2002). If students are asked to create and display functions that are the “same as $4x$,” they are generating a *space* of homogenous objects. In a network, this space is the result of students’ play-full explorations of possibilities, using their individual computing devices, and then using the network to share one’s own examples and learn from the examples of others. Worth emphasizing is the fact that play is not an “anything goes” state of affairs. Indeed, when considered carefully, “anything goes” is not usually a reasonable account of how play actually works for children:

The premise that Durkheim, Vygotsky, and Piaget share ... is that thinking and cognitive development involve participating in forms of social activity constituted by systems of shared rules that have to be grasped and voluntarily accepted. ... The system of rules serves, in fact, to constitute the play situation itself. In turn, these rules derive their force from the child’s enjoyment of, and commitment to, the shared activity of the play-world (Nicolopoulou, 1993, p. 14).

Thinking and cognitive development related to play involve participation: the activity is structured by a “system of shared rules” that need to be “grasped” and “voluntarily accepted.” The power of this form of activity comes precisely from the children’s “enjoyment” and “commitment” related to being part of the “play-world.” Generative teaching and learning as discussed earlier has had this space-creating play aspect, but it has been implicit in a way that has not utilized the “space” of potential mathematical behaviors and artifacts. The student-centered projects

discussed earlier (Section 3.0) advance a sense in which learners “play” in an activity “constituted by systems of shared rules.” In turn, this participation creates a space of objects or emergent behavior that embodies students’ understandings of the mathematical or scientific content.

Dynamic Structure

When students in a networked space create and then display functions that are “the same as $4x$,” the structure of the activity is itself *lived* or brought into being by what the learners do. Unfortunately in mathematics research in particular, the idea of structure has come to be understood in a relatively static way. “Process-object” analyses of mathematics learning have tended to view “structure” as a relatively static kind of object. Social constructivists have tended to respond critically to this idea of structure.

When Soviet psychologists speak of the ‘structure of an activity,’ they have in mind something very different from what has come to be known as ‘structuralism’ in Western psychology (and mathematics education). The units are defined in terms of the function they fulfill rather than of any intrinsic properties they possess (Wertch, 1979, p. 19).

Dynamic structure is intended to point to a functional or operational sense of structure, not fixed or intrinsic attributes. A space of functions is created by the “ $4x$ ” activity, and in a significant sense this space is animated by the learners’ understanding of the mathematical ideas of equivalence. Their mathematical ideas are what *dynamically structure* their learning activity. This socially situated understanding of structure fits well with the ideas Soviet psychologists – and especially Vygotsky (1934/1962) – bring to learning. What may come as a surprise to some, however, is the sense in which this “lived” and larger-than-the individual meaning of dynamics structure may be exactly what Piaget (1970) was pointing to in suggesting constructs like the algebraic group could serve as the “prototype” of what he meant by cognitive structure. Structure, understood in this dynamic way, is a *patterning* or *coordination* in the kinds of operations on elements of the system. It is this larger dynamic sense of structure that allows Piaget to talk about group learning as “co-operation” (Montagnero & Maurice-Naville, 1997, p. 140) and may be what qualifies Piaget as the first major learning researcher to think of mathematics as structuring the social experience of coming to know. Viewed this way, there are striking parallels and forms of complementarity between Vygotsky’s analyses of language and Piaget’s analyses of operational thought that can be brought to the task of thinking about, and designing for, activity in classrooms supported by next generation networked functionality. Generative learning and teaching come to be understood as organized by space-creating play and dynamic structure.

Generative Teaching and Learning from a Socio-Cultural Perspective

This paper proposes that exploring the dialectic that exists between mathematics as a socially constructed domain and the domain-structured, concrete activity of teachers and students in math and science classrooms (MS3) provides researchers and educators an important framework for understanding the implications for design, pedagogy, notions of content, and research methods in classroom-based networks in particular, and classroom teaching and learning more generally. The projects reported on earlier represent important examples of the unique affordances of such networks and designs in that they highlight one aspect of the dialectic, namely the structuring of the classroom social space by mathematics or science proper. However, concomitant examination of these projects in terms of how mathematics is socially constructed by students and teachers engaged in sociocultural activity is necessary to extend the prior analyses of generativity within this framework.

Acceptance of the social construction of mathematics that results from the interactions of teachers and students with content, and with each other, affords a particular, powerful understanding of the nature of what is learned and taught in classrooms. It is also necessary to situate those interactions and the domain more explicitly and in more sophisticated ways within sociocultural activity to truly capture these complex phenomena. Many of the projects using networked classroom environments are focused on traditionally underserved students (e.g., students of color and/or living in poverty), but such things as historical notions of teacher, student, and content (Moll, 1990; Wells, 1999); cultural ways of knowing, interacting, and communicating (Gay, 2000; C. Lee & Smagorinsky, 2000); and the mediating function of language (Vygotsky, 1987; Wertsch, 1991) are under-specified. Thus, agency and participation; what kind of mathematics or science gets constructed, how, and by whom; and the mutually constitutive relation between structure and language need to be more fully elaborated. With this criticism we look to highlight opportunities to build on the important benefits inherent in a focus on the dialectic between mathematics or science as a domain that structures activity and the sociocultural activity through which mathematics or science is constructed. The work described herein does represent significant advancement relative to levels of agency and participation but in each we need to push again relative to situating mathematics in a richer sense of socio-cultural analysis.

Levels of Agency

As detailed earlier, students' opportunities to assume agency in these new classroom social spaces are markedly different from traditional classrooms. Certainly, to be a visible and necessary participant in public construction of knowledge entails a significant form of agency that has been largely missing in mathematics/science classrooms.

Recent work by Sarah Davis looks to make visible some important aspects of student and teacher agency. In the networked classroom, students can submit responses to be considered by the class without their identity being associated with that information. Both a sense of "that's me" and a "right level" of anonymity combine to give students new forms of agency and situated identity. Anonymity facilitates the ability to explore mathematical behaviors in a non-threatening way. Freed from *who* sent in a response, students are able to explore *what* the mathematical activity represents, whether it is sending in functions that are the same as $4x$ or controlling a traffic light in a simulation. Davis has found this anonymity helps advance both the students' and the teachers' sense of agency.

From the teacher's perspective: "It just promotes a lot of discussion and everybody's free to discuss it because kids can be criticizing an equation that they themselves wrote and nobody would know" (Davis, 2002). Students identify with their responses, icons, and data that show up in the group display. As can be seen in the next quote, in time the representation of self in relation to the group space can give the students a sense of how they are doing relative to the class as a whole.

Interviewer: Does that, in the other classes where you don't know how other people are doing (Student 12: Right), you don't know if you're the only one (Student 12: Right), does that raise your anxiety level any...?
Student 12: Yeah it's scary, because I think I'm the only one...I'm looking at my test, I think I'm the only one who got a 60 or whatever. And the couple of kids around me I'll know what they got but then I have no idea how anyone else is doing, because it's all privately done. Not that I need to know their test grades, but I'd like to know, how I stand. Am I the only one who needs help? And then you feel embarrassed to be the one raising your hand all the time, be the one staying after class because you think you're the only one. So, here, it's a lot more comfortable. You're not embarrassed in front of the other kids. (Davis, 2002)

The ability to gather responses on all questions from all students gives important knowledge to the teacher. That knowledge then gives the teacher options for how to proceed in class. And again from the teacher's perspective:

It's great to know, where the kids are, actually it's not always great because sometimes it's pretty depressing to see where the kids are. There was something I did this year in one of my classes and I asked if there were any – I thought I had done a fine job – I asked if there were any questions, nobody had any questions and I just had an inkling, And I said okay well log on and lets check. And I believe two kids got it right so obviously they didn't have a clue what they were doing and I went back and re-taught. (Davis, 2002)

Agency here includes opportunities for both the teacher and students to anonymously reflect on and situate their respective states of understanding. Understanding centers both on the construction of content as such and, just as importantly, on "where everybody is" individually and collectively.

As significant an improvement as all these network-supported forms of interaction are – where historical notions of teachers' and students' relation to content are challenged – historical notions of content itself are still being maintained, tacitly, in the work to date. One might observe that an under-examined, embedded assumption is that math is homogeneous, monolithic; there is no ambiguity in what math is. The very notion that mathematics is socially constructed, especially given attention to the concrete, localized construction processes in classrooms, problematizes that homogeneity or lack of ambiguity. Viewed in this way, even an updated notion of generativity can become an aspect of classroom activity that serves only to lead to a predetermined outcome, rather than something that produces potential insights and growth possibilities for both actors *and* content. Generativity needs to be pushed to have us become more open to the possibility that what it is that ends up mattering to a group, and how this comes to be understood, might not have a greater point of comparison other than that of the group's co-constructed agency.

Participation

Careful consideration of what kind of participation in the construction of what kind of content is important so that classroom learning is generative in the sense that students not only learn in ways that foster powerful, dynamic understandings of mathematics, but that they develop knowledge and skills that foster their successful participation in mathematics in the larger world. If one of the aims of these projects is to open mathematics and mathematical reasoning to students, particularly students who have traditionally been under-served, in more powerful ways, then questions about the nature of the sociocultural activity from which such knowledge emerges are critical. Whose community, culture, history is the focus of the design activities (e.g., PartSim's elevators, traffic gridlock)? What connections to students' lives – lives that are situated in social, cultural, historical, political arenas – do design activities include? If the goal is design of generative activities, how do they facilitate students' development of meaningful mathematics that can help them take action on their world? Linguistic and cultural diversity in

classrooms push notions of participation, as well as historical notions of student and content. (O. Lee's (1999, 2000) work in inquiry-based teaching is germane in science education.) What kinds of considerations need to be made in inviting the contributions of, for example, English as a second language learners as legitimate participants whose language and culturally grounded knowledge are viable, important resources for assessing proof (for example), and for teaching and learning? Further, attention to the fundamental influences of culture and language on learning (similar to Kaput's work (Kaput & Shaffer, in press) on externalized manipulation of formal systems changing the very nature of cognitive activity) may make visible heretofore unexamined notions of appropriate interaction and participation in the new social spaces created within networked classroom environments.

And finally, relative to developing more culturally situated notions of participation, a somewhat uncritical focus seems to be on mathematics as a universal language. Treating language as a mediating tool through which structures emerge poses interesting challenges to universality. To explicate what is truly meant by claims that mathematics is constructed through sociocultural activity, the mutually constitutive relation between the language of mathematics and the structure of classroom activity, as well as between the language of the classroom and the structuring of mathematics, must be explicated more fully. In addition, in networked classrooms, the expansion of language through the production of new artifacts (e.g., real-time public displays of jointly constructed knowledge) and the nature of the conversations that have public construction and display as their focus offer unique glimpses into the development of mathematical knowledge and reasoning. They also have potential to invite diverse modes of communication and interaction into the process. Careful examination of the new avenues for socially and culturally specific communication and interaction can optimize the generative potential of the new social spaces created in networked classroom environments.

To be fair, while a critique of mathematics education as acultural, ahistorical, or apolitical is possible, critiques of sociocultural theoretical frameworks for ignoring content are also important in extending prior analyses, as we begin to do in this paper. Socioculturalists have helped researchers and educators focus on the reality of the social construction of mathematics, however, they have done less well in attending carefully to the dialectic proposed here because of a disconnection from content. There has been (with some exceptions, e.g., Gauvain's (1998) work on cognition being shaped over time by various number systems) one-way attention to the fundamental connections between activity and learning, looking more at activity structuring learning rather than also looking at domains structuring learning through activity. Thus, the dialectic framework proposed here has potential to enrich both sociocultural research and theory and mathematics education by bringing them into conversation with each other.

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