Embodied Artifacts and Conceptual Performances

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Abstract: Learning scientists are only beginning to appreciate the potential of synergy between two concurrent developments—theory of embodied cognition and technology of embodied interaction. We characterize and evaluate this prospective synergy from a sociocultural perspective. First we analyze learning in explicitly embodied cultural practices (e.g., surfing), then analogize to the implicitly embodied practice of mathematics. We next contextualize this analogy via interpreting data collected in a design-based research study, in which twenty-two 9-to-11-year-olds developed notions of proportionality through participating in guided problem-solving activities in an embodied-interaction space. In both surfing and mathematics, we argue, learners develop "embodied artifacts," i.e. body-based and modular rehearsed actions. Embodied artifacts lend individuals entry into disciplinary competence via participation in action, refinement of operations, and integration into activity structures. Furthermore, embodied artifacts may become "conceptual performances," wherein performance not merely augments, but stands for and constitutes understanding.

Introduction

Artifacts—cultural objects embedded in social practice—continue to intrigue scholars of human cognition and development. Reasons are at least threefold: (1) philosophically, artifacts constitute essential cultural objects to think-and-act-with; (2) pedagogically, education involves learning to use the tools humanity has found indispensable; and (3) methodologically, artifacts render learning processes more visible for investigative scrutiny. As design-based researchers of educational technologies, the pedagogical artifacts we investigate are historically young. Nevertheless, we approach these novel artifacts from the same theoretical perspectives as we would a seemingly humdrum manual tool (say, a hammer), and ask, "What learning gains can such an artifact foster? What can it teach us about human learning and the relation between performance and understanding?"

For the purposes of this report we are less interested in material artifacts per se, such as a piano or an abacus; neither are we presently concerned with symbolic artifacts, such as musical notes or numerals. Instead we focus our investigations on *embodied artifacts*—rehearsed performances or trained routines, such as the dynamical physicality of playing Für Elise or manipulating an abacus. As we shall argue, novel motion-sensitive cyber-technologies (e.g., Nintendo Wii, Xbox Kinect) are uniquely geared to both craft and leverage embodied artifacts as means of fostering learning and, for researchers, opening windows onto how learning occurs.

We embark from embodied artifacts within *explicitly* embodied domains—domains wherein the core practices are patently visible to interested observers. We do so for two interrelated reasons. Rhetorically, to assist the reader by providing transparent examples of embodied artifacts and, strategically, to bring attention to the pedagogical traditions of disciplines largely marginalized in the learning-sciences literature. Disciplines such as dance, martial arts, and various crafts are characterized by pedagogical practices apparently different than those of so-called "abstract" disciplines such as mathematics, a difference perhaps most clearly articulated by traditional perspectives on transition from performance to understanding. Namely, in abstract disciplines the transition from procedural to conceptual, often modeled by Piagetian "encapsulation," is viewed as an epistemic necessity. In this process, a procedural performance is "reorganized on a higher plane of thought and so comes to be understood" (Beth & Piaget, 1966, p. 247). In contrast, pedagogical traditions of explicitly embodied disciplines seldom promote "higher planes of thought" as intrinsically necessary or even meaningful for the acquisition of competence (but see "reflective practice" by Schön, 1983). Instead, disciplinary knowledge is viewed matter-of-factly as emerging in, instantiated through, and assessed via performance *per se*.

In the following section, we take the readers on a guided tour of a few decidedly low-tech instantiations of embodied artifacts. Following that, we frame our investigations of embodied artifacts against the backdrop of increasingly ubiquitous technologies leveraging embodied action. Finally, we introduce one of our recent design experiments as an example of what it may look like for learning-sciences researchers to apply *in practice* the notion that understanding emerges in and through performance.

Embodied Artifacts in Action: From Surfing and Kicking to Doing Mathematics

To begin, imagine a first surfing lesson in Honolulu, Hawaii. Despite the endless crowds at the sun-drenched Waikiki beach, a neophyte surfer is eager to get in the water. Doing so immediately, however, is likely to invite disappointment. His inability to distinguish the many types of waves, crowding by dozens of other nearby surfers, neuromuscular fatigue from continuous paddling, an uneasy sense of unspoken social hierarchies among more experienced surfers, and a myriad other factors large and small all conspire to quickly dizzy and exhaust the novice. Yet the beachboys (surfing instructors) of Waikiki are famous for claiming they can make anyone ride a wave—at least, that is, for a second or two. How?

Before getting in the water, the beachboy asks the first-timer to lie down upon the surfboard on the sand. There, the beginner is taught the elementary Stand-Up sequence (SU; our nomenclature) on the surfboard, roughly thus from a prone position: (1) kneel; (2) raise one knee up; and (3) stand up. Only once the beachboy determines the neophyte is capable of executing this basic sequence with confidence does the surfer take to the water, where SU's utility quickly becomes apparent. In the water, the instructor waits for an appropriate wave—a selection process beyond the novice's capacity—then push his charge into said wave at the appropriate moment and velocity. At this point, all the neophyte must do is paddle hard into the wave and (attempt to) execute SU. A complex activity is thus partitioned into: (a) selecting a wave; (b) approaching a wave; and (c) SU. Hence the beachboys accomplish their claim of getting anyone to surf by performing (a) and (b) on behalf of their charge and "sharing" with the neophyte an embodied artifact, (c) the elementary SU sequence.

Note that the function of an embodied artifact is modular, in the sense that it can be taught and learned as a standalone sequence of operations performed outside the ecological domains of deployment, yet later be contextualized via integration into a larger system. For example, the learner becomes more adept at timing, instigating, and performing SU in respect to his distance to the wave (contextualization via integration—recall that the SU sequence was learned on sand); and learns a more optimal knee placement during the kneeling portion of SU (refinement via analysis). Learners therefore approach disciplinary competence by *entering at the level of actions in the form of rehearsed performances*. In other words, embodied artifacts serve as entry into disciplinary engagement—as knowledge through practice (cf. Ericsson, 2002) and reflection (cf. Dewey, 1933; Schön, 1983). Importantly, the learner may rehearse elements of this modular action (SU) independently of any larger activity system (surfing; cf. Leont'ev, 1981). We revisit these critical ideas later in the report.

Due to their modular nature, embodied artifacts tend to be adaptable in their application. Consider the Flying Sidekick, an aerial attack historically used to strike over ground fortifications (e.g., defensive spikes) and dismount enemy combatants off of warhorses and other beasts of war (see Figure 1b). In modern times, neither spike-barricades nor mounted warriors pose a serious concern, yet this artifact continues its existence as more than a quaint text-bound technique. Due to its modular nature, it survived the disappearance of its original context, mêlée warfare.

a. b.

Figure 1. Embodied artifacts in the wild: (a) novice surfer prepares to use the Stand-Up sequence, as coach, seated, prepares to launch him into a wave; and (b) Soo Bahk Do expert demonstrates a "Flying Sidekick."

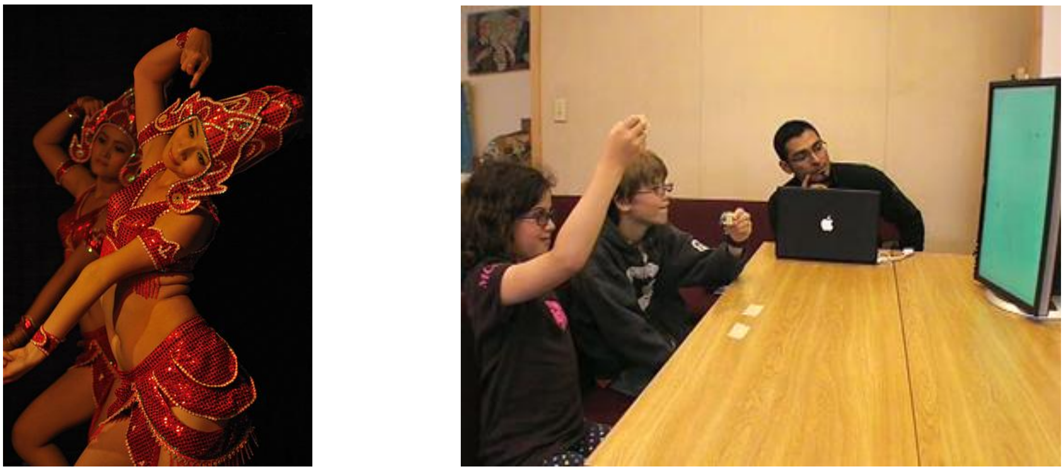
These two brief examples are meant to illustrate some of the variety of embodied artifacts. Whereas embodied artifacts may work in tandem with other artifacts (as in the case of surfing, operating an abacus, or playing the piano), they may merely require space and gravity (such as dance). The critical but common thread is that all embodied artifacts are rehearsed performances, ready-to-hand cultural equipment created by "packaging" procedures for skillfully encountering particular situations in the world. By mediating one's encounters with the world, embodied artifacts constitute an integral part of cultural and individual development. First, humans embody cultural procedures through participating in social activities. Through observation, demonstration, imitation, and training, these cultural procedures become our resources in the form of embodied artifacts. And, as we shall argue, embodied artifacts may become signified as *conceptual performances*, that is, embodied procedures that serve as concepts-in-action. Hence, through embodied artifacts we store cultural knowledge in the body, using the body as both the material for and means of encountering the world (cf. Dourish, 2001; Dreyfus & Dreyfus, 1999).

As researchers, our interest in embodied artifacts is twofold. First, current empirically supported theories of mind suggest that embodiment—having and using a physical body in the world—is fundamentally linked to all reasoning, whether involving "pure" thought or getting one's hands dirty (literally or figuratively). Second, we hold that deliberate use of embodied artifacts in mathematics instruction may render hitherto undetectable learning processes open to both formative assessment in classrooms and empirical scrutiny in laboratories. The idea is simple: if students must perform physically in the service of, say, doing mathematics, then such doing becomes publicly observable rather than hidden away "in their heads."

Yet in addition to our research practice as learning scientists, we are designers of pedagogical artifacts. As designers, we are interested in availing of novel technologies to engineer learning environments wherein students craft embodied artifacts in pursuit of mathematical competence. We then observe students engaged with our design and, hopefully, we learn more about the process of learning. So doing, in turn, we also learn more about designing learning environments.

The design investigations presented here begin from observations about the pedagogical potential of embodied artifacts in light of increasingly ubiquitous motion-sensor technologies. These observations, in turn, form the theme of the following section, where we situate our study in the broader context of research on the role of embodiment in human learning and knowing. From the perspective of educational design, we consider the following question: How, if at all, may novel motion-sensor technologies be pedagogically utilized, particularly in light of recent theoretical advances indicating the fundamental cognitive role of embodiment?

In addressing this question, we present a proof-of-existence educational intervention that leverages novel technology, namely the Mathematical Imagery Trainer (hence, "MIT," see Figure 2b). Working with the MIT for Proportion (MIT-P), students move their hands in an environment that changes its state in accord with the ratio of the hands' respective heights. Via learning to maintain the target state *even as they move about in the environment*—i.e. to perform a "dynamical conservation"—students effectively train an embodied artifact. They then reflect on, analyze mathematically, and ultimately contextualize this artifact as a case of proportionality. Before introducing this design, let us take a moment to briefly frame our work theoretically.



a. b.

Figure 2. Embodied artifacts take many forms: (a) traditional Cham dancers, and (b) Mathematical Imagery Trainer for proportion (MIT-P) in use by two students (with instructor looking on).

Framing: The Body, Mathematics, and Technology

The Rise of Embodiment Theory and Its Application to Mathematics Education

Can conceptual understanding emerge from embodied interaction? One answer is that we are physical beings living in a physical world; hence, attempts to understand the development of conceptual thought need look to physical interaction. Perhaps due precisely to its apparent simplicity, this answer has been ignored by cognitive science throughout the last century. Instead, traditional cognitivist views partition mundane interaction into three mutually exclusive constituent facets: perception, thought, and action (e.g., Fodor, 1975). Thoughts and concepts, per those models, intervene between perception and action and are characterized as distinct from those real-time embodied processes by token of being symbolic–propositional. Yet in alternative views discussed below, cognition is neither secluded nor elevated from perception and action but is rather embedded in, distributed across, and inseparable from these corporeal processes.

Embodiment studies rose fast in prominence towards the end of last century through the converging efforts of numerous pioneers across various fields like robotics, cognitive science, philosophy, psychology, and computer science (Brooks, 1991; Varela, Thompson, & Rosch, 1991; Winograd & Flores, 1987). Though many of these perspectives initially emerged in opposition to then-prevalent symbolic-architecture models of mind, embodiment studies have, over the last two decades, burgeoned into a vast area of investigation in its own right—replete with its own spectrum of proponents. Within this spectrum, one might roughly identify: conservatives, who cautiously posit that the body may have a role in cognition (e.g., Dove, 2009); moderates, who argue that physical action underpins or forms the substrate of cognition (e.g., Barsalou, 2010; Sheets-Johnstone, 1990); and radicals, who hold that cognition itself *is* nothing but action (e.g., Melser, 2004). Indeed, the scope of embodiment studies has grown to the point where not a few scholars concern themselves with defining what, exactly, is meant by "embodied" (Kiverstein & Clark, 2009). In our current work we tend to hold with those who favor the middle ground, and we interpret available empirical evidence as indicating that physical action indeed undergirds thinking, including all so-called "abstract" thinking (e.g., solving for *x*). We are not concerned by the controversy over what precise role corporeality plays in thought: indeed, it gives us something to do. Namely, as interaction designers of mathematical learning we find ourselves in a unique position to contribute toward resolving this theoretical controversy.

Finally, and particularly relevant to our work, embodiment has been presented as a useful framework for theorizing developmental processes inherent to "abstract" disciplinary mastery, including mathematics learning and reasoning (Abrahamson, 2009; Campbell, 2003; Namirovsky, 2003; Núñez, Edwards, & Matos, 1999; Roth & Thom, 2009). One consequence of this view is that observations, measurements, and analyses of physiological activities associated with brain and body behavior can provide insights into lived subjective experiences pertaining to cognition and learning in general, and mathematical thinking in particular. In a strong form, we conjecture that physical action is neither epiphenomenal nor merely supportive to "pure" mental activity. Rather, conceptual understanding—including reasoning about would-be "abstract" contents such as pure mathematics—emerges through and is embedded in actual and simulated perceptuomotor interactions in the world.

Technology for Using the Body

Even as cognitive scientists recognize this essential role of the body, industry has made dramatic advances in engineering technological affordances for embodied interaction. At the time of this writing, Nintendo Wii and Playstation Move players worldwide are waving hand-held "wands" so as to remote-control virtual tennis rackets; iPhone owners are tilting their devices to navigate a virtual ball through a maze; and Xbox Kinect users are controlling video-game avatars with their bare hands—all increasingly commonplace activities hitherto confined to the realms of futuristic fantasy, like flying cars. Moreover, innovative designers tuned to this progress are constantly devising ways of adapting commercial motion-sensor technology in the service of researchers and practitioners (Antle, Corness, & Droumeva, 2009; Lee, 2008). As such, media that only recently appeared as esoteric equipment will imminently be at the fingertips of billions of potential learners. We are excited about the prospect of using these new media to create learning environments, wherein students discover, craft and rehearse embodied artifacts—then subsequently investigate them via mathematics, gaining embodied entry into this disciplinary domain.

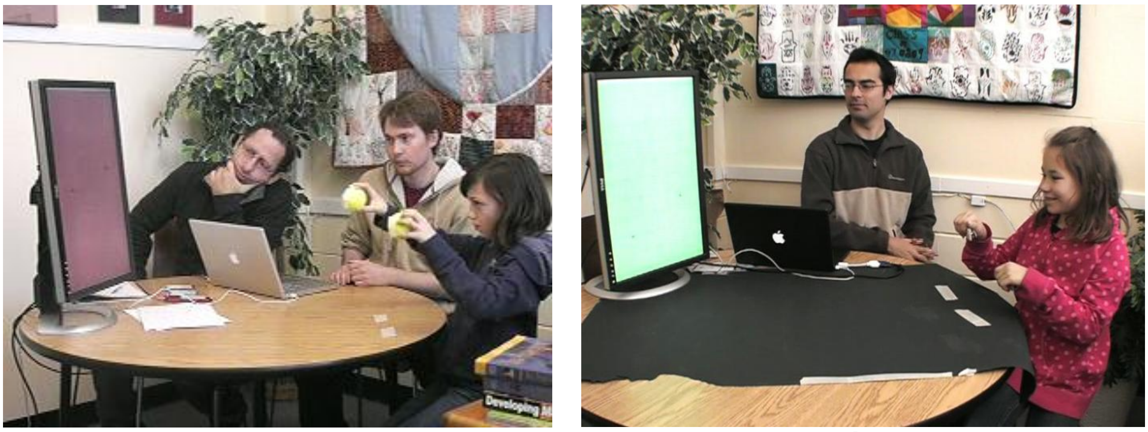
Specifically, our use of new media takes the form of Embodied-Interaction (EI) learning environments. In brief, EI is a form of technology-supported training activity, by which users develop or enhance cognitive resources that presumably undergird specialized forms of human practice, such as proportional reasoning. As is true of all simulation-based training, EI is particularly powerful when everyday authentic opportunities to develop the targeted schemes are too infrequent, complex, expensive, or risky. Emblematic of EI activities, and what distinguishes EI from "hands on" educational activities in general, whether involving concrete or virtual objects, is that EI users' physical actions are *intrinsic*, not just logistically instrumental, to obtaining information. That is, the learner is to some degree physically immersed in the microworld, so that the embodied artifact—instantiated in finger, limb, torso, or even whole-body movements—emerges not only in the service of acting upon objects but rather the motions themselves become part of this learned cultural–perceptuomotor structure. EI is "hands in."

Pedagogical & Experimental Design: The Mathematical Imagery Trainer

Our overarching design conjecture is that some mathematical concepts are difficult to learn because mundane life does not occasion opportunities to embody and rehearse their spatial–dynamical foundations. Specifically, we conjectured that students' canonically incorrect solutions for rational-number problems—"fixed difference" solutions (see Lamon, 2007 for an overview)—indicate students' lack of appropriate dynamical imagery to ground proportion-related concepts (Pirie & Kieren, 1994).

Accordingly, we engineered an EI computer-supported inquiry activity for students to discover, rehearse, and then objectify and signify the physical performance of a particular proportional transformation of our design. Hence, our solution to this general design problem is the Mathematical Imagery Trainer for Proportion (MIT-P, see Figure 3; see http://www.youtube.com/watch?v=n9xVC76PlWc).

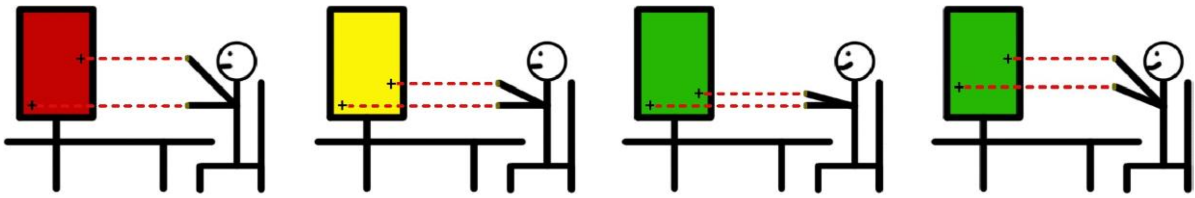
Technologically, our instruction design leverages the high-resolution infrared camera available in the inexpensive Nintendo Wii remote to perform motion tracking of students' hands. Details on technical, interface, and design properties can be found elsewhere (see Howison, Trninic, Reinholz, & Abrahamson, 2011).



a. b.

Figure 3. MIT–P in action: (a) a student's "incorrect" performance (raising both hands up at equal increments, thus maintaining fixed distance between the hands) turns the screen red; (b) "correct" performance (raising both hands up at proportionate increments, i.e. "different" distances) keeps the screen green.

In practice, the MIT-P measures the heights of the users' hands above the desk. When these heights match the unknown ratio set on the interviewer's console, the screen is green (see Figure 4). Hence, the embodied artifact of the MIT activity is the dynamical physical articulation of all the pairs effecting a green screen. As such, the initial purpose of the MIT is to train a particular proportion-relevant embodied artifact.



a. b. c. d.

Figure 4. The Mathematical Imagery Trainer (MIT) set at a 1:2 ratio, so that the right hand needs to be twice as high along the monitor than the left hand: (a) incorrect performance (red feedback); (b) almost correct performance (yellow feedback); (c) correct performance (green feedback); and (d) another correct performance.

Participants included 22 students from a private K–8 suburban school in the greater San Francisco (33% on financial aid; 10% minority students). Care was taken to include students of both genders from low-, middle-, and high-achieving groups as ranked by their teachers. Interviews took place in a quiet room within the school facility. Students participated either individually (see Figure 3) and/or paired with a classmate (see Figure 2b) in task-based, semi-structured clinical interviews (duration: mean 70 min.; SD 20 min.). In addition to the interviewer, at least one observer was present, whose duty included taking written notes in real-time, crewing the video camera, and assisting in operating the technological system.

Study participants were initially tasked to move their hands so as to find a position that effects a green screen and, once this objective was achieved, to keep moving their hands while maintaining a continuously green screen. That is, the participants needed to discover a means of enacting a green-keeping performance. In a sense, the MIT offers students a pre-numerical "What's-My-Rule?" mathematical game. The protocol included gradual layering of supplementary mathematical instruments onto this microworld, in the form of Cartesian grid without and then with numerals. Hence, once students have embodied the proportional-transformation dynamical artifact, semiotic resources (mathematical instruments) and discursive support (the tutor) are present for this embodied artifact to be mathematically signified, elaborated, and analyzed.

The interview ended with an informal conversation, in which the interviewer explained the objectives of the study so as to help participants situate the activities within their school curriculum and everyday experiences. Finally, the interviewer answered any remaining questions participants may have had with the objective that participants achieve closure and depart with a sense of achievement in this challenging task.

Our investigation of the empirical data—ethnographic notes, daily debriefs, and video recordings—was conducted as collaborative, intensive microgenetic analyses (Schoenfeld, Smith, & Arcavi, 1991).

Findings

All students succeeded in devising and articulating strategies for making the screen green, and these initially *a*mathematical strategies came to be aligned with the mathematical content of proportionality (with varying degrees of success). This particular finding serves as a proof-of-existence supporting the conjecture that embodied artifacts *can* create pedagogical opportunities to support student learning of targeted mathematical concepts. Naturally, there existed variations in individual participants' initial interpretation of the task as well as consequent variation in their subsequent trajectory through the intervention protocol. But, importantly, the students progressed through similar problem-solving stages, with the more mathematically competent students generating more strategies and coordinating more among quantitative properties, relations, and patterns they noticed. We now elaborate on this learning trajectory.

Each student began either by working with only one hand at a time, waving both hands up and down in opposite directions, or lifting both hands up at the same pace, occasionally in abrupt gestures. They realized quickly (< 1 min. on average) that the simultaneous actions of both hands are necessary to achieve green and, consequently, that the vertical distance between their hands was critical, although at first they viewed the distance between their hands as fixed. We found this default "fixed distance" approach of importance, as it arguably matches an enduring (mis)conception of students seeing 2/3 as "the same" as 4/5. Indeed, our hope was that by uncovering and addressing such conceptions physically, we could access and affect students' pre-numerical conceptual reasoning underlying their prospective arithmetic competence with this target concept of proportional relations.

In brief, students were given the initial opportunity to *a*mathematically develop embodied artifacts related to proportionally by enacting green-preserving action sequences, i.e., embodied artifacts. Gradually the protocol contextualized students' actions within the broader world of mathematical proportions. This was achieved by providing mathematical tools as semiotic resources for re-articulating the embodied artifact (by way of analyzing and expressing it mathematically). As such, and via the tutor's attentive guidance, the MIT-P introduced an embodied artifact via which students gradually instantiated and complexified the cultural practice of "proportional reasoning." Similar to the Waikiki surfer who embodied, utilized, contextualized, and refined his surfing actions initially learned on sand, the students in our study integrated their initially amathematical moves into the broader world of proportional mathematics. While the pedagogical effectiveness of MIT cannot be determined through this pilot study, we find reason for cautious optimism.

A Brief Study Excerpt

As Liat, a 5th-grade female student, attempted to discover a means of making the screen green, she initially exhibited the "fixed distance" (i.e. "same difference") approach described above.

Liat: I think if I keep them [her hands] apart and keep going up [she keeps a fixed-distance between her hands], it stays the same... [i.e., remains green]

Int.: If you keep them apart and you keep going up it stays the same?

Liat: It's not becoming red, but...

Int: So... how are you thinking about keeping them apart?

Liat: [she attempts to effect green again] Oh maybe it's more. If it's farther up, then it has to be…they have to be more apart.

Above we see Liat coming to verbally articulate her increasingly skillful performance, i.e., embodied artifact that enacts green. Later, upon the introduction of the Cartesian grid and other mathematical tools into the learning environment, she was asked to predict green locations without moving her hands. Referring to the green pair "1" (left cursor) and "2" (right cursor), Liat noted that the vertical interval in between the cursors is "one row." Having objectified and quantified the interval, she said:

Liat: And if you go… 10… if you go up to 10, there's gonna be like 4 or 5 rows. [i.e., if the right hand is at "10," the interval down to the left hand should be 4-5 rows, so as to make "green."]

Thus, prior to the introduction of the grid, Liat's articulation of the embodied artifact for green had been based on the qualitative relation of "farther up" and "more apart," yet once the grid was introduced, she instrumentalized it so as to analyze her action, rendering the description *quantitative*. So doing, Liat reflexively improved her performance by introducing greater precision. However, the embodied artifact was yet to become articulated in an epistemic form supporting the ultimate emergence of the 1:2 ratio. To do so, Liat would need to calibrate her expression from approximate to specific, possibly via assimilating the grid's numerical affordances into her enactment, so that some consistent quantitative principle will obtain across all green locations. Indeed, immediately after, Liat transitioned to a more powerful mode of reasoning that involved multiplicative relations. The transition was prompted by the interviewer's request that she decide between "4 or 5 rows."

Liat: No… five!

Int: Five? How did you do that so quickly? How did you know it was five?

Liat: Half of ten is five.

In above excerpts and the ensuing discussion, Liat came to re-describe and refine her newly developed "green finding" embodied skill via mathematical operations. She does so by, first, noticing an empirical, functional commensurability between the qualitative rule "further up—more apart" and the quantitative halving/doubling property she had just discovered and, ultimately, by reconciling the manual performance as a "case of" proportion. So doing, she comes to see "different differences" as a legitimate mathematical concept, beginning to contextualize it more generally as a case of proportion. As another student chimed in at the end of the session, "Now I know it's OK to be different." Here, we posit, through reconciling rehearsed *physical* performance (moving hands at appropriate rates to make green) with disciplinary content (proportion), an embodied artifact became signified as a *conceptual* performance that stands for and constitutes understanding.

Discussion

We have been threading together two central themes. First is that movement matters. Physically interacting in a physical world is our mode of being and the roots of our thinking. This thread, then, dealt with the relation between performance and understanding: namely, we interpreted existing embodiment studies as suggesting that conceptual understanding—including reasoning about would-be "abstract" contents such as pure mathematics—emerges through and is embedded in actual and simulated perceptuomotor interactions in the world. We introduced the construct of an embodied artifact as a means of articulating how cultural practices are "packaged" and "shared" with novices, enabling their entry into the world of skillful action and disciplinary competence. Second theme is that recent decades have witnessed advances in remote-interaction cyber-technologies. We opined that critical questions have remained unanswered regarding the interaction of these two themes of theory and technology as this interaction pertains to design for learning.

It is in embodied-interaction (EI) design that our two themes meet. We envision EI as a form of physically immersed instrumented activity geared to augment learning by crafting embodied artifacts targeted towards specific disciplinary practice. In pursuing design problems, our strategy has been to engage in conjecture-driven cycles of building, testing, and reflecting on these two themes and their potential synergy. Here we aim to share a conviction that EI offers unique affordances for mathematics learning via introducing and signifying an embodied artifact, such as a dynamical conservation discussed above, as a conceptual performance. Learning in EI occurs in contexts where *participants embody practices in the forms of embodied artifacts and come to make meaning of those artifacts as conceptual performances* (see Lave & Wenger, 1991 on learning as participation rather than acquisition; and Vygotsky, 1982 on the emergence of meaning)

We anticipate that, when coupled with recent technological advances, EI stands to become a focus of design for—and research on—mathematics learning. As our work indicates, EI activities serve as highly useful empirical settings for research on the ontogenesis of mathematical concepts and, more generally, relations between performance and knowledge in mathematics education. In particular, these immersive activities create opportunities for design-based researchers to observe and resolve ostensible theoretical tension between: (a) unreflective orientation in multimodal instrumented spaces, such as riding a bicycle or playing ping-pong; and (b) reflective mastery over symbol-based re-description of this acquired competence, such as in mathematical form.

We hope through this line of investigation to develop models of embodied mathematics instruction. Researchers could look to diverse cultural–historical forms of physical performance, such as music, dance, and the martial arts, as ethnographic entries into traditional and indigenous pedagogical acumen accumulated over centuries or even millennia. The skills inherent to these cultural practices might, at first blush, be viewed as *a*conceptual and, as such, hardly bearing on mathematical reasoning and learning. Yet as recent theoretical and empirical work, including our own, suggests, our shared biology implies that even the most abstract of mathematical concepts may first be embodied, then verbally articulated, and finally reified in conventional semiotic forms (Núñez, et al., 1999; Saxe, 1981). Such issues are more than academic. All too often proverbial lines are drawn in the sand regarding the importance of "conceptual" knowledge versus "procedural" performance (e.g., see Schoenfeld, 2004 on "math wars"). Yet physical actions performed in disciplinary activity constitute *vital* aspects of cognition and knowledge (cf. Alač & Hutchins, 2004; Kirsh, 2009, 2010). Embodied knowledge, we submit, is developed, elaborated, and expressed as situated conceptual performance. In future work, we will continue to investigate the embodiment of mathematical concepts through the reciprocal efforts of developing theories of embodied learning, and designing educational technologies.

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