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# Eye-Tracking Piaget: Capturing the Emergence of Attentional Anchors in the Coordination of Proportional Motor Action

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**Key Words**
Attentional anchor · Coordination · Eye-tracking · Genetic epistemology · Natural user interface · Piaget · Proportion

**Abstract**
The combination of two methodological resources – natural user interface and multimodal learning analytics – is creating opportunities for educational researchers to empirically evaluate theoretical models accounting for the emergence of concepts from situated sensorimotor activity. Seventy-six participants (9–14 years old) solved tablet-based presymbolic manipulation tasks designed to foster grounded meanings for the mathematical concept of proportional equivalence. Data gathered in task-based semi-structured clinical interviews included action logging, eye-gaze tracking, and videography. Analysis of these data indicates that successful task performance coincided with spontaneous emergence of stable dynamical gaze path patterns soon followed by multimodal articulation of strategy. Significantly, gaze patterns included unmanipulated, non-salient screen locations. We present cumulative evidence that these gaze patterns served as ''attentional anchors'' mediating participants' problem solving. By way of further contextualizing our claim, we also present case studies from the various experimental conditions. We interpret the findings as enabling us to revisit, support, refine, and perhaps elaborate on seminal claims from Piaget's theory of genetic epistemology and in particular his insistence on the role of situated motor-action coordination in the process of reflective abstraction.


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The eminent cognitive developmental psychologist Jean Piaget has had a rocky career in the learning sciences. Despite a near-centennial stretch of prodigious, paradigm-changing, academic oeuvre, despite the omnipresence of constructivist educational parlance in pre-K-12 science, technology, engineering and math rhetoric, and despite his indirect yet formative and enduring mark on the design of commercial pedagogical products for discovery-based learning, Piaget's groundbreaking construct of a schema has received some bad press. The construct suffered, perhaps, via its too-convenient association with Piaget's oft-critiqued yet oft-misunderstood Stage Theory or the indefatigable attacks on the validity of his clinical methodologies. But whereas Piaget bashing has generated many a dissertation and built entire research programs, his theoretical constructs and model of conceptual schemata rising from sensorimotor operatory schemes, we posit, have yet to find their match as explanantia for meaningful situated learning. At the very least, we concede, the waning of empirical Piagetian research is hampering our field's intellectual progress and increasingly vitiating its relevance to the changing terrain of educational media [Abrahamson & Sánchez-García, 2015, in press].

We are calling to renew Piagetian discourse specifically on mathematical learning, and more specifically, mathematical learning with state-of-the-art interactive media [Forman, 1988; Lindgren & Johnson-Glenberg, 2013; Marshall, Antle, van den Hoven, & Rogers, 2013; Moreno-Armella, Hegedus, & Kaput, 2008; Sarama & Clements, 2009]. Even more specifically, we are looking for forms of empirical research in environments where both student and researcher respective activities avail of multimodality, with the student engaging in explorative activity that the researcher monitors, measures, and analyzes, even in real time [Martin & Sherin, 2013; Schneider, Bumbacher, & Blikstein, 2015; Worsley & Blikstein, 2014]. This brave new world of multimodality in design, instruction, and research demands theoretical infrastructure for thinking seriously, anew, about situated motor action skill acquisition as it relates to conceptual development. In turn, we are thus also looking to draw on a century of progress in the somatic kinesiological disciplines [Bernstein, 1996; Kelso & Engstrøm, 2006; Newell & Ranganathan, 2010; Thelen & Smith, 1994] as these bear on the action-to-concept learning process [Bamberger, 2013]. In fact, we will argue that Piaget's genetic epistemology is key to populating learning sciences discourse with this diversity of fresh, pertinent, and resonant perspectives. In a sense, we are stepping back to jump forward.

Up front, we wish to clarify that our call is to build on, rather than replace, a research tradition of treating Piagetian themes through qualitative analysis [Abrahamson, 2012; Dubinsky, 1991; Gray & Tall, 1994; Norton, 2008; Sinclair, 1990]. In fact, it is precisely these types of investigations that we wish further to pursue by introducing new constructs and methodological techniques.

To contextualize and substantiate this call, we present and discuss empirical data collected during the implementation of experimental educational interventions, in which young study participants were engaged in technologically enabled embodied interaction activities designed to foster presymbolic proportional reasoning. We will argue for the unique and pivotal traction of Piaget's thesis on our research by way of explaining the critical role that his constructs of sensorimotor scheme and reflective abstraction served in making sense of our data. Namely, the Piagetian perspective enabled us to posit the significance of nuanced changes in children's sensorimotor activity for their conceptual ontogenesis as well as the implications of these findings for both theory and practice of mathematics education.


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## Theoretical Framework

Piaget (1896–1980) was fascinated by children's opportunities for personal development through engaging in the social enactment of cultural practice, such as moral development through game play. However he viewed culture, along with its social agents, practices, norms, and material artifacts, moreso as the setting and playing field of ontogenesis than as its very fabric and constitution.

In the later 20th century, a slew of monographs inspired by the cultural-historical psychology of Lev Vygotsky (1896–1934) impressed upon our intellectual community a set of views not readily perceived as concordant with Piaget's epistemological theory. Instead of foregrounding the child's piecemeal construction of cognitive structures, these views underscored the critical role of sociocultural activity structures as shaping individual disciplinary enskillment, such as mathematical competence. These alternative views include the theorization of: (a) artifact appropriation and contextual adaptation as the *sine qua non* of mediated maturation into communal techno-scientific practice, including visualizations, orientations, and discourse [Newman, Griffin, & Cole, 1989; Saxe, 2012; Wertsch, 1979]; (b) learning as legitimate peripheral participation in the social co-enactment of purposeful cultural practice [Lave & Wenger, 1991; Rogoff, 1990]; and (c) discourse as the vehicle and substance of knowing [Sfard, 2010].

Scholars holding constructivist views of cognition have retaliated that, notwithstanding, meaning must be grounded in tacit, presymbolic sensorimotor routines and innate early cognitive capacity [Allen & Bickhard, 2013; Denison & Xu, 2014; Harnad, 1990] and concepts are built painstakingly by coordinating multiple personal and situated resources for *ad hoc* productive engagement [Case & Okamoto, 1996; Noss & Hoyles, 1996; Smith, diSessa, & Roschelle, 1993]. In the fray of this grand altercation some scholars are looking to forge dialogue between these die-hard entrenched camps [Abrahamson, 2012; Cole & Wertsch, 1996; diSessa, Levin, & Brown, 2015]. By and large, though, the field is at a stalemate, with each faction chiding the other, "Show me!"

We have something new to show that might jostle the field out of its stalemate. We reasoned that if only we could demonstrate empirically student behaviors that are better accounted for by constructivist than sociocultural theory, then we would be in a better position to argue for a dialectical view of mathematical learning – a view of learning as action-based ontogenesis in facilitated settings that we regard as culturally-historically evolved instrumented fields of promoted action [Abrahamson & Trninic, 2015]. Moreover, by way of reemphasizing the critical role of sensorimotor activity in conceptual development, we could justify an introduction, into learning sciences discourse, of emerging models of teaching and learning imported from disciplines focused on motor action skill development and methodology [Abrahamson & Sánchez-García, 2015, in press; Hutto, Kirchhoff, & Abrahamson, 2015; see also Beilock, 2008].

Our renewed interest in Piaget's theory of learning, and in particular his model of reflective abstraction, emerged from unexpected quarters. Namely, philosophers of radical enactive cognition [Chemero, 2009; Hutto & Myin, 2013], who reject exclusively representationalist epistemologies, have been seeking corroboration via partnerships with social scientists engaged in the empiricism of skill acquisition.[1] For example, Hutto and Sánchez-García [2015], respectively a philosopher of cognition and a sociologist

[1] For a survey of key concepts in enactivism as it relates to mathematics educational research, see Reid and Mgombelo [2015].


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of sport, collaborated in articulating a radical enactivist interpretation of athletic performance. In particular they developed the construct of an attentional anchor, which then became central to our own work [Abrahamson & Sánchez-García, 2015, in press]. Let us explain this construct, as it will be central to our line of argument here.

An *attentional anchor* is a phenomenological aspect of the agent's goal-oriented interaction with the environment. Attentional anchors may be a specific object (real or imagined), area, or other pattern or behavior of the perceptual manifold that an agent detects, invokes, selects, and uses to enact the activity at hand. For example, a juggler might imagine a tall rectangle rising in front of her, and aim for the vertices, so as to better organize the complex performance of simultaneously managing the coordinated trajectories of multiple balls. The attentional anchor emerges and interpolates itself into the agent-environment relation to serve as an enabling task constraint – it becomes a new systemic element that hones and channels attention during perception-action couplings. The attentional anchor reduces operational complexity, rendering ergonomic and feasible an otherwise overwhelming task. The agent acting on the attentional anchor experiences it as a "steering wheel" overlaid upon the perceptual field – the attentional anchor becomes the mediating proxy both for operating on the environment and interpreting feedback from the environment. Specifically, the attentional anchor brings forth to the agent new latent affordances by objectifying, specifying, and foregrounding the environment's task-oriented invariants [see also Chow, Davids, Button, Shuttleworth, Renshaw, & Araújo, 2007; Kelso & Engstrøm, 2006; Newell & Ranganathan, 2010].

We are intrigued and motivated by an apparent resemblance of the constructs *motor action coordination* and *attentional anchor* from ecological dynamics theory to those of *coordination* and *category* in Piaget's genetic epistemology model of reflective abstraction. Our line of argumentation in this paper draws heavily on this apparent axis we are discerning between ecological dynamics and genetic epistemology, and so it is incumbent upon us to dwell, below, on a brief sketch of reflective abstraction. Whereas the notion of reflective abstraction is key to Piaget's theory [Kitchener, 1986], it is a broad construct, too, and so in the following we will treat this construct in a manner that brings out those ideas from Piaget and his interpreters most relevant to our research design, empirical data, and line of argumentation, and with particular contextual reference to the learning of mathematics.

*Reflective abstraction* is the highest of three abstraction levels distinguished by Piaget. The first level is empirical abstraction, which develops from the specific to the general, deriving knowledge about properties of objects. The second level is pseudo-empirical abstraction, which concerns deriving properties of actions the child performs on objects. Piaget [1985] cites the example of one-to-one correspondence between two sets of objects. Noticing this correspondence is pseudo-empirical because what is abstracted here into appreciating the one-to-one relation is mainly the actions performed on these objects (a process). Reflective abstraction goes beyond the witnessing of actions per se to construct coordination of actions. This type of abstraction is constructive, in the sense that new syntheses emerge by which, in turn, new meaning can be derived from regularities. In short, reflective abstraction is "the construction of mental objects and of mental actions on these objects" [Dubinsky, 1991, p. 101].

Why is this type of abstraction called reflective? Kitchener [1986] explains that the operation abstracted from the child's actions is transposed on a higher plane *(réfléchissement)*. This projection is a psychological reconstruction *(réflexion)*. Thus it bears aspects that are both proactive (projecting on a higher plane) and retroactive (seeing the lower-level action from a higher plane). Importantly, reflective abstraction subsumes the lower-level actions.

The three levels of abstraction are interdependent. As Dubinsky [1991, p. 98] writes:

> Empirical and pseudo-empirical abstraction draws knowledge from objects by performing (or imagining) actions on them. Reflective abstraction interiorizes and coordinates these actions to form new actions and, ultimately new objects (which may no longer be physical but rather mathematical such as a function or a group). Empirical abstraction then extracts data from these new objects through mental actions on them, and so on.

Different forms of reflective abstraction are relevant to mathematics learning: interiorization, coordination, encapsulation, generalization, and reversal. Actions on objects can be interiorized into processes, as the one-to-one correspondence example illustrates. If a student has formed schemata for several processes, these can be coordinated and reversed. But students can also reflect on processes and form new, more abstract objects; this phenomenon is called *encapsulation*. The overall formation of new schemata, by means of interiorization of action processes and encapsulation of processes, can eventually lead to *generalization*.

The construction processes of reflective abstraction have been described and illustrated via intensive analyses of empirical data gathered in clinical task-based settings [Dubinsky, 1991]. However, it has not been possible to study the phenomenon of reflective abstraction in such microlevel as is now possible given new forms of technology for gathering multimodal data, such as eye-tracking instruments. What this technology in particular adds to the picture is perception. Gaze data supplement the empirical portfolio with an investigative window into the child's visual attention to objects and processes. Here we are particularly interested in the role that the hypothesized attentional anchors play in accomplishing the performance of new motor actions on objects (coordination of sensorimotor schemes) and in the emergence of new objects (encapsulation), as these may all bear on the development of mathematical concepts.

The inherently subjective and phenomenological quality of the child's reflective abstraction processes raises questions for educational programs bent on seeking points of contact and leverage with the child during this process so as to support particular directions and forms of reasoning congruent with targeted cultural practices. Perhaps this is the enduring question of constructivist pedagogy: How does one steer conceptual construction? Indeed Kitchener [1986] writes that, "when an action successfully attains a goal, one is aware only of the result of one's actions and not of the means (action schemes) used to attain it" (p. 63). We thus cannot expect that students who accomplish our educational tasks will be aware of the conjectures and attentional anchors they have implicitly used. And yet it is precisely these tacit qualia that we would target in our interventions. Rather, it is the child's failure to accomplish a task that instigates breakdown, awareness, and reflection:

> Such reflection must occur at a higher level than that of the action, since it involves reasoning about the underlying mechanism and this entails a representation or conceptualization of it … Such a conceptualization of the action schemes is a case of reflective abstraction … and involves a structure of concepts for understanding why action schemes succeed or fail. [Kitchener, 1986, p. 63; see also Koschmann, Kuuti, & Hickman, 1998]

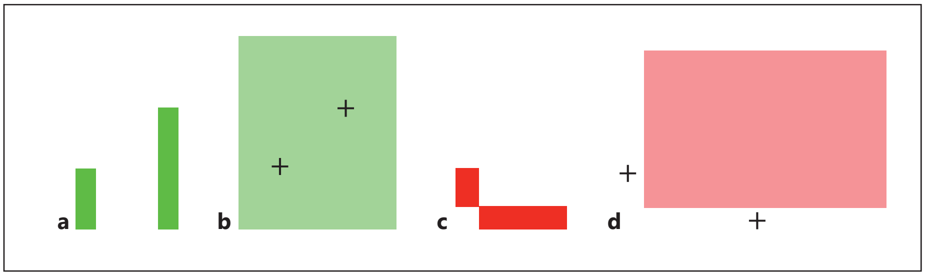


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**Fig. 1.** Sample screenshots from enacting four activity modules in the touchscreen tablet application. To make the screen green, participants had to manipulate either the extension of bars (**a**, **c**) or positions of cursors (**b**, **d**) along parallel (**a**, **b**) or Cartesian axes (**c**, **d**). **a** Parallel Bars: The user drags bar tips each along its vertical axis to extend or shorten bars; color feedback on bars. **b** Parallel Pluses: The user slides pluses each along its vertical axis to reposition them; full-screen color feedback. **c** Orthogonal Bars: The user drags bar tips along their vertical (left) and horizontal (right) axes to extend or shorten bars; color feedback on bars. **d** Orthogonal Pluses: The user slides pluses along their vertical (left) and horizontal (right) axes; full-screen color feedback.

When students construct and use attentional anchors for motor action coordination serving our educational tasks, we expect these tacit qualia to surface for intervention precisely where they fail. In the introductory activities, we deliberately set students up to experience this cognitive conflict. Typically our interventions demand new forms of motor action coordination to solve problems that emerge in the task environment. Coming from the systemic views of ecological dynamics, we implicate motor action coordination as the very stuff that Piaget is talking about in his discussion of sensorimotor schemes whose emergence, failure, and adaption instigate reflective abstraction.

It should come as no surprise that Piaget's work resonates with dynamical systems theory, given his deep commitment to anti-representationalist, situated structuralism [Piaget, 1970; Turner, 1973]. In the remainder of the paper we will discuss an empirical study, in which we have tracked what appears to be children's sensorimotor activity that marks a new coordination focused on an attentional anchor; a coordination leading to the reflective abstraction of a higher-order functional structure and its conscious articulation as a new phenomenal entity. As we explain, this situated cognitive process is pivotal to a designed activity on proportional relations.

## Methods

In total, 76 volunteering students from the Netherlands participated in two studies. Study 1 included 30 students in 5th or 6th grade (mean age = 11;3 years; 13 male, 17 female) from five elementary schools. They all worked on the Parallel Bars activity (fig. 1a). Study 2 included 46 students in 7th or 8th grade (mean age = 13;5 years; 29 male, 17 female) from two prevocational schools. Of these, one group (26 students) only worked on "parallel tasks" – Parallel Pluses (fig. 1b) followed by Parallel Bars (fig. 1a); the other group (20 students) only worked on "orthogonal tasks" – Orthogonal Pluses (fig. 1c) followed by Orthogonal Bars (fig. 1c). In a very rough sense, the older group is academically on par with the younger group, being both verbally weaker and less articulate than their academic track peers.

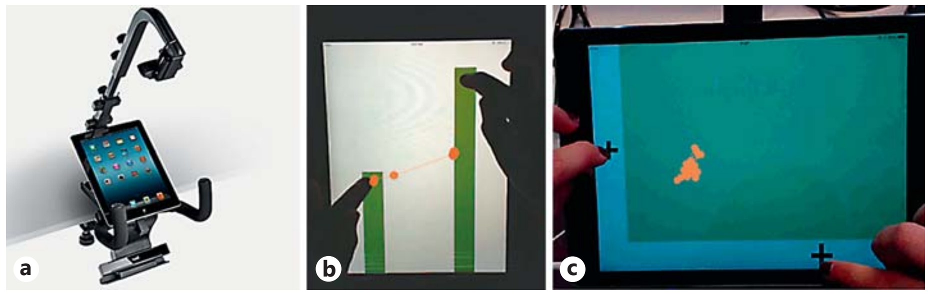




**Fig. 2.** Devices used in the experiment. **a** Tobii Mobile Device Stand for X2. The stand is attached to the edge of a desk. The iPad is positioned in the center. The eye-tracker is placed on the stand base, with the camera on the top. **b** Sample integrated eye-tracking and video data from the Parallel Bars condition. **c** Orthogonal Pluses data sample.

The tasks were variations on the Mathematical Imagery Trainer for Proportion (MITP) [Abrahamson, Gutiérrez, Charoenying, Negrete, & Bumbacher, 2012; Abrahamson & Howison, 2008; Abrahamson & Trninic, 2011; Abrahamson, Trninic, Gutiérrez, Huth, & Lee, 2011; Howison, Trninic, Reinholz, & Abrahamson, 2011]. The MITP is an interactive technological device designed for students first to develop new sensorimotor operatory schemes underlying mathematical concepts and only then mathematize these schemes using standard frames of references (e.g., a grid, numerals). The task is implemented in a multitouch tablet, with each hand (or each index finger) controlling one element on the screen, either a plus-shaped cursor (the "plus" task conditions) or the edge of a stretch/shrink rectangle (the "bars" task conditions). The task objective is to move these elements on the screen so as to achieve a specified goal state: keeping green either the whole screen ("plus") or elements thereof ("bars"). The software mediating user action input and screen color output instantiates mathematical functions. In this study, the device records, for each of the two manipulated interface elements, its distance from a datum point (e.g., 10 and 20 cm, respectively, above the screen base) and then calculates their quotient (e.g., 10/20). A match with a preset ratio (e.g., 1:2) makes for green, otherwise red (e.g., see fig. 1c). Thus in the case of a 1:2 ratio, users might move their index fingers along the screen constantly keeping the right-hand double as high as the left ("parallel" conditions, fig. 1a, b) or double as far from the origin ("orthogonal" conditions, fig. 1c, d), or they might attend to other properties of the performance, such as the distance between their hands or their speeds [Shayan, Abrahamson, Bakker, Duijzer, & van der Schaaf, 2015].

The intervention and analysis followed principles of task-based, semi-structured clinical interviews [Clement, 2000; diSessa, 2007; Ginsburg, 1997; Goldin, 2000]. Our data set comprises videography (of student actions and multimodal student-tutor discourse), streaming logs of touchscreen activity, and eye-gaze tracking (fig. 2). This complex data constellation was designed so as to serve us in developing a more detailed and comprehensive theoretical model for the spontaneous emergence of new sensorimotor coordinations grounding mathematical conceptions. We used visualization software that superimposes the eye-tracking paths onto the videography, so that we could see which particular locations on the screen were in the users' foveal vision as they were manipulating the virtual objects in dialogue with the researcher. Computational analyses of users' visual pathways on the screen fed into microethnographic analyses of their concurrent actions and multimodal utterance [Siegler, 2006]. These analyses enabled us to discern general patterns in students' search for, and articulation of, effective bimanual manipulation strategies. We were particularly interested in implicating the emergence of attentional anchors that first support the bimanual motor action, then come forth into dyadic discourse as new mathematical objects and solution procedures. Furthermore, we evaluated whether Piaget's four phases of reflective abstraction – interiorization, coordination, encapsulation, and generalization – could be genuinely discerned and differentiated in the data as depictive markers parsing students' activity flow.

The first round of analysis treated the videography by focusing primarily on the overall interaction and paying little to no attention to the eye-gaze paths. This round suggested certain commonalities across participants in terms of the types of phases observed and the timing of their appearance along the protocol. Next, the overlaid eye-tracking data were studied in search of distinct visual patterns. This round enabled a refining of the phase logging so as to include certain distinct phases: exploration, enhanced coordination, rule discovery, verification by grid and number, etc. One can think of this segmentation in terms of phases in the development of knowledge and skill: the children's action at one level came to serve as objects of reflection at the next level [Simon, 2006]. So in a continuum of action sequences, each phase was dependent on the previous phase.

Focusing on the enhanced coordination and the discovery phase, both researchers logged any visual patterns that included noticeable screen locations other than the fingertips. Summative discussions among the research team led to the consolidation of the phases and patterns reported below in the Results section.

For reliability, the total corpus of video data was split in two equal parts. Two researchers each watched one part, then shared their findings, and finally watched the videos repeatedly until reaching agreement over all their observations.

## Results

### *Cumulative Findings: A Piagetian Analysis of Learning Proportion as Reflective Abstraction*

Both within and across age and condition groups, students differed along several dimensions relevant to the study, including: (a) duration of time elapsed until discovery of a first effective interaction routine; (b) time to complete the whole task; and (c) pace of finger movement (fast or slow) at the initial exploration phase. Participants also differed in the incorrect rules they initially posited, their eye-gaze patterns accompanying successful hand coordination strategies, and their lines of reasoning toward effective solutions. These individual differences notwithstanding, the progress of *all* participants through the activity bore the pattern presented in table 1. The discoveries students made en route to figuring out "green" interaction rules replicate our earlier findings [Reinholz, Trninic, Howison, & Abrahamson, 2010]. However adding eye-tracking visualization into the data manifold now enables us better to model the emergence of these discoveries from students' interactions and characterize the discoveries in terms of reflective abstraction phases.

In figure 3, circles represent focal gaze points, lines are gaze paths. *Triangulated with our tablet action logging and clinical data, we interpret these gaze patterns as evidence for ecologically coupled, sensorimotor attentional anchors mediating effective enactment of problem solutions for the embodied interaction task.* In our collaborative analysis sessions, as we watched the superimposed gaze video data, we were compelled by the dynamical evolution of these forms, in particular when we played these movies in fast motion: It is as if bits and pieces of a would-be instrument – a handle or steering wheel – assemble in the task environment as solution means; as actionable media "between" student and objective. These media, the attentional anchors, emerge via co-evolving dialectical process: attentional anchors are invented *for and by* the sensorimotor scheme that wields it as a means of accomplishing the situated task ob-

**Table 1.** Participants' cross-condition prototypical behavioral sequence follows Piaget's reflective abstraction phases

| Reflective abstraction phase | Participants' prototypical behaviors |
|---|---|
| 1. Interiorization: exploring task environment | Students (a) explored the task environment without any clearly discernable plan or strategy; (b) found greens haphazardly; (c) could not replicate green positions; (d) attempted strategies that did not bear out, e.g., moving fingers in equal pace; (e) realized there should be a spatial relation between the hands; and (f) attempted to coordinate actions. Concurrently, eye gaze shifted between the moving fingertips. |
| 2. Coordination: stable sensory patterns emerge concurrent with effective motor action performance | In the course of attempting to develop an effective bimanual dynamical motor action scheme for keeping green, gaze patterns emerged (see fig. 3) that (a) followed tentative localized discovery of effective positions and constraints on action; (b) manifested as iterated rapid shifts among specific interface elements; (c) included at least one unmanipulated point; (d) settled on consistent, stable, and reoccurring forms; (e) coincided with significant improvement in overall performance; (f) coincided with more continuous as opposed to abrupt motor action; (g) enabled to reconstruct/replicate/repair previous green locations; and (h) preceded logical-mathematical reflective reasoning, discovery, or articulation of rules. |
| 3. Encapsulation: articulating sensorimotor patterns results in objectifying tacit elements, enhanced performance | Probed to articulate their strategy, students objectified the attentional anchor and then elaborated on it, forming new conjectures. Initially, though, their conjectures tended to belie their actions, such as speaking of a fixed distance between the moving fingers in the Parallel conditions, whereas in fact they had been changing the distance covariate with height. As they enacted their thoughts, however, they gradually came to appreciate the error, such as noticing that the distance in fact increased with height. After several replications they expressed their inference, such as saying, "No I was wrong." At times, the experimenter guided this process by either challenging students or orienting them on critical features in the visual display. In turn, the process of articulating and evaluating effective strategies resulted in better performance. |
| 4. Generalization: from iterated, qualitative process rule to explicit functional rule: articulating a latent mathematical relation as a constant property | Once students had validated an effective strategy, their actions were no longer explorative. Their gaze pattern intensified, e.g., more consistent and more rapid eye-gaze shifts along the triangular attentional anchor in Parallel Bars. Concurrently, their utterance included qualitative properties of objects and prospective actions. Introducing the grid precipitated a shift toward quantitative reasoning, e.g., "When they are lower they are one line apart, when they are in the middle they are more lines apart, and when the right hand is at the very top they are most apart." Supplementing the numerals resulted in students unpacking the bimanual composite into ordered pairs, e.g., left at 1, right at 2; left at 2, right at 4; etc. Eventually they recognized a constant intra-pair quantitative (multiplicative) relation, e.g., "Oh wait it's a half … I know it's a half, the left is always half of the right." They thus shifted from a scalar, inter-position process rule for iterated enactment (the higher you go, the bigger the distance) to an explicit intra-position functional rule with predictive power (wherever right is, left is half). That is, they articulated the notion of a constant ratio that underlies proportional equivalence. |

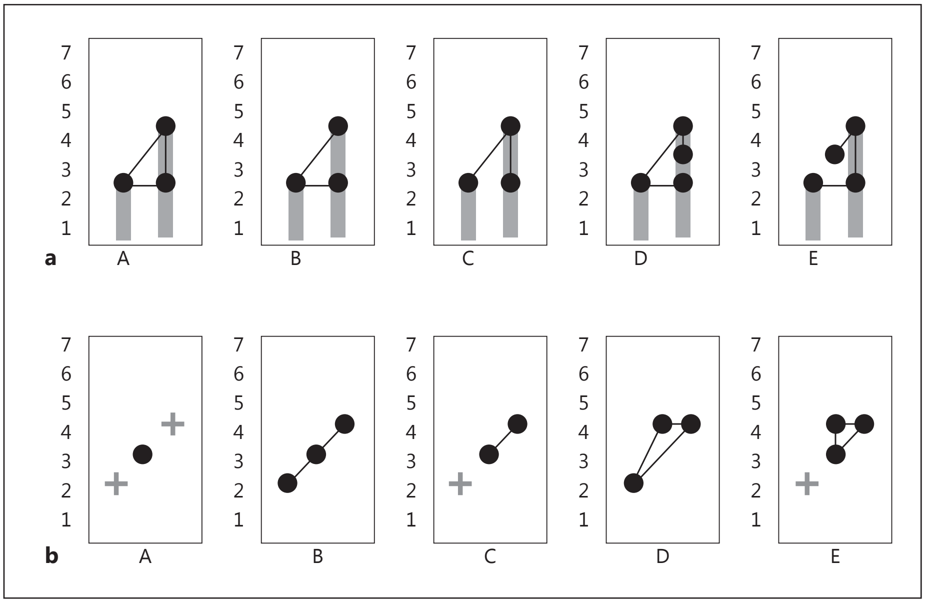


**Fig. 3. a** Schematic overview of the variety of emergent dynamical gaze patterns in the Parallel Bars condition reveals attentional anchors. The top of the left bar is "projected" to an unmanipulated point on the right bar. Students dynamically calibrated the height of the left bar relative to half the height of the right bar, and vice versa. Triangle A was the most prevalent among participants. **b** Schematic overview of the variety of emergent dynamical gaze patterns in the Parallel Pluses condition reveals attentional anchors. The focal gaze point between the pluses is an unmanipulated location. It emerged and was constructed in relation to the two plus signs, constantly moving them and moved by them. Pattern B was the most prevalent among participants.

*(For fig. 3c, d see next page.)*

jective. As the interview advances, the attentional anchors ascend: from latent aspects of the task environment, to tacit, dynamical, ecologically coupling patterns, to bona fide articles of discourse and reasoning.

### *Case Studies*

From the viewpoint of reflective abstraction, study participants who engaged in the interaction problem of keeping green while moving the hands progressed along similar phases (table 1). All participants eventually converged on some stable gaze path pattern. Within conditions these patterns were of comparable form. For all participants the dynamical perceptual patterns coincided with improved task performance (more green); in turn, the perceptual, motor, and performance changes preceded multimodal description of the new sensorimotor schemes.

Despite progressing along similar phases, participants differed in various properties of their respective discovery process along these phases. These differences were discernable within conditions and, moreover, across conditions. Analyzing these dif-

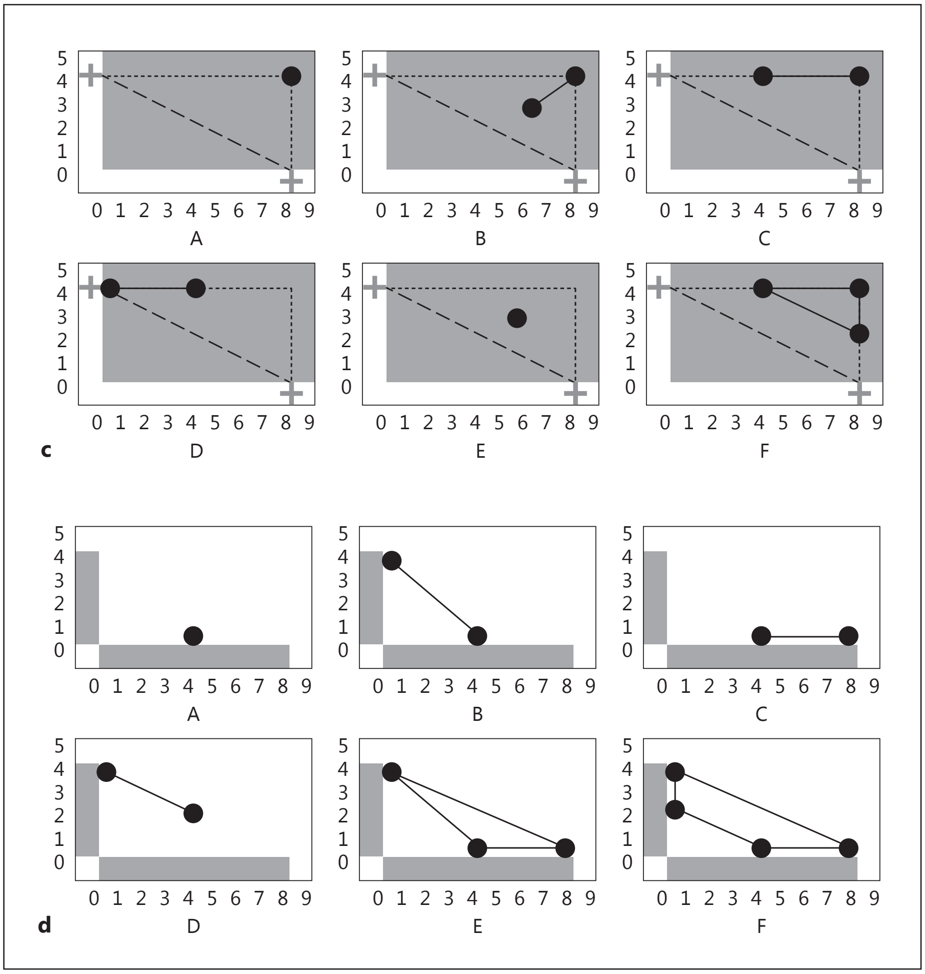


**Fig. 3. c** Schematic overview of the variety of emergent dynamical gaze patterns in the Orthogonal Pluses condition reveals attentional anchors. All focal gaze points that are not on the pluses themselves lie on unmanipulated locations. These locations emerged and were constructed in relation to the two plus signs, constantly moving them and moved by them. **d** Schematic overview of the variety of emergent dynamical gaze patterns in the Orthogonal Bars condition reveals attentional anchors. All focal gaze points that are not on the bar edges lie on uncued locations. These locations emerged and were constructed in relation to the two plus signs, constantly moving them and moved by them. Pattern B was the most prevalent among participants.

ferences could help our field understand student experience with these new forms of educational technology as they bear on conceptual development in the disciplines. In particular, microethnographic descriptions of integrated multimodal data could shed light on the didactical problem of supporting students in transitioning from pre-reflective enactment to reflective mathematization.

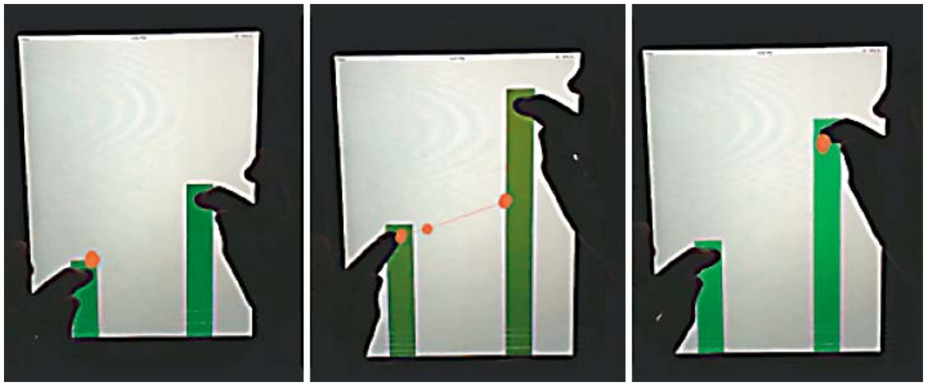

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**Fig. 4.** Three consecutive frames of a time-sequenced triangular attentional anchor – top left, middle right, top right – for keeping the bars green while moving the hands. The middle frame shows where the eye gaze (on the right bar) anticipates the next height of the left bar (in a subsequent frame not shown here).

Therefore, we will now present two sets of brief case studies selected from the younger group working on the Parallel condition and the older group working on the Orthogonal condition. We will use these cases both to demonstrate and attempt to explain cross-participant variability in learning process. We attribute within-condition variability to the open-ended nature of the activity task, which may enable diverse entries and solution paths. We attribute between-condition variability to the available interaction means and their consequences for the discovery process.

We wish to remind readers at this point that in the current build of our technological platform, the interviewer-tutor did not have a real-time view of students' eye-gaze. Our case studies will include descriptions of intriguing attentional anchors that, if known by the tutor in real time, may well have informed her intervention.

All names are pseudonyms selected so as to be gender and culturally appropriate.

*Case Studies: Parallel Condition.* The following two individuals all progressed through the four phases depicted in table 1 toward discovering and articulating an effective rule, and yet they varied in their pace toward task completion. We begin with Anna, who was slower than average, and then continue with Kate, who was faster than average.

Anna (age 10;9 years) advanced through the interiorizing phase with slow hand movements. As her gaze shifted between her two controlling index fingers, she mostly focused on the right bar finger. At 5:08, with both bars green, Anna first looked halfway up the right bar. This event marked for us the onset of her coordination phase.

Over the subsequent several minutes, Anna frequently repeated the triangular gaze pattern "top left bar, mid right bar, top right bar" (fig. 4). Anna struggled to encapsulate her sensorimotor coordination. Even as her gaze and action patterns were stabilizing, concurrent with improved performance outcomes, she reported on various action theories that did not appear to reflect her actions nor represent effective

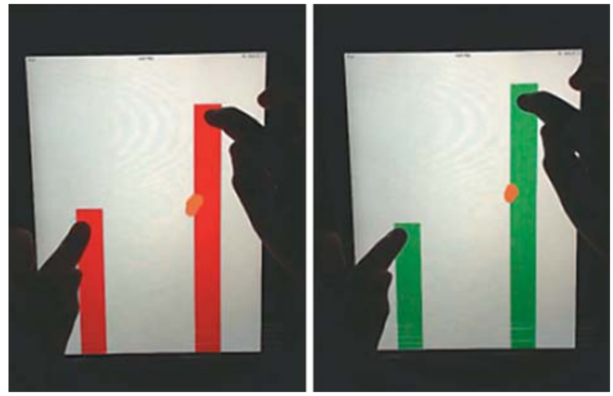



**Fig. 5.** Two consecutive frames of gaze patterns before and after finding the next green. The orange dot indicates the fixation of the eye gaze, expecting the half and turning the red to green.

or sufficiently detailed strategy. For example she said, "The bars turn green when they move in the same pace," even as she was patently moving the two bars at different paces. We marked a generalization at 14:33: With the grid on the screen, Anna found an effective rule that represented her actions, stating, "The smallest is half of the other." Concurrently, she pointed at a location upon the right bar that was as high as the left bar, the very location where she had been gazing as she enacted the triangular pattern.

Kate (age 11;10 years) proceeded through the interiorizing phase much faster than Anna. She demonstrated effective bimanual movement from early on. At 5:22 we observed the onset of her coordination phase, when her gaze pattern began cycling through a triadic structure of screen locations. At 6:48, over a minute later, the encapsulation phase was marked by an "Aha" moment: Focusing more frequently and for longer durations at the midpoint along the right bar (fig. 5), Kate generalized, "When you keep the left bar little, half [of] the right bar, then they both stay green, and if you make the left bigger than half [of the right, then] they turn red."

*Case Studies: Orthogonal Condition.* Participants in the Orthogonal condition were two years older than those in the Parallel condition and studied in a different education system (vocational track). These students demonstrated a greater variety of attentional anchors (fig. 3c, d).

Cross-condition differences in experimental effects are expected regardless of underlying design rationales or experimental hypotheses. Yet understanding the relation between interaction conditions and processes can be valuable to educational designers of task environments for content learning. We will now dwell shortly on these differences and their possible causes, and then we will continue to the cases themselves.

In making sense of observed cross-condition differences in participants' process toward achieving effective motor coordination, we have implicated three points of difference between the two conditions' interaction settings and mechanics.

To begin with, our pedagogical design rationale was for students to confront their own implicit "additive" assumption, by which the difference between two quantities should remain constant as these quantities increase, that is, 1:2 = 2:3 due to equivalent difference of one. The Parallel condition is a better fit than the Orthogonal condition for realizing this design rationale, because the Parallel condition is more conducive for attending to this quantitative difference that is at the center of the confusion. In the Parallel condition, the difference lies directly between the fingertips, embodied in the vertical distance between the fingertips. But in the Orthogonal condition this difference is not directly measurable – to "see" this difference as a distance the user might mentally rotate and project one of the distances onto the other. As such, the Orthogonal condition disadvantages participants with respect to this particular learning process.

Second, we discuss a "technical" difference. Whereas Parallel tasks were performed with the tablet in portrait orientation, Orthogonal tasks were performed with the tablet in landscape orientation. Parallel tasks were agnostic to orientation, because the interaction was constrained to one and the same axis and therefore the screen's global properties were irrelevant. However Orthogonal tasks likely draw attention to the screen's aspect ratio, because it is perceptually and conceptually related to the task objective of finding and keeping a particular ratio between spatial extensions along the screen's vertical and horizontal axes. Let us elaborate on this point.

The tablets used in this study (Apple iPad Air) were of 3:4 aspect ratio. This unequal size of screen height and width appears to have favorably constrained students toward productive search patterns during the interiorizing phase. Searching for a 1:2 ratio within a 3:4 space, we believe, students were implicitly cued to position their fingertips at unequal rather than equal distances from the origin, with the horizontal extension greater than the vertical extension. The participants' motions along the screen's axes tended to align to its proportions – moving faster along the longer tablet dimension – even as they were still unconscious of the proportional rules underlying the task solution. As such, the would-be neutral properties of the experimental settings – the screen dimensions – may have jumpstarted Orthogonal as compared to Parallel participants toward satisfying the task objectives even despite their relative difficulty in implicating the difference as a distance. This could explain their apparently shorter durations toward performing green stably.

In any case, the unwitting advantage of Orthogonal participants during the coordination phase later boomeranged during the encapsulation phase. As compared to Parallel participants, it took Orthogonal participants longer durations of time to discover that there is a relation between the two hand movements and to ascertain what this relation is. Compared to Parallel students, Orthogonal students hardly spoke about the difference in distances traversed by the two hands. Moreover, Orthogonal students showed far greater variability in expressed strategy as compared to Parallel students. The following cases were selected to demonstrate this variability. We stress this variability by selecting for our case studies participants who developed similar, though not identical, attentional anchors.

A word on our technical language. In order to describe the participants' behaviors, we will be referring to particular screen locations of their eye-gaze and gesture. We do this by using a grid system that is based on 12 units along the $y$-axis and 18 along the $x$-axis.[2] This is the grid system that later is illuminated on the screen, but we wish to underscore that we will use these convenient indexes in our descriptions even before the grid is visible to the participants.

[2] Even though the iPad Air screen is of dimensions 3:4, the grid layout created a 2:3 Cartesian field.

*Illustrative Example: Bram.* Bram (13 years old) is a mid-level prevocational student.[3] Bram advances through the interiorizing phase by moving the right finger only, while the left finger remains at a fixed position just above the corner (the axes' origin). He finds the first greens fairly quickly, then moves the right finger farther away from the corner at equal steps, pausing in between to correct with the left finger to the next green. Next he switches the fingers' roles: now he fixes the right finger and moves the left finger only, correcting to green with the right finger. Bram's gaze always follows the moving finger. For the next 50 seconds, Bram continues moving one finger at a time, finding more "pairs of green." Suddenly his gaze focuses at the Cartesian point *[x:y]*, where $x$ is the position of right finger and $y$ is the position of left finger (fig. 6a). This point can be imagined as the fourth corner of a rectangle that is formed by the origin and the two finger positions. Concurrently, Bram's fingers begin moving more smoothly along the screen.

We viewed this multimodal data cluster as an event marking the onset of the coordination phase, where the Cartesian point emerged to serve as an attentional anchor. The Cartesian point is an attentional anchor, because it is not located at any of the action points (i.e., the locations of the fingertips' contact with the interface). The Cartesian point cannot be manipulated directly but only indirectly – it is structurally related to action points and dynamically updated by these action points so as to cue the next action points.

As Bram continues moving his fingers concurrently along their respective axes and farther away from the origin, his gaze tracks and mobilizes the continuously updating Cartesian point. As a result, a new structure emerges from the eye-tracking data: a diagonal line that instantiates the function $y = ½ x$. We are not at all suggesting that Bram is aware of this line let alone its mathematical significance. Rather, we are noting this new structure and perhaps musing about its implications for future directions of educational design.

Bram's movements now appear more assertive and purposeful yet still explorative. Whenever he "hits red," Bram returns to the last found green. At times, he moves his fingers one at a time, venturing forward into red with one and then correcting to green with the other then moving *it* forward only to correct with the first finger, and so forth.

The experimenter asks Bram to begin once again from the origin and move both fingers simultaneously, maintaining green all along. Bram complies. Beginning from the origin, Bram immediately creates the attentional anchor to guide a sweep across the screen, with the attentional anchor tracing the $y = ½ x$ diagonal line (fig. 6b). When his right-hand fingertips exhaust the horizontal extent of the screen, Bram stops his left finger too, suggesting his awareness that no further greens lie ahead.

Bram pauses. He says, "Ohh I get it, you have to keep it the same, both length and width the same, otherwise it is red, you have to keep them good." We thus witness how Bram's mathematical reasoning is lagging behind his sensorimotor skill. Bram is at the encapsulation phase, yet he cannot as yet describe his own action patterns with parity – he cannot mathematize his attentional anchor. We view such moments of failed mathematization as prized didactical opportunities, not problems. It

[3] In the Dutch system, about 60% of students are tracked into prevocational tracks after elementary school. Within this track, there are four levels that designate the amount of academic work interleaved with the training of professional skills.


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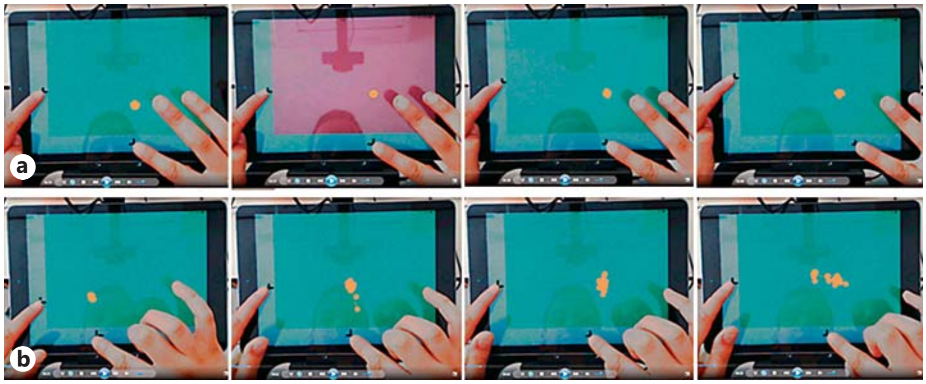

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**Fig. 6. a** Bram's gaze focus is on and around the Cartesian point on the $y = ½\ x$ line. **b** Bram's gaze focus is again on the Cartesian point as he starts over again.

is precisely these emergent disparities between competent pre-reflective enactment and deliberate reflective reasoning brought forth via discourse that our educational process strives to foster. In a deep Piagetian sense, embodied action is at the vanguard of mathematical learning.

Once again, Bram begins from the origin, and again he generates and uses the very same attentional anchor, with increasing dexterity, to manage the dynamical production of green. And once again Bram states that his fingers are at equal distances from the corner, even as the data patently belie that statement. Rather than challenge his theory directly, the experimenter decides to introduce the grid onto the screen as an objective frame of reference for evaluating the statement.

At 8:31 Bram begins a trial yet again, now with the grid on display, and at 9:35 the experimenter probes him, so that the following exchange ensues. Note that when Bram speaks of a "block," he is referring to a grid square:

EXP: What do you think?
Bram: Both pluses have to be at the corner. Then this one on the right has to go one block, this one on the *y*-axis has to move farther, I think. Then you have to move them together like this. (He moves the two fingers simultaneously rather than sequentially.)
EXP: And what about the other one? The low bar? (She is referring to the left finger and Bram understands her.) Will this one (the left finger) also move one block up?
Bram: Yes. Just guessing. But the other one goes one block to the right. (Bram disambiguates the clause "the other one" by slightly wiggling his right finger.)

The experimenter and Bram have been discussing the size of the displacements, to the right and up, that the right hand and left hand should shift, respectively, so as to generate a green screen. Yet, note how the experimenter will now attempt to draw Bram's attention to the distances of the horizontal and vertical from the origin and, moreover, to the difference between these two distances. Bram will become disoriented. Where this scene begins, Bram's left finger is up on 3, and his right finger is across at 6. The screen is green.

EXP: Yes, that one moves only to the right, that's correct. But is the right one also larger than the left one – the distance between the plus and the corner?
Bram: Yes, I think it is a bit longer.
EXP: How much? Do you know?
Bram: Well I don't understand what you mean.
EXP: Well, for instance you have three blocks at the (*y*-axis) right here ok? (pointing to 3 on the *y*-axis)
Bram: Yeah.
EXP: How many blocks do you have on the other side, at your right?
ram: Six.
EXP: So six, and …
Bram: Oh it is twice!
EXP: Ok so the right one is the larger one?
Bram: Yes.

Recall that prior to illuminating the grid on the screen, Bram had enacted continuous green, with his right hand double as far along the horizontal axis as his left hand was up along the vertical axis, and yet Bram believed that his fingers were equidistant from the origin. Guided by the experimenter, the grid served Bram in realizing that the fingers were not equidistant – he realized that the horizontal span was greater than the vertical, and he determined the multiplicative relation of one green pair. Still, Bram has not generalized this multiplicative rule.

Bram continues iterating from one green pair to the next by displacing his left and right hands alternately. His gaze follows the moving finger, as he counts the blocks. Next the experimenter illuminates the numerals along the *y*-axis and at 12:41 urges Bram as follows:

EXP: So you were just talking about 3 and 6. I want you to start over again and tell me when exactly the screen turns green.
Bram: I think it is always twice, so here (along the *y*-axis) you are one, and here (along the *x*-axis) you are two, and here and so here … (He iterates from one green pair to another: 1 & 2, 2 & 4, 3 & 6, etc.)

Finally, Bram has generalized a rule for the relation between the hands. He then generates more green pairs to verify his theory, and the interview ends.

In the early phases of the interview, Bram had used the moving Cartesian point – the *[x, y]* of his right and left hands – as his attentional anchor for coordinating his bimanual motor action satisfying the task objective. Yet though he skillfully enacted green, Bram could not encapsulate the Cartesian strategy. Encouraged to reflect on his actions within a gridded frame of reference, Bram initially stated that his moving hands were maintaining equidistance from the origin. He changed this belief gradually, through further trial and error. Supported by the grid and eventually by numerals along the grid's vertical axis, Bram determined and generalized the multiplicative rule.

Thinking forward, one might speculate how this particular interview might have looked different if the tutor could see Bram's Cartesian gaze pattern in real time. Moreover, imagine how Bram might have responded to seeing his own Cartesian gaze pattern. What would happen when the actual grid were illuminated?

*Illustrative Example: Milan.* Milan (14 years old) is a student in the lowest level of the prevocational track. He begins the interiorizing phase by quickly moving both fingers to the end of their axes. The screen never turns green. Milan then moves his fin-


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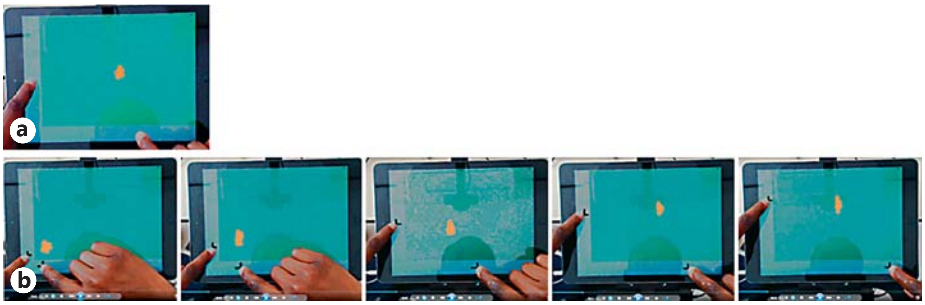


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**Fig. 7. a** Milan's first green, 40 seconds into the game, concurrent with the invention of an attentional anchor at the [6, 10] coordinate. **b** Milan's gaze points on the $x = y$ line, where the height of the left hand position *(y)* is mapped on the $x$-axis and can be compared with the length of the right hand position.

gers at different paces back toward the origin, left finger faster and right finger slower, his gaze shifting between the two fingertips. At 1:35 Milan strikes his first green at $x = 12$ and $y = 6$, his gaze already focused at about [10, 6] (fig. 7a). Milan soon says:

> Milan: Ohh I see. This (left finger) is exactly in the middle and this (right finger) almost in the middle … maybe a bit farther.

Milan is referring to the screen dimensions as a spontaneous frame of reference. He next moves his hands to $y = 2.5$ and $x = 5$, and the screen turns green. Milan's gaze is at about [6, 3].

> Milan: Now I have another green. This is almost the other way around. This one on the $y$-axis is not in the middle now.

Clearly Milan is still using the screen's dimensions as his frame of reference, and yet doing so appears to impede him from arriving at a generalization. The screen dimensions cause Milan to attend to properties of his green pairs that mark these pairs as dissimilar ("almost the other way around") rather than similar. In particular, he is not attending to the fingers' distance from the origin. This perceptual pattern persists: When Milan generates green at $x = 9$ and $y = 4.5$, the screen turns green and Milan says yet again:

> Milan: Here also, now I have this one ($x$-axis) in the middle and this one ($y$-axis) a bit under (the middle).

Next Milan begins over from the point of origin and moves both fingers slowly. Immediately the attentional anchor emerges, indicating for us that Milan has arrived at the coordination phase. His focused gaze moves along the $y = x$ line, as if he is mapping the position of his finger on the $y$-axis, onto the $x$-axis (fig. 7b). Once his right finger has reached the end of the horizontal extent, Milan explains that the left finger should not move any farther up; there is always this much (gesturing with his left thumb and index at the spatial interval between a point on the $y$-axis and the top of the axis) to the top yet you should not move your finger up to there, because it does not make it green. This is as far as Milan can encapsulate his idiosyncratic strategy at this point, but it suggests that Milan is moving toward generalization.

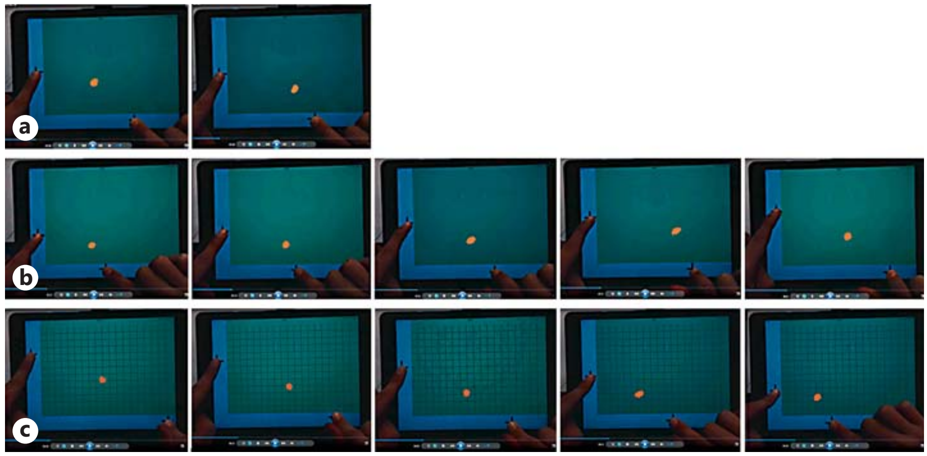


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**Fig. 8.** **a** Daan's gaze point sometimes falls on, and moves along the $y = x$ line projecting from the origin and sometimes on and along the $y = ½\ x$ line from the origin. **b** Daan starts from the beginning and moves his hands forward and looks at points that are roughly on the $y = ½\ x$ line. **c** Daan starts at the ends of the axes. He moves both hands simultaneously toward the origin, consistently looking at points that are on the $y = ½\ x$ line.

When the grid is introduced, Milan repeats the same bimanual movement, his gaze still moving along the $y = x$ line. When the numeral appears along the $y$-axis, Milan begins anew from the origin. He performs the task adroitly, only this time his gaze is focused almost exclusively at [9, 9]. Once he completes the task, the experimenter probes him one last time:

| | |
|---|---|
| EXP: | So anything you noticed? Where are your fingers now? |
| Milan: | 18 and 9, my left hand is at 9 and my right hand at 18. |
| EXP: | And do you notice anything about that? |
| Milan: | Yes, 9 is half of 18 … It is always the half then … |

Milan then verifies the theory by iterating through the green-making locations.

*Illustrative Example: Daan.* Daan (14 years old) is at the highest level of the prevocational track. He begins the interiorizing phase by moving his fingers rapidly, at the same pace, up and across the screen to its full extent. He finds no greens. Daan then moves his fingers back toward the origin, again at the same pace, and again without finding any greens. His gaze shifts between the moving fingertips. On a subsequent trial, he first finds green at $y = 7$ and $x = 14$. He stops moving and decides to move back. Just then the attentional anchor emerges on the $y = x$ line, remaining there for a few seconds. This is the onset of the coordination phase. Daan appears to coordinate his movements better while moving backward than forward. Curiously, his attentional anchor sometimes shifts from $y = x$ to $y = ½\ x$ (fig. 8a). For example, at 7:20, his hands at $y = 3$ and $x = 6$, Daan suddenly adopts a $y = ½\ x$ attentional anchor (fig. 8b).

When the grid is illuminated on the screen, Daan's hands are at the screen edges. He begins moving back toward the origin, his gaze moving downward along the $y = ½\ x$ line (fig. 8c). Against the grid as a frame of reference, the attentional anchor


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now appears to hover in the middle of the hypotenuse of a right-angled triangle. When the numerals appear, Daan proceeds silently to evaluate whether the "half" strategy indeed obtains throughout the screen. At 12:00 he finally generalizes that the *y* values are always half the *x* values.

*Illustrative Example: Lars.* Lars (14 years old) is a vocational track student in the least academic level. He begins the interiorizing phase with quick fingertip action, switching his gaze between the left and right fingertips. Two minutes later, his fingers slow down, and he then first succeeds in making the screen green. Still alternating his gaze, Lars finds another green pair of finger locations, then another.

At 3:00, Lars's fingers are at the origin. He then moves his fingers away from the origin along their respective axes. He is moving smoothly, generally keeping the screen green. By 3:15 an attentional anchor is formed on the fourth corner of the rectangle (see fig. 3c, part A). At 3:20 Lars stops halfway along the axes. Maintaining the same focal point, and all the while keeping the screen green, Lars slides both fingers back to the origin. After a break of several seconds, Lars resumes from the origin. This time he does not quite keep the screen green, possibly because he is attempting to move his fingers faster than before. He stops abruptly and reflects as follows:

Lars: In the corner you have to move them together. They have to be the same … so that … Let's say that the angle has to be the same … (Sliding his index finger across the iPad screen, Lars depicts an imaginary line connecting the fingertips' *y*- and *x*-axes points, where his fingertips have just been) so that they stay on one line.

EXP: So you have to keep them in one line, uhum, ok. So then how do you move your fingers?

Lars: You put them on these two sides and move them on one line. They have to stay on one line. (Lars gestures on the screen. To the analysts it appears as though Lars might be imagining a right triangle with the left index, right index, and the bottom-left corner. Lifting his fingers off the screen, it is as though Lars "expands" this triangle. His hand becomes the hypotenuse, and he moves that hand up and to the right, keeping it parallel, at a constant angle to the *x*-axis.) Yes, because if you put one farther than the other one [it] becomes red again. (He returns his fingers onto the screen and moves them farther.) Yes, with one I push harder than the other. I try let's say to keep it as good as possible, so you push with more pressure. (Lars moves both fingers, apparently applying greater pressure with his right finger along the horizontal line. The work seems almost painful!)

Lars has determined an appropriate action plan for keeping constant the angle between his finger-to-finger line and the screen base. However, his implementation of this action plan is not normative. Whereas he appears to have realized that the left-hand and right-hand fingers should move differently each along its respective axis, he currently assumes that this difference lies in the dimension not of speed but effort, as though greater effort in the right-hand motion would implement the plan of maintaining a constant angle. Applying greater downward force in the right hand, he plods along, not quite keeping the screen green. He does not appear to realize that a "green" angle demands longer stretches along the *x*-axis as compared to the *y*-axis. Or perhaps he believes that the green angle is about 45 degrees, which would mean equivalent spans.

At 5:50, after a brief rest, Lars attempts the task again. This time he moves smoothly in green. Concurrently, a focal gaze point appears. Yet his one is not quite


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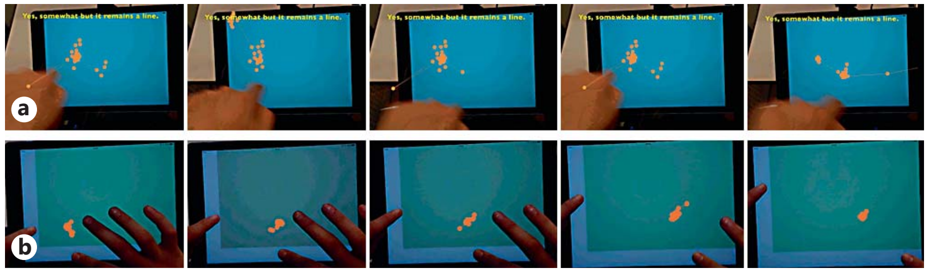


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**Fig. 9. a** Lars, a 14-year-old low-track prevocational education student, gestures an imaginary diagonal line connecting his projected points of contact on the axes. **b** Lars uses an emergent attentional anchor to guide proportional bimanual coordination: he is keeping parallel the imaginary line between his fingertips.

halfway between the left and right fingertips, but always up and to the right of that point, as though anticipating the prospective midpoint (fig. 9).

> Lars: Yes, because with this one (*y*-axis) you should go down, and up, and this one (*x*-axis) should stay on one line. Because you should, for example, really move at the same time, and sometimes not. That is a bit odd because if you try to move at the same time and stay on one line, it becomes red, and so you try to be a bit closer.

Lars's strategy is to move his left hand up and down along its vertical axis, with the right hand keeping parallel the imaginary diagonal between the fingertips. And yet Lars still believes that the hands should move at the same pace. From an educational perspective, this is a fascinating moment. Constructivists view such moments as bearing an implicit cognitive conflict that may or may not instigate conceptual change: Lars is in transition. From an embodied perspective, we are intrigued that once again we witness an effective enacted solution that the learner cannot as yet articulate mathematically. His mathematical knowledge thus acts as a bottleneck on his meaning making. Perhaps if Lars were steered to acknowledge the conflict and confront it, he could be guided to use the available resources so as to reinvent this pedagogical activity's target notion. Perhaps some symbolic artifact could now be introduced into the task space, which Lars might utilize to evaluate his uncontested beliefs.

At 8:00 the grid appears. Again, Lars sets off from the origin, using the same attentional anchors as before. The experimenter asks Lars whether he found the grid useful.

> Lars: Yes, I am now looking at these little blocks.
> EXP: Can you use them, for example, to find the distance between your fingers?
> Lars: Yes.
> EXP: Do you have any idea now why the screen turns green sometimes and red some other times?
> Lars: No.
> EXP: Nothing? You can't tell anything about where your fingers are?
> Lars: Maybe a line (gestures the diagonal connecting the fingertips … On this (gestures to *x*-axis) I need to have more blocks farther.

Lars apparently realized that the hands' locations or motion along their respective axes can be quantized with the grid and that, furthermore, the right-hand quanta are greater than the left-hand quanta.

EXP: Okay, how many more?
Lars: (looking at the *y*-axis) Four blocks on (*y*-axis), then I am at a 7 … It's about half. (His left-hand index finger is at 4, and his right index is past 7, near 8. His gaze darts between the fingers. He continues moving, trying to count more blocks and showing how he still has to "go more blocks" along the *x*-axis than the *y*-axis.)
EXP: Let's bring the numbers for you. (Numerals appear along the *y*-axis.) Ok, so where was it that you saw a half?
Lars: (points to 7 on the *x*-axis). Like look now: 1 on here (*y*-axis) and 2 on here (*x*-axis); here it's 2 on here (*y*-axis) and 4 here (*x*-axis); now 6 here (*x*-axis) and 3 (*y*-axis); now 8 (*x*-axis) and 4 (*y*-axis); now 10 (*x*-axis) and 5 (*y*-axis); now 18 (*x*-axis) and … (on the *y*-axis places right finger at 7 then 8 then 9; screen turns green) 9. It's the half.

The appearance of the grid enabled Lars to determine a quantitative rule governing the paired "green" locations of the left and right hands. That said, it is not clear whether with the articulation of this multiplicative rule Lars ever revisited the earlier question regarding the hands' respective increments along the two axes (i.e., that the right hand proceeds two blocks for every one block the left hand proceeds).

Lars turns to the second task (Orthogonal Pluses, without grid or numerals). He places his left finger at about a quarter of the way up along the vertical bar and his right finger halfway along the horizontal bar. He thus immediately finds a green. He then moves his fingers smoothly along their axes, keeping the bars green. His gaze is initially focused on the top of the vertical bar. Twelve seconds later, his gaze skips along a path: midway along the horizontal line, the far-right tip of the horizontal line, the top of the vertical line, and again midway along the horizontal line. Forty-eight seconds later, Lars says:

Lars: Maybe this (points with a left finger to the vertical bar) is half of this (points to the horizontal bar, while rotating his left thumb from the top of the vertical bar onto its extent along the horizontal axis, as though measuring the vertical extent onto the horizontal axis).

The experimenter asks Lars to move his fingers one more time from the start point to the end. Moving the bars smoothly in green, Lars's gaze shifts between the top of the vertical axis, the middle of the horizontal axis, and the tip of horizontal axis. "Yes," says Lars, "it is the half."

We have presented four prototypical case studies from the Orthogonal condition. In all cases, our volunteering prevocational participants generated an idiosyncratic attentional anchor as their spontaneous solution to the emergent problem of coordinating the enactment of a challenging, environmentally coupled motor action. In all cases, introducing new symbolic artifacts into the task environment created for the students guided opportunities to evaluate and attempt to articulate an effective strategy. In all cases we witnessed a "hook and shift" [Abrahamson et al., 2011, 2012]: The students recognized in the symbolic artifacts utilities – frames of action and reference – for better enacting, explaining, or evaluating their strategy (the hook); but, in so doing, the students accommodated their strategy as they assimilated the enhancing features (the shift). And so we see at the interface of premathematical and mathematical work certain productive fissures in students' progress along the phases of


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reflective abstraction. In a sense, we are witnessing case studies of ontogenesis recapitulating phylogenesis, as students adopt the powerful cultural tools placed into their dominion. Following Vérillon and Rabardel [1995], we perhaps are seeing reflective abstraction in instrumented activity situations.

*Comment: Beyond Representations – Appreciating Piaget as a Non-Cognitivist*

The attentional anchor, that is, the structure that the child constructs through goal-oriented engagement in the task environment, is not a "representation" in the sense of some accessible mental content in her head. Rather, the structure is a cognitive construct, a tacit relation that emerges between the subject and the objective world through adaptive efforts toward equilibrating effective engagement. The structure functions as a dynamical systemic reciprocity, by which are formed both the subject's schematized action routines of engagement with the world and, reciprocally, those worldly categories being engaged – aspects of the world toward which this schematized sensorimotor activity is oriented and transforming; categories by which the child is effecting aspects of the environment.

In the particularities of the child's engagement with the MITP technological system, the emergent operatory schemes are correlational. For example, in the case of the Parallel Pluses the reciprocally emergent category upon which these schemes are operating is often the interval between the hands. The emergent correlational manipulation of the interval coordinates two operations upon it – transforming its elevation, transforming its size – so that the higher the interval is (or the farther it is along the screen), the bigger it should be, so as to effect and maintain the desired worldly state (making and keeping the screen green). This correlational coordination is created through a process Piaget called reflective abstraction, that is, the construction of a higher-order operational structure – the organization of a new phenomenal invariance that breaks away from, yet contains and coordinates, existing routinized operations that hitherto had been sufficient for productive engagement with simpler categories yet hence prove insufficient. To iterate, this coordination is centered on the new category, the interval between the hands.

Looking at results from implementing the MITP system in an eye-tracking study, we have attempted to make sense of our data from this Piagetian perspective. In particular, we have been curious about shifts in students' visual attention toward the objects they are manipulating – shifts that co-occur with, or briefly anticipate, an apparent organization of new action patterns as well as the multimodal discursive articulation of these patterns into proto-mathematical propositions. Emblematic of these pattern shifts is that students will incorporate into their emerging routine new visual attention toward a location on the screen that is not a constituent part of the objects being manipulated. For example, they may stare at a point between two objects that they are manipulating – a point that apparently is strategic for constructing and effecting the new coordination, such as the "higher-bigger" dynamical correlation discussed above. Whenever these new coordinations constitute schemes that we evaluate as proto-conceptual, such as schemes leading to proportional reasoning, it is very tempting to state that the children are re-inventing mathematical concepts within our designed fields of promoted action. That is, we seem to be witnessing the process of reflective abstraction, and this process is mediated by the child's participation in the enactment of a cultural practice, a practice they are never shown but are steered toward.

We are thus offering an explication of mathematical learning as a Piagetian constructivist process embedded in a Vygotskian cultural-historical framework. In so doing, we are also endeavoring to redress a lacuna in Piaget's theoretical thesis, namely his little concern for sociocultural enframings of children's logico-mathematical ontogenesis. As Turner [1973] writes:

> Piaget's model of psychogenesis is formulated in an artificial sociological vacuum; he has never confronted the question of the socio-cultural components of the mind at the level of the basic structure of the psychogenetic process itself. (p. 364)
> [Piaget] has, in other words, not yet come to grips with the problem of the specific social and cultural mechanisms through which cultures and societies participate in and control the genetic development of the individual psyches of their members. (p. 369)

## Conclusions and Implications

When Piaget began publishing on cognitive developmental psychology a whole century ago, clinical interviews were the cutting-edge scientific method. By detecting systematic patterns in children's action and utterance during interviews, as they attempted to respond to his questions and solve his puzzles, and building on a colossal battery of cross-sectional studies, Piaget put forth a cognitive theory of genetic epistemology. Central to this theory was a painstaking explanation for individuals' subjective construction of psychological objects – new phenomenal categories that come forth to enhance, mediate, and regulate effective worldly transactions. These new categories and their attendant sensorimotor schemes coalesce as the child's cognitive adaptations – emergent interaction routines enabled yet constrained by innate cognitive architecture. That is, the mind constructs a new category and, whenever doing so, extends and tightens its grip on the world.

Though much water has since flowed under Geneva's Mont-Blanc bridge, the learning sciences have not advanced much in evaluating Piaget's central claims respecting the child's construction of new psychological objects as solutions to problems of sensorimotor interaction. To be sure replication, qualification, and elaboration have been offered aplenty, and yet abstraction itself – the construct and process – has not been validated via independent measures.

These are early days in our quest to witness the psychological construction of new objects as it occurs. And yet our findings to date cited earlier and reported herein embolden and impel us to submit that we are literally seeing reflective abstraction. Empirical data from our task-based interviews, and in particular children's eye-gaze patterns triangulated against their tablet actions logs and audio-video recordings, are aligning remarkably well with Piaget's constructivist explication of cognitive development. What more, by seeing what the children are looking at and manipulating we now can understand far better our own successes and failures as educational designers in guiding the children to mathematize these tacit constructions. In sum, this study has opened a new window onto the implicit black box of students' learning process, making visible some of its invisible cognitive components. We also described a procedure for triggering reflective abstraction, which could prove useful for educational purposes.

More broadly, we have demonstrated alignment between a core construct from Piaget's theory of genetic epistemology – reflective abstraction – and tenets of enactivism, dynamical systems theory, ecological psychology, and socio-kinesiology. We thus join Allen and Bickhard [2013] in challenging and encouraging our colleagues to revisit Piaget's seminal contributions; to see for themselves the emergence of conceptual categories; to understand what this might all mean in practice; and make that practice a reality.


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