EMBODIED MATHEMATICAL IMAGINATION AND COGNITION (EMIC) WORKING GROUP

Mitchell J. Nathan
Univ. of Wisconsin, Madison
MNathan@wisc.edu

Erin R. Ottmar
Worcester Polytechnic Inst.
EROttmar@wpi.edu

Dor Abrahamson
Univ. of California, Berkeley
dor@berkeley.edu

Caroline Williams-Pierce
Univ. at Albany, SUNY
CWilliamsPierce@albany.edu

Candace Walkington
Southern Methodist Univ.
CWalkington@smu.edu

Ricardo Nemirovsky
San Diego State Univ.
Nemirovsky@mail.sdsu.edu

Embodied cognition is growing in theoretical importance and as a driving set of design principles for curriculum activities and technology innovations for mathematics education. The central aim of the EMIC (Embodied Mathematical Imagination and Cognition) Working Group is to attract engaged and inspired colleagues into a growing community of discourse around theoretical, technological, and methodological developments for advancing the study of embodied cognition for mathematics education. A thriving, informed, and interconnected community of scholars organized around embodied mathematical cognition will broaden the range of activities, practices, and emerging technologies that count as mathematical. EMIC builds upon our 2015 working group, and investigations in formal and informal education and workplace settings to bolster and refine the theoretical underpinnings of an embodied view of mathematical thinking and teaching, while reaching educational practitioners at all levels of administration and across the lifespan.

Keywords: Classroom Discourse, Cognition, Informal Education, Learning Theory

Motivations for This Working Group

Recent empirical, theoretical and methodological developments in embodied cognition and gesture studies provide a solid and generative foundation for the establishment of **an Embodied Mathematical Imagination and Cognition (EMIC) Working Group** for PME-NA. The central aim of EMIC is to attract engaged and inspired colleagues into a growing community of discourse around theoretical, technological, and methodological developments for advancing the study of embodied cognition for mathematics education, including, but not limited to, studies of mathematical reasoning, instruction, the design and use of technological innovations, learning in and outside of formal educational settings, and across the lifespan.

The interplay of multiple perspectives and intellectual trajectories is vital for the study of embodied mathematical cognition to flourish. Partial confluences and differences have to be maintained throughout the conversations; this is because instead of being oriented towards a single and unified theory of mathematical cognition, EMIC strives to establish a philosophical/educational “salon” in which entrenched dualisms, such as mind/body, language/materiality, or signifier/signified are subject to an ongoing and stirring criticism. A thriving, informed, and interconnected community of scholars organized around embodied mathematical cognition will broaden the range of activities and emerging technologies that count as mathematical, and envision alternative forms of engagement with mathematical ideas and practices (e.g., De Freitas & Sinclair, 2014). This broadening is particularly important at a time when schools and communities in North America face persistent achievement gaps between groups of students from many ethnic backgrounds, geographic regions, and socioeconomic circumstances (Ladson-Billings, 1995; Moses & Cobb, 2001; Rosebery, Warren, Ballenger & Ogonowski, 2005). There also is a need to articulate evidence-based findings and principles of embodied cognition to the research and development communities that are looking to generate and disseminate innovative programs for promoting mathematics learning through movement (e.g., Petrick Smith, King, & Hoyte, 2014). Generating, evaluating, and curating empirically validated and reliable methods for promoting mathematical development and effective instruction through embodied activities that are engaging and curricularly relevant is an urgent societal goal.

The EMIC Working Group: A Brief History

The first meeting of the EMIC working group took place in East Lansing, MI during PME-NA 2015. It has a somewhat longer origin, however, growing out of several earlier collaborative efforts to review the existing literature, document embodied behaviors, and design theoretically motivated interventions. One early event was the organization of the 2007 AERA symposium, “Mathematics Learning and Embodied Cognition” (Nemirovsky, 2007). This and other gatherings led to a funded NSF “catalyst” grant to explore a Science of Learning Center, which was to involve scholars from multiple institutions and countries. Though unfunded, those SLC efforts shaped a subsequent 6-year NSF-REESE grant, “Tangibility for the Teaching, Learning, and Communicating of Mathematics,” starting in 2008. Interest from the International PME community in this topic grew, and led to special issues of *Educational Studies in Mathematics* (2009), *The Journal of the Learning Sciences* (2012), and an NCTM 2013 research pre-session keynote panel,“Embodied cognition: What it means to know and do mathematics,” along with a series of academic presentations, book chapters, and journal articles, as well as several masters’ theses and doctoral dissertations. By now, several research programs have formed to investigate the embodied nature of mathematics (e.g., Abrahamson 2014; Alibali & Nathan, 2012; Arzarello et al., 2009; De Freitas & Sinclair, 2014; Edwards, Ferrara, & Moore-Russo, 2014; Lakoff & Núñez, 2000; Radford 2009), demonstrating a “critical mass” of projects, findings, senior and junior investigators, and conceptual frameworks to support an on-going community of likeminded scholars within the mathematics education research community.

It was within this historical context that approximately 22 members of PME-NA 2015 came together for three 90-min sessions of semi-structured activities. On **Day 1,** the organizers engaged attendees in some of the body-based math activities used in their research on proportional reasoning and geometry. We discussed how embodied theories are advancing our understanding of mathematical thinking, and how these ideas are shaping a new class of educational interventions. During **Day 2**, we used hands-on activities to expand our own understanding of topology. We then built on the emerging rapport among the group to hold a facilitated discussion of the potential intellectual benefits of forming a self-sustaining Working Group on embodied cognition, along with the necessary infrastructure it would need to maintain. Several concrete proposals led to the list of Future Steps on **Day 3**. However, before we tackled those matters, participants began the session doing math games and activities in small groups, including Spirograph, Set, Rush Hour, Tangrams, and Mastermind. We reflected on how some games and activities draw people into rich mathematical thinking and actions, and how we naturally engage in math through these activities. Day 3 culminated in an organized list of Future Steps, with some working group members assigned to specific tasks.

Since our first meeting at PME-NA 2015 **our accomplishments** include:

1. Creating a contact list with names and emails of attendees (n = 22) and other interested scholars who could not attend PME-NA 2015 (n = 25);
2. Developing a group website using the Google Sites platform to support ongoing interactions throughout the year
3. Joint submission of an NSF DRK-12 by members who first met during the 2015 EMIC sessions
4. Some senior members joining a junior member’s NSF ITEST grant proposal
5. Submitting a proposal for the continuation of the EMIC WG to PME-NA 2016

6. Examining the potential for an NSF Research Coordination Network (RCN)

Focal Issues in the Psychology of Mathematics Education

Emerging, yet still influential, views of thinking and learning as embodied experiences have grown from several major intellectual developments in philosophy, psychology, anthropology, education, and the learning sciences that frame human communication as multi-modal interaction, and human thinking as multi-modal simulation of sensory-motor activity (Clark, 2008; Hostetter & Alibali, 2008; Lave, 1988; Nathan, 2014; Varela et al., 1992; Wilson, 2002). These views acknowledge the centrality of both unconscious and conscious motor and perceptual processes for influencing conscious awareness, and of embodied experience as following/producing pathways through social and cultural space. As Stevens (2012, p. 346) argues in his introduction to the *JLS* special issue on embodiment of mathematical reasoning,

> it will be hard to consign the body to the sidelines of mathematical cognition ever again if our goal is to make sense of how people make sense and take action with mathematical ideas, tools, and forms.

Four major ideas exemplify the plurality of ways that embodied cognition perspectives are relevant for the study of mathematical understanding: (1) Grounding of abstraction in perceptuo-motor activity as one alternative to representing concepts as purely amodal, abstract, arbitrary, and self-referential symbol systems. This conception shifts the locus of "thinking" from a central processor to a distributed web of perceptuo-motor activity situated within a physical and social setting. (2) Cognition is for action. This tenet proposes that things, including mathematical symbols and representations, are understood by the actions and practices we can perform with them, and by mentally simulating and imagining the actions and practices that underlie or constitute them. (3) Mathematics learning is always affective: there are no purely procedural or "neutral" forms of reasoning detached from the circulation of bodily-based feelings and interpretations surrounding our encounters with them. (4) Mathematical ideas are conveyed using rich, multimodal forms of communication, including gestures and tangible objects in the world.

Alongside these theoretical developments have been technical advances in multi-modal and spatial analysis, which allow scholars to collect new sources of evidence and subject them to powerful analytic procedures, from which they may propose new theories of embodied mathematical cognition and learning. Just as the "linguistic turn" in the social sciences was largely made possible by the innovation that enabled scholars to collect audio recordings of human speech and conversation *in situ*, growth of interest in multi-modal aspects of communication have been enabled by high quality video recording of human activity (e.g., Alibali et al., 2014; Levine & Scollon, 2004), motion capture technology (Hall, Ma, & Nemirovsky, 2015; Sinclair, 2014), and developments in brain imaging (e.g., Barsalou, 2008; Gallese & Lakoff, 2005).

Plan for Active Engagement of Participants

Our formula from PME-NA 2015 proved to be effective: By inviting participants into math activities at the beginning of each session, we were rapidly drawn into those very aspects of mathematics that we find most rewarding. Facilitated discussions (and we now have many effective members who can trade off in this role!) then help us all to "pull back" to the theoretical and methodological issues that are central to advancing math education research. Within this structure of beginning with mathematical activities and facilitated discussions, on **Day 1** we plan to introduce our new website, demonstrate the online resources for building sustained community, and revisit and further develop the items listed in our Future Steps, including assigning roles to EMIC members. On **Day 2**, we will discuss concrete goals and products. One example is the NSF Research Coordination Network (RCN), as a potential compliment to the PME-NA Working Group. The RCN is not intended to promote any one particular research program, but rather to build the networked community of international scholars from which many fruitful lines of inquiry can emerge. Commensurate with the aims of the RCN, we will explore ways to

> share information and ideas, coordinate ongoing or planned research activities, foster synthesis and new collaborations, develop community standards, and in other ways advance science and education through communication and sharing of ideas.

This sharing and coordination will continue into **Day 3**. One proposed activity is to perform a live concept mapping activity that is displayed for all participants to explore the range of EMIC topics and identify common conceptual structure. Harkening back to the four major ideas that we developed earlier, sample seed topics for organizing this activity will be explored, such as:

1. Grounding Abstractions
    a. Conceptual blending (Tunner & Fauconnier, 1995) & metaphor (Lakoff & Núñez, 2000)
    b. Perceptuo-motor grounding of abstractions (Barsalou, 2008; Glenberg, 1997)
    c. Progressive formalization (Nathan, 2012; Romberg, 2001) & concreteness fading (Fyfe, McNeil, Son, & Goldstone, 2014)
    d. Use of manipulatives (Martin & Schwartz, 2005)
2. Cognition is for Action: Designing interactive learning environments for EMIC
    a. Development of spatial reasoning (Liu, Uttal, Marulis, & Newcombe, 2008)
    b. Math cognition through action (Abrahamson, 2014; Nathan et al., 2014)
    c. Perceptual boundedness (Bieda & Nathan, 2009)
    d. Perceptuomotor integration (Nemirovsky, Kelton, & Rhodehamel, 2013)
    e. Attentional anchors and the emergence of mathematical objects (Abrahamson & Sánchez–García, in press; Abrahamson, Shayan, Bakker, & Van der Schaaf, in press)
    f. Mathematical imagination (Nemirovsky, Kelton, & Rhodehamel, 2012)
    g. Students' integer arithmetic learning depends on their actions (Nurnberger-Haag, 2015).
3. Affective Mathematics
    a. Modal engagements (Hall & Nemirovsky, 2012; Nathan et al., 2013)
    b. Sensuous cognition (Radford, 2009)
4. Gesture and Multimodality
    a. Gesture & multimodal instruction (Alibali & Nathan 2012; Cook et al., 2008; Edwards, 2009)
    b. Bodily activity of professional mathematicians (Nemirovsky & Smith, 2013)
    c. Simulation of sensory-motor activity (Hostetter & Alibali, 2008; Nemirovsky & Ferrara, 2009)

Finally, we will introduce the EMIC website (see Figure 1) and invite members to join, and to encourage their interested colleagues to email Caro at cwilliamspierce@albany.edu for access. On this website, we have a list of members with their emails and bios, information about our PME-NA presence, and short personal introduction videos. We've also created a space for members to share information about their research activities – particularly for videos of the complex gesture and action-based interactions that are difficult to express in text format. In addition, we have a common publications repository to share files or links (including to ResearchGate or Academia.edu publication profiles, so members don't have to upload their files in multiple places). At our 2015 working group, some junior members expressed particular interest in this literature support for their pending theses, while more senior members were eager to share and organize the emerging body of work on embodied math education. We've also linked the Google Sites platform directly to a Google Group, so members can participate in online forums (or the linked listserv), and discuss cutting edge topics, share in-progress working papers for review, or advertise for conferences, special issues, or other EMIC-relevant opportunities.

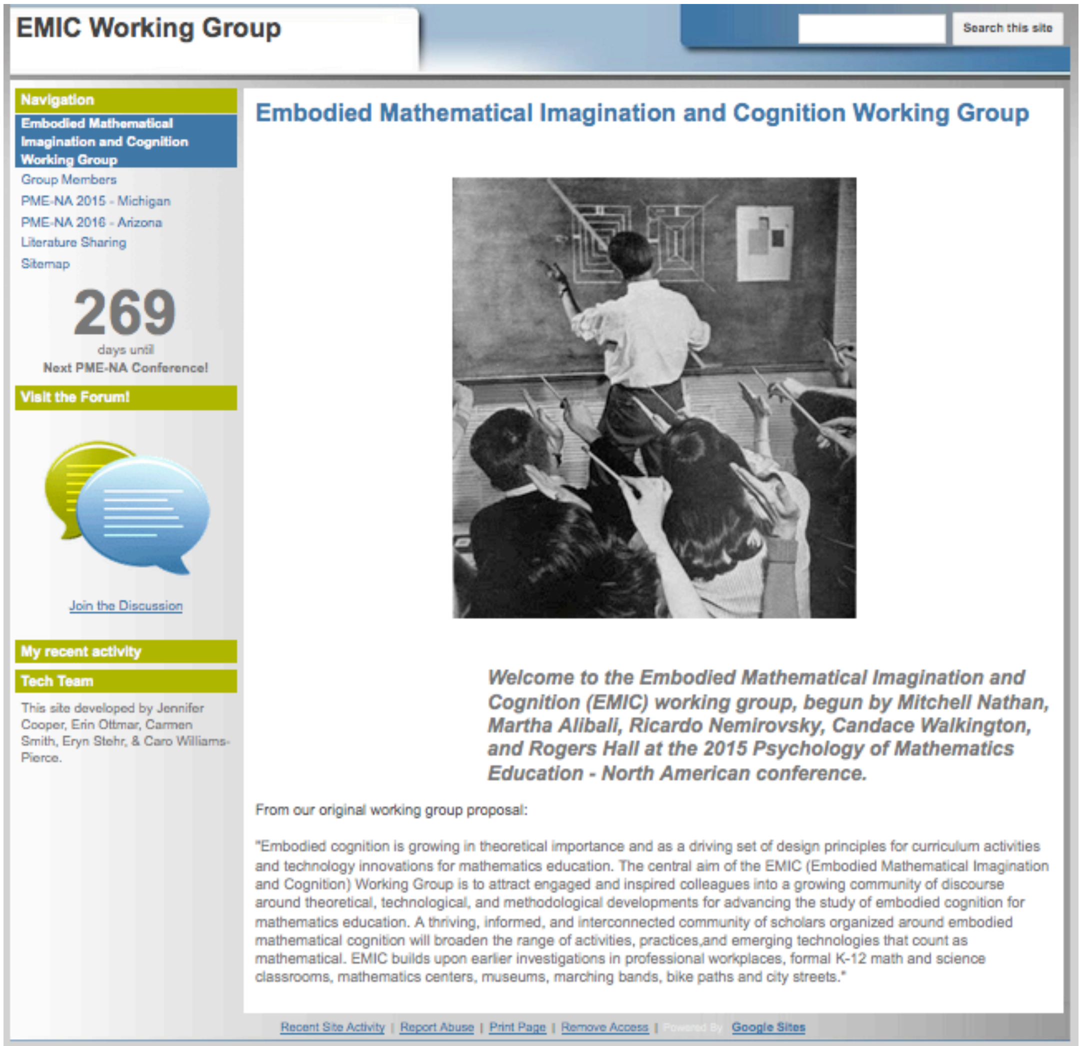

Figure 1. The EMIC website landing page serves as one of the ways EMIC members can share information about themselves and their work, support a common paper repository, post relevant announcements, and coordinate emerging collaborations.

Follow-up Activities

Even prior to our first anniversary, we have already seen a great deal of progress. This is perhaps best exemplified by coming together of the EMIC website and this proposal submission, which draws across multiple institutions. We envision an emergent process for the specific follow-up activities based on participant input and our multi-day discussions. At a minimum, we will continue to develop a list of interested participants and grant them all access to our common discussion forum and literature compilation. Those that are interested in the NSF RCN plan will work to form the international set of collaborations and articulate the intellectual topics that will knit the network together. One additional set of activities we hope to explore is to introduce educational practitioners at all levels of administration and across the lifespan to the power and utility of the EMIC perspective. We thus will strive to explore ways to reach farther outside of our young group to continually make our work relevant, while also seeking to bolster and refine the theoretical underpinnings of an embodied view of mathematical thinking and teaching.

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