Old Tricks Revisited: Studying Probabilistic Reasoning through Incorporating Computer Modeling into Piagetian Research

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Being able to recognize the impossibility or the very weak probability of an outcome, such as winning the lottery, is an important challenge in probabilistic reasoning. We reproduce a classical Piagetian experiment using computer-based modeling to test if we can replicate Piaget and Inhelder's findings on the idea of "chance as the negotiation of miracle" beyond early childhood. We interview adults by asking some initial questions with physical coins and then asking them to write a simple computer-based model that flips millions of virtual coins. We create a miracle for adults through secretly modifying the source code of the NetLogo agent-based modeling environment, in which the interviewees create their models, and make its random number generator slightly biased. Our findings show that Piaget and Inhelder's theory has some explanatory value on adults' intuitive reactions to unexpected outcomes. Our findings also illustrate how computer-based modeling activities can help us expand the scope of Piagetian research.

People often face challenges that require using some sort of probabilistic reasoning (Gigerenzer, 2005). Recognizing the impossibility of some outcome is one such challenge. Many times, people miss the great unlikelihood of events such as winning the lottery. In this paper, we revisit some Piagetian theory on intuitive reasoning about highly unlikely events. In their book "The Origin of the Idea of Chance in Children", Piaget and Inhelder argued that "*chance is, in fact, the negotiation of miracle – that is, to understand the nature of an uncertain distribution will mean for the child,* ***as of us****, admitting the very weak probability of an exclusive succession of heads or tails*" (1975, p. 96). However, they actually never conducted research with adults to support the "as of us" part of their argument. Their research focused on whether this *negotiation of miracle* happens after early childhood. We revisit this theory and the associated Piagetian experiment to find out if it has any explanatory value for adult reasoning. We also use this study as an opportunity to demonstrate another way to incorporate computer-based modeling in psychology research (e.g., Abrahamson and Wilensky, 2005; Abrahamson, Wilensky and Levin, 2007).

In the original experiment, Piaget and Inhelder use a simple trick. They use 15 white counters (as coins), each marked with a cross on one side and a circle on the other side, to play heads and tails with children. The procedure is roughly like this: the interviewed child is asked to predict the outcome when all counters are tossed at once, the interviewer tosses all the coins at once and the child reacts to the outcome. In the middle of the experiment, the interviewer secretly substitutes the fair counters with the fixed ones, which have a cross marked on both sides, and keeps repeating the same procedure. According to Piaget and Inhelder, *negotiating miracle* means that the child may react to this sudden miracle in two ways: (1) rejecting the outcome because it is impossible and trying to find a trick (e.g., picking up a coin and looking at the other side), (2) accepting it because it is possible and trying to find logical explanations for it (e.g., saying that all the coins came up crosses because the interviewer was holding the coins in a specific way).

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In our revised experiment, we wanted to work with adults because we wanted to do something Piaget and Inhelder did not: testing if we can observe a *negotiation of miracle* with adults. Our research participants were four PhD students from a Midwest university. We did not use the interview protocol as is because the interviewees would quickly check the counters and figure out the trick. We needed something a bit subtler, so we extended the interview protocol with a computer-based modeling activity. We started an interview with 10 physical fair coins, asking each interviewee to predict the outcome, tossing the coins and getting their reactions. After asking a few simple questions such as "Is 6 heads, 4 tails less likely than 5 heads and 5 tails?", we started asking questions about highly unlikely events such as "Is it possible to get 10 heads, 0 tails?". All our interviewees answered these questions based on a distinction between small sample sizes, large sample sizes and the law of large numbers (e.g., Tversky and Kahneman, 1971). For example, an interviewee gave the following explanation:

> A: [After being asked if she would be surprised if she gets 10 heads and 0 tails] *If you didn't give me ten coins, if you gave me 100 coins and you said flip all those and 'would you be surprised if all 100 of them came out heads?' I would be like 'hells yes!' I think that is close enough for me to say 'impossible!' because the probability of that happening is so so so tiny.*
> Q: I see, so is it like as the numbers grow?
> A: *But as the numbers, ok, so as the numbers grow you are going to get closer and closer to the* ***true expected probability****.*
> Q: Hmm, what's true expected probability?
> A: *Yeah, right? Probability I think is, so probability, the expected probability is almost just a theoretical construct that you'd never actually see it happen. That's kind of the issue with probability, right? Like,* ***you will never be like "let me give you a million coins"****.*

We used these responses as an opportunity to ask them to develop a coin-flipping experiment in the NetLogo agent-based modeling environment (Wilensky, 1999) because NetLogo allows the writing of very short code snippets that can run in parallel, many times in a short time. In other words, we indeed gave them millions of *virtual* coins so that they can prove to us that it is indeed equally likely to get heads or tails. All four interviewees were competent in NetLogo programming but they were not aware that we sneakily modified the source code of the NetLogo application and made its random number generator slightly biased. When they wrote a simple piece of NetLogo code that would normally produce a ~50-50% outcome and ran it, they saw that their models generated a 49% to 51% outcome instead. In other words, they wrote every single line of the model's code correctly only to see their expectation fail thanks to our subtle trick.

To our surprise, we observed that some adults reacted very similarly to Piaget and Inhelder's children. Below is another snippet from the same interview illustrating how the interviewee negotiated this subtle miracle and decided to formulate a logical explanation instead of rejecting the outcome:

> Q: It's at 49%, not at 50% and we are at about 5 million trials. Why is this the case?
> A: *As the number of trials approaches infinity, you are going to get closer and closer to a perfect 50%. 49.04% and 50.96% is very very close to 50%, though it is not quite there.*
> Q: Should it get closer and closer to 50% as the number of trials go up? It's right now 49.04%. Should it increase like .491, .492, .493?
> *A: And it does not necessarily go linearly, like, you do toggle a little bit but the bigger the number of trials, the closer you will get to ...*
> Q: We stopped at about 5 million trials. What if we get to 10 million or 20 million?
> A: *Still not going to get perfect 50% but it* ***will be closer****.*
> Q: It will be closer than 49%?

A: *Yeah. That's what I expect.*

As her answers suggest, the interviewee believed that the outcome of her model was going to be 50-50% before running it. However, when she got 49%, she did not think this was impossible. Instead, she used a sophisticated mathematical explanation to justify the outcome. Later in the same interview, we tested her explanation by asking her to keep the model running continuously. Even after seeing that the outcome was approximately the same after tens of millions of trials, she did not change her mind:

Q: It's not really getting closer to 50%!
A: *It actually got closer for a while and then it got farther away and it gets closer for a while and it gets farther away. It is not hugely surprising. I'm not going like "Did I do my model wrong?" because these are very* ***within the ballpark*** *numbers.*
Q: So, are you thinking of whether there is a problem in your model?
A: *No!*

Three of our four interviewees could not figure out that there was a trick. Two of them accepted the outcome and thought that it was "*close enough*", while one of them strongly rejected it but could not pinpoint the cause of "the error", as he put it. Only one interviewee rejected the outcome and successfully deduced that the NetLogo application was biased. These findings provide preliminary evidence on the possibility of Piaget and Inhelder's theory of "*negotiation of miracle"* having some explanatory value in adults' probabilistic reasoning. More importantly, our findings support Wilensky's (1993; 1995) previous findings that people can powerfully express their mathematical understandings through programming computational models. Here, we showed that even a very simple modeling activity can help us observe a complex thinking process. Incorporating computer-based modeling activities, particularly with "low threshold" languages such as NetLogo, is a promising approach for expanding the scope of Piagetian research.

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