Part I

Opportunities and Challenges for Integration and Implementation

Proof

1 The Monster in the Machine, or Why Educational Technology Needs Embodied Design

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PROLOGUE: A CAUTIONARY TALE ABOUT DISEMBODIED DESIGN

A while ago I consulted for a large-scale, federally funded effort to develop educational media for young children to learn mathematics. The project was based out of Hollywood, where the studio was abuzz with highly creative animators, scriptwriters, songwriters, and joke experts. The studio was still agog from a recent international award for their flagship product, and they could not wait to brainstorm the 'merch' that would surely emanate from the new project. It was all very flattering and alarmingly lucrative. As I was taxied and flown down and up California, wined and dined on cocktails and sushi, I began to affect a certain hero persona. Assistant professor in the Bay, big-time maven in LA. Abrahamson, Dor Abrahamson. And stir that latte, don't shake it. My job title was 'Education Expert.' Yet gradually, I began to suspect this title was little more than a euphemism for 'Imprimatur.'

On one memorable occasion, we were huddled in a glass room, with collaborating team players video-conferencing in from Juno, Tampa, and Providence, to consider how we might feature simple arithmetic expressions, such as 2 + 3 = 5, in video snippets and interactive tablet games. I witnessed a screening of a mock-up cartoon scene fresh from down the hall. A grinning Cyclops held up a bubble with a "2" in one hand and a bubble with a "3" in the other hand. Gurgling, he moved his two hands toward each other until the bubbles mashed and merged—and of course the SFX were there, stars and ribbons galore—only to reveal a consolidated bubble with a "5" in it.

"But," I ventured brazenly, "how might this help the children actually understand the meaning of addition?" Silence fell. Lowering his brow quizzically, clinically, the creative director proceeded to explain: "The kids will *see* that adding is bringing two things together to get something else." "Well, yes," I persisted, "but what are those *things* that are coming together so that we see how 3 and 2 make 5?" Now I was losing him. "Say," I continued, "that instead of—or along with—those symbols, the monster held up two fingers in one hand and three in the other and then brought all the fingers together? In fact the monster could offload the two fingers from the one hand onto the other hand, so that all five fingers are now unfurled and splayed in one manipulation. The monster could then use the free hand to count up the total of five fingers on the other hand. That is something the kids could make sense of, imitate with their own hands, and even show their parents and friends—it lets them build on their counting skills to learn the meaning of adding rather than just trying to memorize all those sums." "No go," was the peremptory reply. "And why not?" I rebutted. "It's an old animation rule," they explained. "Cartoon characters can only have four fingers on each hand. It's just that they look really bad with five."

I was aghast. We had apparently reached an irrevocable disagreement about a fundamental issue that was very important to me. In a matter of days, though, this design impasse was resolved brilliantly: I was fired.

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Fallen from grace, I bereaved my consultant identity, substituted taco for sushi, and then reinvented myself as a reflective practitioner seeking lessons learned. Never mind the octodigital ogre—may he long simper in cartoon heaven, I muttered. Something deeper was at play that I could not quite put my fingers on. All ten of them.

My brief misadventure was a wake-up call. I had witnessed firsthand an apparent disconnect between research and industry and had begun to fathom the enormity of this disconnect: the lost opportunities, the questionable ethics of funding, the tragic inequity of kids ultimately missing out on big chances because us adults cannot talk to each other. How could my dearest pedagogical convictions be of so little interest to these engineers of commercial products whom we the people, via our governmental agencies and reviewing colleagues, had endowed with the means and mandate to educate our children?

Because guilt is the mother of all creativity, I opted for 'It's not you, it's me' therapy. The blame is on us, educational designers and design theorists, who are not getting our message through. Or perhaps our message is incoherent. Perhaps, even worse, we have no effable message at all, but only tacit skill that we bring to bear anew on each project. Perhaps we delude ourselves into believing that our creative process is deductive, explicable, and transferable. Perhaps design is a know-how that cannot be articulated in such forms and granularity as are necessary for delegation and replication. Perhaps designers have terminal expert blind spots (Nathan & Petrosino, 2003). So, is design unprincipled, more art than craft? Perhaps we should attempt—heroically if not quixotically—to demystify and essentialize what it is we do when we design for learning (Schön, 1983).

Where do we begin? We have 'big' theories of learning that we often implicate as our cynosures, and yet we need mid-level guidelines for implementing these theories in the form of materials, activities, and other instructional resources that ultimately enable teachers to support students in developing target concepts (Ruthven, Laborde, Leach, & Tiberghien, 2009). The field has been taking measures to pool our resources in ameliorating this dearth of heuristic design frameworks (e.g., Abrahamson, 2009a, 2013; Barab et al., 2007; Ginsburg, Jamalian, & Creighan, 2013; Kali, Levin-Peled, Ronen-Fuhrmann, & Hans, 2009). However, a momentous implementation gap still remains between target concepts and learner actions—between the content we want kids to learn and what we should have them do with the media so as to learn that content. This gap is exacerbated by recent technological development in industry coupled with inchoate messaging from academe with respect to reform-oriented constructivist pedagogy (Glenberg, 2006; Marley & Carbonneau, 2014; Sarama & Clements, 2009).

Good tools evolve at a glacial pace, but we live in impetuous times. In evolutionary scale, computers have sped up from 0 to 100 mph in a nanosecond. This accelerated evolution, however, has caused the historical precedent of pedagogy falling behind production: effective educational design principles are not evolving fast enough to catch up with each new sea change in software engineering, social media, and human-computer interaction (HCI). As a result, to paraphrase Seymour Papert's (2004) assessment of educational technology, "we are sailing on a ship that has no rudder on a journey to nowhere." So what are the captains to do? What about us lowly deckhands?

Harboring optimism, still one might welcome these times as bearing great prospects; perhaps pedagogical designers must first witness the dire consequences of under-theorizing and under-explicating their practical acumen before they take action. Perhaps such communication breakdowns not only alert us to the need for design principles but also spur us to reflect on, diagnose, and respond to this need. This chapter is an attempt to do just so, with my consulting ignominy serving as both motivation for change and case study for analysis. I set off by thinking deeper about the grinning monster incident, because I view that particular encounter with the diligent animators as paradigmatic of the theory-to-practice hiatus that ultimately results in suboptimal learning materials that underserve our end-clients, the students. I will then draw on theories of learning to offer several design principles. These principles amount to what I call 'embodied design,' a framework for creating activities that enable students to build meaning for the mathematical ideas they are learning (Abrahamson, 2009a, 2013; Abrahamson & Lindgren, in press). Carrying these principles forward, I then turn to present two exemplars of embodied design that are based, respectively, on learners' naïve perceptual and motoric capacities.

Just before I begin, let me make a brief statement about technology, because this chapter appears in a book on learning technologies and the body. This polysemous semantic unit, 'technology,' bears ancient etymology, and yet in modern times it primarily evokes fossil-fueled steam engines, hydraulic machinery, automatized industry, nuclear plants, electrical appliances, electronic devices, and so on. As a researcher of learning, however, I lean toward a broader definition of technology as technique—any technical procedure in which we deliberately apply available resources—cognitive, corporeal, natural, or artificial—in the pursuit of an objective we envision. From this theoretical perspective, in which fork, iPhone, and algorithm are all exemplars of technology, the colloquial differentiation of technology as 'low' or 'high' according to its material composition is unproductive to a discussion of learning, even if it is centrally germane to the practice of implementing and developing design in the form of products. In fact, I will be exemplifying one and the same design framework, embodied design, with both paper-based and computer-based artifacts. In so doing, I wish to focus our discussion of technology and the body on the learner's phenomenology through technology rather than on technical specifications of media and devices. I will discuss the conceptual change that may come about when technology extends the body—that is, the embodied mind.[1]

TOWARD GROUNDED MATHEMATICS—ISSUES AND PRINCIPLES

Let us step back to examine why I found that four-fingered friend so monstrous. This section draws on theories of mathematics education in an attempt to make progress in broaching the breach between educational researchers and commercial designers.

Meaning: Making Sense before Symbol

The mathematical proposition '2 + 3 = 5' is just about as simple as subject matter content gets in grade school curriculum. And yet, I maintain, the ostensible simplicity of single-digit arithmetic operations makes that generic mathematical proposition rhetorically useful. In particular, analyzing this proposition and how the creative director proposed to instantiate it—to wit, monstrously—I can mount my jeremiad against what Thompson (2013) calls "the absence of meaning" in mathematics education. Thompson submits that thinking about this mercurial thing called meaning is an effective analytic strategy for researchers to unpack and diagnose the ills of mathematics education at large. In particular,

> attending to issues of meaning allows us to see problems of mathematics learning as emergent from fundamental cultural orientations as much as from epistemological problems of learning sophisticated ideas.
>
> (Thompson, 2013, p. 57)

So what or where is the meaning of '2 + 3 = 5'? Kant (2007) thought that a mathematical proposition such as this is "synthetic a priori." It is *synthetic*, because it denotes a non-tautological assertion regarding a state of affairs, in this case the numerical equivalence of two mathematical expressions. In other words, the proposition is synthetic because its predicate ('5') cannot be determined solely through analyzing elements of its subject ('2 + 3')—some supplementary knowledge and process must be brought to bear to generate or evaluate this proposition. On the other hand, Kant denied that the validity of this proposition is contingent upon humans reflecting on worldly interaction. Rather, the proposition's validity is *a priori* to the agent's psyche or dealings, based on universal laws of nature that transcend and anticipate experience. To evaluate this proposition, an individual enlists what Kant called a *schema*—an intuitive psychological framing that imposes a specific type of structure on the sensory manifold.

Are mathematical schemas indeed part of our innate gear? If not, where do these intuitions come from? And what would that mean ultimately for mathematics pedagogy? We turn to a 20th-century giant, the cognitive-developmental psychologist Jean Piaget.

Piaget (1968) investigated the epigenesis of schemas, those would-be intuitive framings of perceptual input. For his research, Piaget used innovative experimental methodology, which included qualitative analysis of young children's behavior as they participated in naturalistic cultural practices within domestic settings, as well as in task-based clinical interviews in laboratory settings. Based on his findings, Piaget implicated goal-oriented sensorimotor interaction as the experiential origin from whence cognitive schemas emerge. Knowledge is not some would-be mental archive of static pictures depicting the-world-as-we-find-it but, rather, dynamical mental activity drawing on what-we-learned-about-the-world-as-we-interacted-with-it. Piaget wrote, "Knowing does not really imply making a copy of reality but, rather, reacting to it and transforming it (either apparently or effectively) in such a way as to include it functionally in the transformation systems with which these acts are linked" (Piaget, 1971, p. 6). Adults, Piaget concluded, perceive and interpret the world in ways that are fundamentally different from children, and this adult capacity reflects radical cognitive reorganization of naïve viewpoints, a reorganization that is achieved gradually, painstakingly, through reflexive generalization of functional regularities latent to myriad worldly interactions. Eventually, Piaget asserted, conceptual development gives rise to formal logical reasoning that is no longer manifest in external sensorimotor behavior.

Philosophical stances regarding Number's transcendent, a priori qualia notwithstanding, *subjective* numerical knowledge is thus hard earned via schematizing worldly interaction into formal operations. As such, mathematics "uses operations and transformations ('groups,' 'operators') which are still actions although they are carried out mentally" (Piaget, 1971, p. 6). Even if we wish to philosophize mathematical laws as preexisting the cognizing agent, Piaget reasoned, still "what is involved [in the development of numerical knowledge] is the *actual perceiving* of correspondence" (p. 311, italics added). Piaget maintained that rudimentary correspondences are those of inclusion (perceiving the unit 1 within the compound 1+1 known as 2) and order (perceiving quantitative increase from 1 to 1-+-1 to 1-+-1-+-1, etc.—i.e., 1, 2, 3, etc.) (p. 310).

An agent's knowledge that '2 + 3 = 5' emerges, therefore, not from moving cryptic symbols on paper, as in the proverbial 'Chinese Room' (Searle, 1980), but from interacting with objects. These symbol-grounding worldly interactions (Harnad, 1990) involve not the symbol string '2 + 3 = 5' per se but concrete instantiations of each of the symbols '3,' '2,' and '5,' as well as the operational '+' and empirical '=.' In particular, understanding the proposition '2 + 3 = 5' involves understanding equivalence as a relation—a reversible transformation of putting together and taking apart sets of quantities (Jones, Inglis, Gilmore, & Dowens, 2012). It is thus that mashing up the symbolic notations '2' and '3' into a *deus-ex-machina* '5' is hardly too effective for pre-K low-SES students to first learn the meaning of numbers, let alone arithmetical operations upon numbers. At best, it might support meaningless exercising and, in so doing, implicitly foster a lifelong belief in the arbitrariness of mathematical activity.

Still, one may wonder why society so ardently perpetuates those educational practices that give rise to a predicament of students learning to execute mathematical procedures meaninglessly. We have discussed Thompson's diagnosis for the symptoms of this predicament as pointing to a prevalent systemic malady he named "the absence of meaning." We now turn to Nathan (2012), who psychoanalyzed society in an attempt to get to the bottom of this psychosis, which he named "formalisms first" (FF). Nathan's investigation implicated an implicit societal belief,

> an apparent conflation of the structure of a discipline and the developmental trajectory by which newcomers gain mastery of that discipline. The FF view uses the disciplinary structure as its developmental roadmap: What is foundational to the discipline is also deemed developmentally primary; what constitutes secondary and peripheral topics to the field then follow in the learning experience; and applications of disciplinary knowledge to practical problems comes last in the scientific process, and therefore are expected to occur later developmentally.
>
> (Nathan, 2012, p. 135)

Nathan's analysis of this vicious cycle offers extenuating character evidence in support of my LA colleagues' sorry approach to teaching arithmetic. "Let's first teach kids the actual mathematics, with its formal symbols and operations," they meritoriously yet naïvely muse, "and then, perhaps, if we have time and budget, we could also show them how all this applies to the world; perhaps we will help them visualize the math."

Nathan counters FF with a grounded approach to establishing mathematical meanings, in which formalisms come not first but last; formalisms are absolutely necessary for effective participation in advanced cultural practices, and yet formalisms initially gain their meaning from embodied experiences, not vice versa.

> It is through grounded relationships that connect to our direct physical and perceptual experiences (or through chains of relations that connect to things that connect to our experiences) that these formal entities attain their meaning. Once meaning is established, however, it is the abstract and form-based properties of formalisms that imbue them with capabilities for quantitative modeling across a broad range of domains, as well as high-speed and high-capacity computation. Formal systems are powerful culturally established tools for advancing our reasoning capacity and for institutionalizing cultural knowledge. Yet they need to be mapped to the world they purport to model to be meaningful and valid.
>
> (Nathan, 2012, p. 139)

Nathan's fiats resonate with the long convoy of reform-oriented mathematics education scholars and designers, such as Friedrich Fröbel, Maria Montessori, Constance Kamii, Hans Freudenthal, Jerome Bruner, Caleb Gattegno, Zoltan Diénènes, Seymour Papert, and Jeanne Bamberger. Going back to the dawn of modern pedagogy, we find this quotation in a 1762 treatise about a boy called Émile:

> What is the use of all these symbols; why not begin by showing him the real thing so that he may at least know what you are talking about?. . . . As a general rule—never substitute the symbol for the thing signified, unless it is impossible to show the thing itself; for the child's attention is so taken up with the symbol that he will forget what it signifies.
>
> (Rousseau, 1979, Book III, p. 170)

Fine, you may concede, by showing "the thing itself" we enable the students to establish a bottom-up sense of what the lesson is *about*. But what about "all these symbols"? How does that top-down signification become part of reasoning? That is, if cognitive development begins with goal-oriented sensorimotor activity, and if indeed this physical–perceptual experience is the root of all understanding and competence in the disciplinary domains, such as mathematics, then two issues follow—an inference and a query.

The inference is that learning environments should provide opportunities for students to participate in goal-oriented sensorimotor activity where they engage or develop schemas pertaining to the target content. The query relates to how such tacit, unarticulated skill might give rise to explicit, articulated knowledge that is expressed in formal semiotic systems, such as symbolic notations. That is, it is one thing to experience certain sensations, such as sensing the relative likelihood of an event in a random experimental trial, but it is a whole other thing to be able to describe that sensation numerically. To wit, Kahneman (2003) differentiates between the respective activity of two cognitive systems, "the automatic operations of perception and the deliberate operations of reasoning" (p. 697). Lo, is there not an inherent epistemic hiatus between these two systems? If so, how do we mind this gap? If we cannot, then the formalism-first folks may still have the upper hand and final word.

We are discussing the pedagogical challenge of helping students ford a would-be epistemic gap between intuitive and formal descriptions of quantitative phenomena. And yet is this gap for real? It appears to be warranted from a constructivist perspective, and yet from the sociocultural perspective this putative gap is pooh-poohed as mere cognitivist chimera. Per sociocultural theory, that which we call concepts are not intricate cognitive structures standing upon deep naïve foundations. Instead, what we call concepts are our rationalizations of essentially mundane, contextualized routines that individuals enact so as to accomplish their respective roles within larger activity structures that lend meaning to their actions. I shall now elaborate on the sociocultural worldview.

Learning: Making Sense of Symbol

The research problem of how individuals learn to participate effectively in human activities that are populated with arbitrary signs, such as words and symbols, has been central to sociocultural scholarship. Notably, Vygotsky (1978) re-theorized conceptual knowledge as a form of competence in enacting cultural practice: What others call conceptual learning is rather the individual's acculturation into and appropriation of cognitive routines inherent to disciplinary discourse. This appropriation is both motivated and mobilized via participating in social activities that involve enacting those cultural practices and thus implicitly mediate their inherent cognitive routines. As Wertsch (1979) clarifies, "It is not the case that the child first carries out the task because he shares the adult's definition of situation. It is precisely the reverse: he comes to share the adult's definition of situation because he carries out the task (through other-regulation)" (p. 20). Looking specifically at classrooms, Sfard (2007) submits that "[students] have no other option than to engage in the leading discourse even before having a clear sense of its inner logic and of its advantages" (p. 607). Note how radically different this 'top-down' conceptualization of learning appears to be as compared to the Piagetian 'bottom-up' conceptualization.

Central to Vygotskian as well as neo-Vygotskian theory is the construct of an artifact. Artifacts are, writ large, any cultural–historical form, such as adze, abacus, algebraic algorithm, android apps, and so on, that evolved socio-genetically to serve individuals as means of accomplishing personal goals that address enduring and emerging collective problems (Saxe, 2012). All forms of technology, writ large, are necessarily artifacts.

Artifacts extend the naked eye and hand, but mastering them requires adopting their inherent, idiosyncratic grip on the world they engage. To use the terminology of Vérillon and Rabardel (1995), even as we instrumentalize objects to accomplish an objective, we dialectically instrument ourselves with its inherent utilization schemas. Each time we tune our actions so as to avail of a new artifact, our self-adjustments perforce mediate cultural–historical acumen: ways of doing things that necessarily involve particular ways of seeing ~~and~~ as well as acting on what we see. Thus sociocultural theory emphasizes the interactionist nature of conceptual ontogenesis: Novices are indoctrinated, via guided participation in co-enacting expert cultural practice, into vocational discourse that radically reshapes their worldview into forms that differ from their naïve conceptualizations (Newman, Griffin, & Cole, 1989).

But what about the constructivist decree we discussed earlier, by which all disciplinary knowledge should be grounded in naïve operatory schemes? Ostensibly, this theoretical discord between constructivism and sociocultural theory is irredeemable (Cole & Wertsch, 1996). That is, if formal mathematical parsing of reality is incompatible with tacit naturalistic visualization of the same reality, how can mediated concepts possibly be grounded in unmediated phenomenology? (Clue: via objectification.)

Luis Radford (2000, 2013) interprets mathematical inscriptions—for example, the symbolic notation *x*—as semiotic–cultural artifacts. Let us consider that fleeting moment when, as we sit at your computer, hands hovering above the keyboard, searching for *just* the correct word to express an idea, all at once we recall a word that *feels* right. This word is our *semiotic means of objectifying* that ineffable presymbolic notion we had just borne—the word is an externalized utterance that embodies, instantiates, and thus gives form to that particular yet nebulous sensation we were struggling to articulate. In turn, per the sociocultural litany, uttering the sign at once conventionalizes and enables participation in larger activity structures, including the consumption and generation of oral and written communications in the general discursive communities.

Radford's theory of objectification, which emerged from analyzing observations of classroom events, in turn has implications for teaching and design. Mathematics instructors can indirectly evoke students' presymbolic notions by having them participate in problem-solving activities. The learning environment, including its material and human composition and the regimes of instructional practice, should be set up such that the students both experience some presymbolic notion via sensing the properties of a phenomenon they are investigating and then feel compelled, given the prevailing social norms, to select some available resource as means of objectifying this notion. This resource should appear to the students as bearing cognitive–discursive utility for capturing the ephemeral felt sense rising from the activity; the resource should take on those notions as its assigned meanings. It is thus that students are to adopt the discursive norms and implicit worldviews inherent to the practices of the mathematics discipline. That is, even as students assign meaning to a new resource, they resign themselves to the worldview it implicates; even as students instrumentalize the resource, they instrument themselves with its enfolded disciplinary visualization of the phenomenon at hand.

And yet all this still leaves open that issue of minding the epistemic gap. Namely, *how* do learners recognize that some unfamiliar resource, such as a perceptual display, bears the desired utility of promoting their local objectives? Let us consider once again that scene where we are seated at the keyboard, searching for the perfect word. Imagine that we are searching in vain—we stammer, gesticulate emphatically, grope for that elusive ~~word,~~ but cannot come up with just the right one that would adequately objectify our presymbolic notion. Now, what if someone came along at this very moment and offered us some resource that we have never quite seen before, at least in this context, but that somehow *appears* to afford us the very utility we are searching for? Chances are we will adopt that resource—we will appropriate it as extending our schema.

Creating such moments, in which learners figure out how to assimilate a new resource as a means of realizing their enactive or discursive intent, is the purview and mandate of educational designers. Design-based *researchers*, moreover, use the empirical data emerging from this design practice to investigate learning processes (e.g., Lobato, 2003). I, for one, strive to better understand the design features, instructional tactics, and cognitive processes that enable learners to mind the epistemic gap between tacit and cultural views of natural phenomena (Abrahamson, 2012).

We have completed our theoretical journey, in which I advocated for design that makes sense—that is, design that initially fosters learners' felt sense, and only later introduces symbols that the learners make sense of. It is thus that the student is to adopt a new worldview. We now examine two designs that attempted to instantiate this pedagogical technique, which I call embodied design.

EMBODIED DESIGN: MINDING THE EPISTEMIC GAP BY FIRST MAKING SENSE, THEN MAKING SENSE OF SYMBOL

Learning materials are interesting artifacts. To students, they can serve as the focus of activity. To researchers, however, they can serve as apertures into their designer's implicit epistemological beliefs, underlying pedagogical philosophy, guiding theory of learning, and heuristic design framework. As an educational designer, I have come to articulate my own modus operandi as *embodied design*.

> Embodied design is a pedagogical framework that seeks to promote grounded learning by creating situations in which students can be guided to negotiate tacit and cultural perspectives on phenomena under inquiry; tacit and cultural ways of perceiving and acting.
>
> (Abrahamson, 2013, p. 224)

The two exemplars of embodied design to be discussed in this section demonstrate a proposal to foster grounded mathematical understanding. Broadly, embodied design is a sense/symbol two-step:

- *Making sense before symbol*. First, we present learners with a problem whose solution draws on their 'unschooled' capacity—either a perceptual-judgment task or a motor-inquiry task. Through solving the problem, the learners 'make sense'—that is, they experience unarticulated sensations.
- *Making sense of symbol*. Then, we provide further resources. Learners recognize these resources as potentially enhancing their performance, and so they adopt these resources. The learners now have two different methods of accomplishing the task—the intuitive method that relies on unmediated engagement, and the mathematical method that relies on mediating structures. The instructor leads the learner through a negotiation and ultimate reconciliation of these two methods.

As such, and by way of juxtaposition with the ~~questionable~~ '2 + 3 = 5' ~~monstrous~~ design rationale, embodied designs, such as two ahead, aim for students to build meaning before they signify this meaning in mathematical form.

Low-Tech: The Combinations Tower for Probability—Engaging Perception

The *Seeing Chance* project (Abrahamson, 2009b) took on the design problem of students' perennial difficulty with random compound events. For example, when students are asked to list all the possible outcomes of flipping two coins, they very often fail to consider both 'heads, tails' and 'tails, heads.' Consequently they err in constructing the classicist sample space for predicting outcome distributions from actual experiments with compound-event random generators. In particular, they expect a mixed heads/tails event to occur a third of the time, whereas in fact it occurs half of the time. Our two-step embodied-design plan was thus twofold: (1) invent some random compound-event experiment that would elicit children's naïve-yet-*correct* predictions; and (2) present the formal mathematical analysis of this experiment in a form that would enable the children to accept the variations (i.e., both 'heads, tails' *and* 'tails, heads').

In our design solution, the instructor presents the student with a small tub full of marbles—a mixture with equal amounts of green and blue marbles—accompanied by a utensil for drawing out exactly four marbles set in a 2-by-2 square configuration (Figure 1.1). Students are asked to indicate the four-marbles event they believe is most likely to be drawn from the tub. The instructor then provides the students cards as well as a green and a blue crayon and guides them through combinatorial analysis of the stochastic experiment. This process results in the construction and assembly of the experiment's event space—a collection of 16 iconic representations of all possible outcomes, organized in five stacks according to k (# of greens).

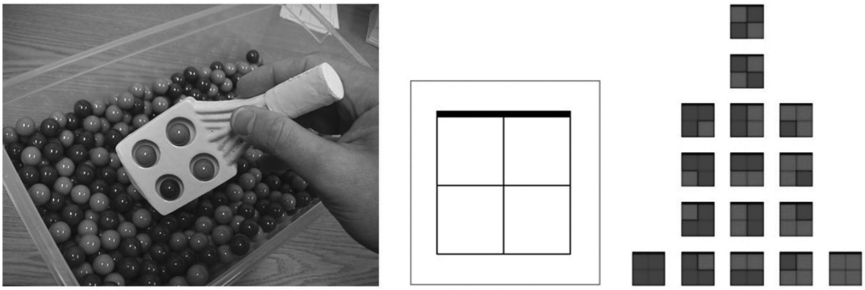

Figure 1.1 Selected materials from a design for the binomial. From left: an open urn full of green and blue marbles with a scooper for drawing out four marbles; a card for indicating possible outcomes using green and blue crayons; the event space made up of 16 such cards.

In our studies, Grade 4–6 students who had not formally studied probability judged that the most likely four-marbles draw from the tub would have two green and two blue marbles. This is precisely what mathematicians would predict via probability theory, and yet the students did so based not on combinatorial analysis but on a perceptual capacity to infer the representativeness of samples based on comparing color ratios in a sample and its source population (Denison & Xu, 2014; Tversky & Kahneman, 1974). The students further judged that an all-green or all-blue draw would be the rarest type of draw, and so on. Importantly, these naïve inferences were couched in terms of the five possible combinations, with no reference to the variations on these combinations. Nevertheless, and critically, students were ultimately able to make sense of the event space as triangulating their naïve expectation, even though the event space does include those variations they had been ignoring.

High-Tech: The Mathematical Imagery Trainer for Proportion—Engaging Action

The *Kinemathics* project (Abrahamson, Trninic, Gutiérrez, Huth, & Lee, 2011) took on the design problem of students' enduring challenges with proportional relations. When students look at 6:10 = 9:x, for example, they are liable to make sense of these symbols through an 'additive lens' instead of a 'multiplicative lens.' That is, they might attend only to the *differences* among the numbers: seeing a difference of 4 between 6 and 10 (or seeing a difference of 3 between the 6 and 9), they would infer that the other pair has the same difference, so that the unknown number is 13 (whereas it should be 15). We assumed that students have scarce sense of what proportional equivalence is or looks like. Our two-step embodied-design plan was for students to: (1) develop foundational images of proportional equivalence; and (2) describe these images mathematically.

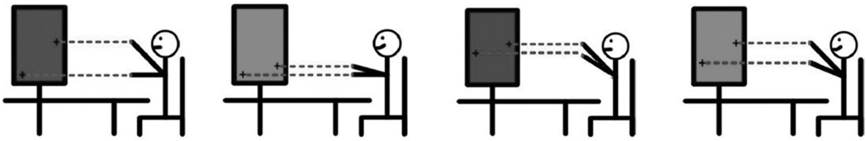

Figure 1.2 The Mathematical Imagery Trainer for Proportion (MIT-P) set at a 1:2 ratio, so that the favorable sensory stimulus (a green background) is activated only when the right hand is twice as high along the monitor as the left hand. This figure encapsulates the study participants' paradigmatic interaction sequence toward discovering the proportional operatory scheme: (a) while exploring, the student first positions the hands incorrectly (red feedback); (b) stumbles on a correct position (green); (c) raises hands maintaining a fixed interval between them (red); and (d) corrects position (green). Compare 2b and 2d to note the different vertical intervals between the virtual objects.

Our design solution was the Mathematical Imagery Trainer for Proportion (MIT-P). We seat a student at a desk in front of a large, red-colored screen and ask the student to "make the screen green." The MIT-P remote-senses the heights of a user's hands above the datum line (see Figure 1.2a). When these heights (e.g., 2" and 4"; Figure 1.2b) relate in accord with an unknown ratio set on the interviewer's console (e.g., 1:2), the screen is green. If the user then raises her hands in front of the display maintaining a fixed distance between them (e.g., keeping the 2" interval, such as raising both hands farther by 6" each, resulting in 8" and 10"), the screen will turn red (Figure 1.2c), because the preset ratio of 1:2 has thus been violated. But if she raises her hands appropriate distances (e.g., raising her hands farther by 3" and 6", respectively, resulting in 5" and 10"), the screen will remain green (Figure 1.2d). Participants are tasked first to make the screen green and, once they have done so, to maintain a green screen while they move their hands.

The activity advances along a sequence of stages, each launched by the introduction of a new display overlay (see Figure 1.3) immediately after the student has satisfied a protocol criteria. For example, consider a student who is working with the cursors against a blank background (Figure 1.3b). Once he articulates a strategy for moving his hands while keeping the screen green, the activity facilitator introduces the grid (see Figure 1.4c).

We implemented the MIT-P design in the form of a tutorial task-based clinical interview with 22 Grade 4–6 students, who participated either individually or in pairs. Qualitative analyses of video data collected during those sessions suggest that the activities created opportunities for students to struggle productively with core conceptual challenges pertaining to the target content of proportions. Before we overlaid the grid, the students discovered a strategy that relates between the hands' elevation and

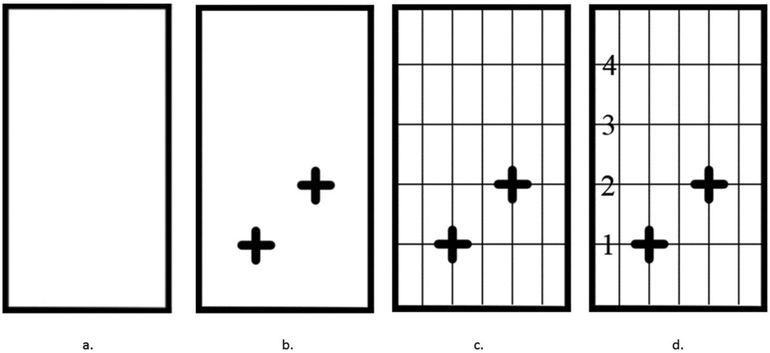

Figure 1.3 MIT-P display configuration schematics, beginning with (a) a blank screen, and then featuring a set of symbolical objects incrementally overlaid by the facilitator onto the display: (b) cursors; (c) a grid; and (d) numerals along the *y*-axis of the grid. These schematics are not drawn to scale, and the actual device enables flexible calibrations of the grid, numerals, and target ratio.

interval—for example, "The higher you go, the bigger the distance needs to be between them to make it green." Next, students engaged the overlaid tools to enhance their performance, discourse, and inquiry. For example, they engaged the grid as a frame of reference that appeared better to enable an enactment of the 'higher-bigger' strategy. Yet in so doing they modulated into a new strategy: in the 1:2 setting they said, "For every 1 unit I go up on the left, I need to go up 2 units on the right" (the '*a*-per-*b*' strategy). In a final activity, students were able reconcile additive and multiplicative visualizations of the solution schema (Abrahamson, Lee, Negrete, & Gutiérrez, 2014).

Comparison: Zooming in on the Crux of the Epistemic Gap

The two designs surveyed ~~earlier,~~ the *Seeing Chance* design for probability and the *Kinemathics* design for proportion, are markedly different in their selection of media—marbles, paper, and markers as compared to programmable remote-sensing devices—and yet they are similar in their use of media. In both of them, students first engage what Bamberger and Schön (1983) call "knowledge-in-action" and only then adopt available symbolic artifacts that enhance, shift, and formalize this knowledge.

In Abrahamson (2013) I proposed a taxonomy of "design genres" according to the type of presymbolic capacity that a design draws on, and I cited these two designs to suggest, respectively, perception-based design and action-based design. To explain in broadest terms possible the psychological mechanism whereby learners are willing to adopt a symbolic reformulation of their presymbolic inference or strategy, I proposed the idea of 'epistemic grounds.' The idea is that learners who engage in an embodied-design problem-solving task will adopt a symbolic artifact contingent on its evaluated fit with their presymbolic schema. In perception-based design, learners make sense of a symbolic artifact contingent on achieving *inferential parity* between the immediate and mediated views on a source phenomenon. For example, in the design for probability, the students achieved a visualization of the combinations tower that was analogous to their perceptual judgment. In action-based design, learners make sense of a symbolic artifact contingent on achieving *functional parity* across a naïve and an instrumented strategy for effecting the targeted goal state of a technological system. For example, in the design for proportion, the students evaluated that the *a*-per-*b* grid-based strategy functioned similar enough to the higher-bigger presymbolic strategy.

Both inferential-parity in perception-based design and functional-parity in action-based design constitute for learners epistemic grounds for appropriating the mathematical signification of their embodied skill. When we design educational activities, we should implicate what epistemic grounds students might draw on so as to ford the epistemic gap between naïve and scientific presentations of phenomena. And we should take measures to ensure that the students are guided to draw on and apply these epistemic grounds.

LOOKING FORWARD: EDUCATIONAL THEORY AND PRACTICE AS MUTUALLY INFORMING

So what should educational technologists do so that children will learn that '2 + 3 = 5'? Commercial designers of educational technology for conceptual learning need not become fluent in theoretical models of instruction. That is usually the province of academic researchers of teaching and learning. But it could be dramatically beneficial for all stakeholders in the educational-technology enterprise if commercial designers and educational researchers were in constructive dialogue. In particular, researchers should articulate for designers theoretically coherent, empirically based heuristic frameworks that guide the process from envisioning a product to its implementation. The researchers, in turn, stand to be inspired by evolving technological developments and their emerging cultural practices.

A message of this chapter to all stakeholders in the educational practice is that we should remain constantly vigilant to ensure that students are making sense of mathematics—not just the symbols but what the symbols are about. One way of monitoring for understanding is following the guidelines of embodied design. Embodied design is a framework for fostering learners' grounded understanding of mathematical symbols and procedures. Embodied-design learning activities are two-stepped. First, learners *make sense*—that is, they experience presymbolic notions that are evoked in their embodied minds as they engage in sensorimotor problem-solving tasks. Second, they *make sense of symbols* when they signify the presymbolic sense in mathematical artifacts that they engage in an attempt to promote their in-situ task-based objectives.

Dyson (1996) claimed that "The great advances in science usually result from new tools rather than from new doctrines" (p. 805). The same applies to instructional design. We cannot but stand in awe as new technologies evolve that challenge our implicit assumptions about what might be possible for young people to know and do. In particular, technologies for embodied interaction, such as remote-sensing immersive systems, are breaking down would-be epistemological barriers between knowing and doing, between the ghost and the machine. There is no mathematical ghost in the body machine—that would be a category error (Ryle, 2009). Instead, mathematical knowing is the individual's socially mediated cultural signification of goal-oriented interaction. Educational designers should avail of new opportunities to nurture doing into knowing.

NOTE

1. Heidegger (1977), however, did not believe that modern technology has changed our historical phenomenology of instruments as means of manipulating the world. Instead, he believed modern technology shifted our ontological relationships with the natural ecology, from Romantic apotheosis of natural entities to utilitarian cupidity of natural resources—the Rhine was his case in point.

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