

The interactional work of configuring a mathematical object in a technology-enabled embodied learning environment

Virginia J. Flood, University of California, Berkeley, flood@berkeley.edu
 Benedikt W. Harrer, San José State University, benedikt.harrer@sjsu.edu
 Dor Abrahamson, University of California, Berkeley, dor@berkeley.edu

Abstract: We present a detailed account of interactional mechanisms that support participation in STEM disciplinary practices as an adult and a child explore a technology-enabled embodied learning environment for mathematics. Drawing on ethnomethodological studies of technology-rich workplaces, we trace the process of transforming a vague reference into a mutually available mathematical object: a covariant variable. Our analysis reveals that this mathematical object is an interactional achievement, configured via a *reciprocal* process of instructing one another's attention. In particular, we demonstrate how participants' explicit responsiveness to indexical and multimodal resources achieves this object.

Keywords: objectification, collaborative imagining, ethnomethodology–conversation analysis, multimodality, technology-enabled embodied learning environments.

Introduction

Discovery-based learning environment designs are increasingly enlisting new user interface technologies (e.g., Microsoft Kinect) to incorporate the body into computer-mediated, collaborative explorations of STEM phenomena. Technology-enabled embodied learning environments (TEELEs) engage groups of learners in a wide range of topics, from models of meteor orbit trajectories (Lindgren & Moshell, 2011) and atomic interactions (Enyedy et al., 2012) to polar bear energy expenditure (Lyons et al., 2012), and even mathematical objects (Nemirovsky et al., 2012). As these technology-rich environments for inquiry gain popularity in classrooms and informal learning centers, there is a growing need to investigate how social interactions in TEELEs can support or constrain children's opportunities for participation in STEM disciplinary practices.

In this paper, we contribute a close examination of a productive sequence of interaction between a child and an adult tutor as they use a TEELE called the Mathematical Imagery Trainer for Proportions (MIT-P). Inspired by ethnomethodological workplace studies of technology-rich environments (e.g., Goodwin, 1996; Hindmarsh & Heath, 2000), we uncover participants' methods for transforming an initially vague reference into a ratified, mutually established mathematical object. Our fine-grained analysis allows us to reconstruct the interactional work that leads to the joint accomplishment of the covariant variable *distance*. We find two key interactional mechanisms involved in this process: (1) responsiveness to indexicality leading to increased specificity; and (2) responsiveness to multimodality in order to demonstrate mutual understandings.

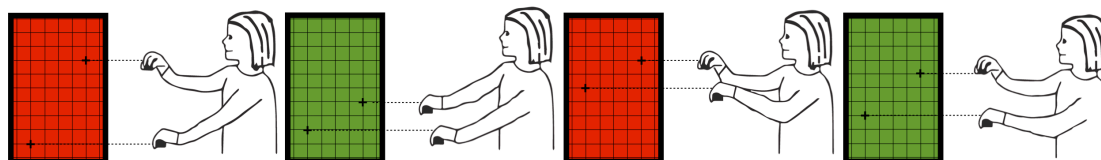


Figure 1. When the Mathematical Imagery Trainer for Proportion (MIT-P) is set to a 1:2 ratio, the screen is green only when the right hand remote is twice as high as the left hand remote; otherwise it is red.

The work of negotiating objects in complex perceptual fields

"The unity of a geometrical configuration or a melody is a general problem of organization."
 (Gurwitsch, 1964, p. 55)

The MIT-P provides an interactive context for users to explore ideas about proportionality through sensorimotor activity. Children operate two independent, hand-held remotes to manipulate the heights of two corresponding

cursors on a computer screen (see Figure 1). The system generates green feedback only when the cursor heights correspond to a particular pre-set, concealed ratio (e.g., 1:2 in Figure 1). Whenever the cursor heights do not fulfill the secret ratio, the screen is red. Children engage in guided exploration as they develop and articulate strategies for turning the computer screen green. Adult tutors guide children's discoveries and facilitate shifts in their strategies for generating green feedback by introducing a series of canonical mathematical artifacts—first a Cartesian grid, then numerals to label that grid. Children enlist these artifacts to generate more robust descriptions of strategy, identify patterns, and develop sophisticated quantitative models that account for the behavior of the device (Abrahamson et al., 2012).

The MIT-P constitutes a complex, dynamic perceptual field of potentially relevant phenomena. Learners discover many perceptual patterns (e.g., haptic, visual) in this phenomenal field of potential relevancies, but a key challenge is getting others—either peers or adult tutors—to perceive these collections of features as significant patterns, as well. In the rich multisensory, multimedia landscapes of TEELEs, this does not merely entail directing one another's attention to visually-present physical features (e.g., in the case of the MIT-P, the cursors on the computer screen). Perceived patterns consist of visually available features in the environment, but they may also incorporate imaginary realms of possibilities that move beyond the immediate spatio-temporal circumstances (Nemirovsky et al., 2012).

To illustrate this, consider, for example, the famous Rubin Vase (see Figure 2). Perceiving the objects *face* or *vase* is not merely determined by the invariant, physical sense data (e.g., the colors, or the boundary between the colors). The same physical features support both imaginary perceptual possibilities. In order to perceive one possibility over the other, the same data has to be *organized* into different configurations (Gurwitsch, 1964). Organizing particular features into certain relationships with one another gives an assemblage of details coherence as an object (Garfinkel, 2008), whether it is real or imaginary.



Figure 2. Rubin Vase

Configuring objects for one another by instructing each other's attention is a ubiquitous practice in technology-rich technical and scientific workplace settings (Garfinkel et al., 1981; Goodwin, 1996; Hindmarsh & Heath, 2000; Stevens & Hall, 1998). Identifying patterns in vast amounts of information and interpreting their significance is a daily endeavor at these sites. Personnel work together to monitor multiple, heterogeneous streams of information within complex multimedia environments. They must actively negotiate, constitute, and render particular physical and imaginary features of the scene relevant to the projects at hand in order to orient each other to potentially significant patterns (Garfinkel et al., 1981; Hindmarsh & Heath, 2000). Professionals use a variety of situated, embodied strategies as part of this *collaborative imagining* (Murphy, 2009)—including purposefully vague references (Garfinkel et al., 1981; Goodwin, 1996) and gestural mimicry (Becvar et al., 2005)—to configure and mutually establish objects. Finding creative ways to highlight and render phenomena of interest publicly inspectable from an abundance of data is an important STEM disciplinary practice, essential to building scientific and mathematical knowledge together (Latour, 1999).

In the guided-discovery learning that is characteristic to TEELEs like the MIT-P, participants must work to coach each other in how to attend to the rich haptic and visual phenomena they deem relevant and important in a particular moment. Following the lead of Stevens and Hall (1998) and Koschmann and Zemel (2009), we believe that much can be learned about processes of discovery and objectification in pedagogical environments by looking for parallels in the interactional methods employed by professional scientists and technical workers. Inspired by studies of technical workplaces, we attempt to understand the complexity of the interactional work that goes into configuring mutually available objects in the complex field of potential relevance in discovery learning with the MIT-P. We approach learning as a process of producing intersubjectively-achieved ratified understandings (Koschmann & Zemel, 2009; Schegloff, 1991; Stevens & Hall, 1998). Our goal is to reveal the specific *interactional methods* employed by a learner and an adult tutor to accomplish a ratified understanding of a mathematical object.

A methodology for tracing intersubjectivity as an interactional achievement

A powerful architecture for *intersubjectivity* is built into the sequential nature of interaction: Each conversational turn consists of an updated display of the speaker's current understanding of the project underway, which is designed in response to an antecedent display of understanding by the prior speakers (Schegloff, 1991). To trace how an initially vague, proposed perceptual pattern was transformed into a clearly defined intersubjectively-achieved mathematical object, we engage in a fine-grained analysis of the participants' embodied interactional practices. Our praxeologically-based study of interaction is informed by

ethnomethodology, an analytical approach that focuses on the *in situ* methods that people use to coordinate mutually intelligible courses of action together (Garfinkel & Sacks, 1970).

Following ethnomethodology, we adopt the view that the meaningfulness of the situations in which participants find themselves is not given a priori and need not be accounted for by partially overlapping or matching psychological states (Schegloff, 1991). Instead, what enables participants to coordinate a given course of action (e.g., having an argument, making a discovery) in any actual situation is a sense of mutual intelligibility that must be built sequentially, moment-by-moment, with the local resources at hand (e.g., turns at talk, bodies, spatial arrangements, tools) through a relentless process of incremental, public displays of revisions and ratifications of meaning (Garfinkel & Sacks, 1970; Goodwin, 1996; Sacks, 1992; Schegloff, 1991). Our analysis seeks to identify the particulars of the interactional resources that participants mobilize to create these scenic displays of the sense they are making of a situation (Garfinkel and Sacks, 1970).

From evidently-vague reference to reified mathematical object

Our episode is drawn from a corpus of task-based interviews with 4th, 5th, and 6th-grade children using the MIT-P. It occurs about 40 minutes into an interview, in which Boaz, a 5th-grader, has been working with two adult tutors, Dean and Devon. The MIT-P is currently set to the secret ratio of 1:2. In the prior 40 minutes, Boaz has offered several accounts of how to make the screen green (e.g., the right-hand cursor always has to be higher than the left-hand cursor). With the newly introduced Cartesian grid, Boaz proposes he could use "the squares" to see how much higher the right cursor is compared to the left cursor.

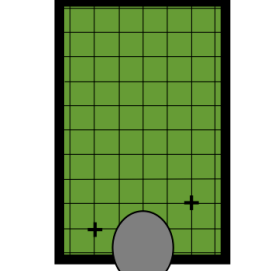
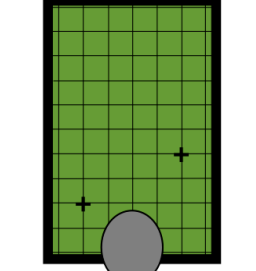
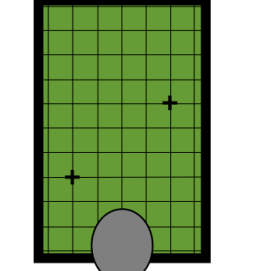
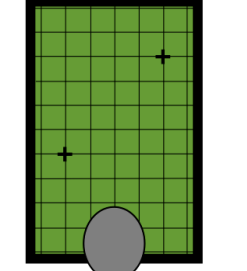
Devon suggests that he and Boaz try this out. Devon moves the left cursor up one grid box at a time, and Boaz adjusts the vertical position of the right cursor to make the screen green (see Figure 3). They advance the cursors up the screen in this fashion, until suddenly Boaz gasps "OHHhhhhh!", marking what comes next in Excerpt 1 as newly revealed information: a *proposed discovery* (Koschmann & Zemel, 2009).



Figure 3. Devon (adult tutor) and Boaz (child) each operate a remote to control a cursor on the MIT-P screen, while Dean (a second adult tutor) looks on.

Excerpt 1¹

- 1 **Boaz:** Each time it's (.) increasing the square (1.0)
- 2 so that one was one then we got to two (.)

Boaz moves cursors to...	...then moves them to...	...then he moves to...	...and finally...
			
3 Boaz and I'm thinking °sort of like° o::ne	4 Boaz and then this one's two	5 Boaz then like here this one is (.) THREE	6 Boaz and then this one is (3.5) it goes up to FOUR
7 SO:o it increases the number of squares each time.			

In Excerpt 1, line 1 (1.1), after both Boaz and Devon have lowered the remotes, Boaz offers this candidate proposal of a discovery: "Each time it's increasing the square." With this vague reference, the sense of this discovery is not yet publicly available to Devon and Dean. The statement functions as a *prospective indexical* (Goodwin, 1996) that projects reference into the future toward a forthcoming resolution: its sense will need to

be further specified and elaborated as the sequence unfolds. A word, phrase, or expression is *indexical* if its intelligibility is only recoverable by considering the context in which it is embedded (Garfinkel & Sacks, 1970).

Boaz's turn in 1.1 initiates Devon and Dean into attending to and examining subsequent turns in the sequence for the sense of his vague reference. The prospective indexical sets up an initial framework for interpretation: It renders what follows (in 1.2 and 1.4–1.6) as *instructions for perceiving* the specific phenomenal features that make up the sense of "it's increasing the square." By pointing forward toward something whose details are not yet specified or ratified by participants, prospective indexicals give objects-in-progress a quasi-presence in the interaction, allowing for their ongoing negotiation (Goodwin, 1996). Garfinkel et al. (1981) first revealed the function of such *evidently-vague reference* in the context of professional scientific work, tracing its key role in the embodied, social negotiation of a certifiable scientific object of knowledge (the astronomical discovery of a pulsar). Koschmann and Zemel (2009) showed how, in a similar fashion, prospective indexicals allow students to negotiate and establish ratified kinematics discoveries while exploring a pedagogically-designed physics microworld.

In 1.2, Boaz offers a first, verbal elaboration. Then, he picks up both remotes and in 1.3 through 1.6, he narrates a sequence of activity with the MIT-P. He pauses rhythmically after positioning both cursors and matches each position to a verbalized numerical value. Starting at 1.4, he also verbally demarcates each configuration with "this one." Through this multimodal performance, Boaz offers an ordered series of parallel cases of *it's increasing the square* for inspection. Boaz has narrowed the complex perceptual field of the MIT-P and configured an ephemeral, animated realm of possibilities for Devon and Dean to search and examine for potentials of the proposed discovery. In 1.7, Boaz uses a similar statement to 1.1 to now *retrospectively* index his just-prior performance (he creates a *retrospective indexical*).

Despite his embodied demonstration, Boaz's instructions for how to appreciate his proposal are not unequivocal for his audience. The numbers Boaz calls out are consistent with at least two possibilities during each event: (1) the positions of the left cursor in each case; and (2) the total number of squares between the vertical positions of the left and the right cursors. There are also several "increases" one might perceive and attend to: (1) the height of the left cursor; (2) the height of the right cursor; (3) the heights of both cursors; and (4) the distance between the cursors. Notably, all these potential patterns contain imaginary features: The properties height and distance have no material instantiation (as opposed to, for example, the cursors or the lines of the Cartesian grid on the screen) in the phenomenal field. A sufficient amount of interpretive work is still necessary to negotiate what features constitute the pattern Boaz is proposing. In Devon's adjacent, subsequent turn, he shows responsiveness to the ambiguity of Boaz's explanation (Excerpt 2).

Excerpt 2

- 1 **Devon:** And (1.2) so when you say it increases what- if you were speaking
- 2 to someone on the phone and they can't- they can't see what's
- 3 going on here (.) and they say it inc- WAIT WOAHH woah woah, what
- 4 are you- what are you talking about, what is increasing?

In Excerpt 2, Devon laminates the immediate spatio-temporal present with a hypothetical situation by voicing an imaginary interlocutor who would have limited perceptual access to the current scene (no visual or haptic access) at the other end of a telephone line. We can appreciate the function of this turn as a "repair initiator" (Schegloff, 1991). It marks a source of trouble in understanding Boaz's just-prior formulation. Participants in interactions have a variety of coordinated repair practices at their disposal to mark and resolve troubles in speaking, hearing, and understanding. These practices provide mechanisms for incrementally establishing intersubjectivity turn-by-turn (Schegloff, 1991).

In this case, the use of "what" has multiple functions. (1) It pinpoints the exact location of the trouble: Devon has swapped Boaz's "it" with "what" to build the hearably parallel construction, "*what* is increasing" (2.4), which targets the *it* as what needs repair. (2) The use of "what" also displays a claim of understanding: It operates as a candidate understanding—an ontological interpretation of what Boaz originally meant. The understanding Devon displays here is that Boaz's *it* is some kind of *thing* (as opposed to a person—who; a time—when; or a place—where). Finally, (3) Devon's "what" projects a format for how Boaz should re-design his initial description (how he should do the third-turn repair): He should further specify the nature of this *thing*.

Devon's turn explicitly attends to the prospective indexical nature of Boaz's prior turns and *renews* Boaz's antecedent coda (from 1.7) as prospectively indexical: The lamination with the hypothetical telephone situation positions 1.7 as *continuing to be* prospectively indexical. It renders Boaz's explanation as still

unresolved, unfinished, and in need of elaboration. Devon's turn encourages Boaz to elaborate the instructions for perceiving the phenomenon in a way that would be sensible for someone not immediately part of the current scene (the person at the other end of the phone line; 2.2). A hallmark of the work of scientists involves the design of representations to capture and archive features of phenomena, and to make them available in places and times beyond the immediate material circumstances in which they were first discovered (Latour, 1999). Thus, this turn creates an opportunity for Boaz to participate in the disciplinary practice of generating a context-independent representation of a phenomenon. In what follows, Boaz does re-design his description, but this does not immediately result in consensus about the object-in-progress.

In Excerpt 3.1, Devon displays his orientation to Boaz's initial description of the pattern ("by one") as a potential object ("that thing"). He makes a strong claim that his understanding is the same as Boaz's and that all that remains for them to figure out is what to call *it* (3.2). At first, Boaz does not contest Devon's development of his idea. He suggests naming *it* "plus" and goes on to fit "plus" into Devon's construction from 3.1: "plus is increasing" (3.3). Devon's repetition in 3.4 is another third-turn repair initiator, now locating trouble in "the plus" and providing an opportunity for Boaz to rephrase or elaborate what the plus is in the subsequent third turn. However, Boaz does not elaborate in 3.5 and instead simply says "yeah." In 3.6, Devon reveals his understanding that "plus" refers to only one cursor at a time. Boaz responds that "both" should be called plus (3.7). Again, there are multiple possible interpretations of Boaz's assertion: It could refer to *the pair* of the left and right cursors, or something else entirely, such as the increasing space the cursors bound when they are moved together in the configurations from Boaz's demonstration (cf. Excerpt 1).

Overall, in turns 3.6–3.9, Boaz and Devon display significant difficulties in understanding each other. Unable to project the end of each other's turns, both Boaz (3.7) and Devon (3.8) interrupt each other mid-sentence. This trouble projecting the boundaries of turns signals a failure to appreciate their ideational content.

Excerpt 3

- 1 **Devon:** How should we call that thing that is increasing by one?
- 2 I can see it also but I'm not sure what to call it.
- 3 **Boaz:** Hmmmm call it plus, plus is increasing=
- 4 **Devon:** =The plus? =
- 5 **Boaz:** =°Yeah° =
- 6 **Devon:** =Wa-wh- ss the one on the right? Or the-
- 7 **Boaz:** -All right, both of them should be called plus cause that's-
- 8 **Devon:** -Yeah, yeah, we call these things crosshairs, (°but whatever°), but
- 9 oka::y so they're increasing by one
- 10 **Boaz:** °Right°. NO not they increase- the first one increases by one and it
- 11 gets higher and then it goes two, three, four, five, si- it goes
- 12 um um odd even odd even each time

The breakdown in Excerpt 3 powerfully demonstrates that mutual understandings are incrementally renewed and re-achieved on a turn-by-turn basis and can be quite suddenly lost. Participants *ongoingly* generate resources for their partner(s) to determine whether they are still oriented to the same phenomenon in the same way *and* their interlocutor(s) must continually monitor these resources for evidence of misalignments. Each subsequent response contains further evidence for the original speaker that their instructed perceiver is or is not orienting appropriately to the features, displaying successful or unsuccessful apprehension of the intended phenomenon (Hindmarsh & Heath, 2000). In our case, Boaz is instructing Devon's perception. He must carefully attend to Devon's responses for evidence that Devon is apprehending the same phenomenon, and move to remedy detected displays of misunderstandings.

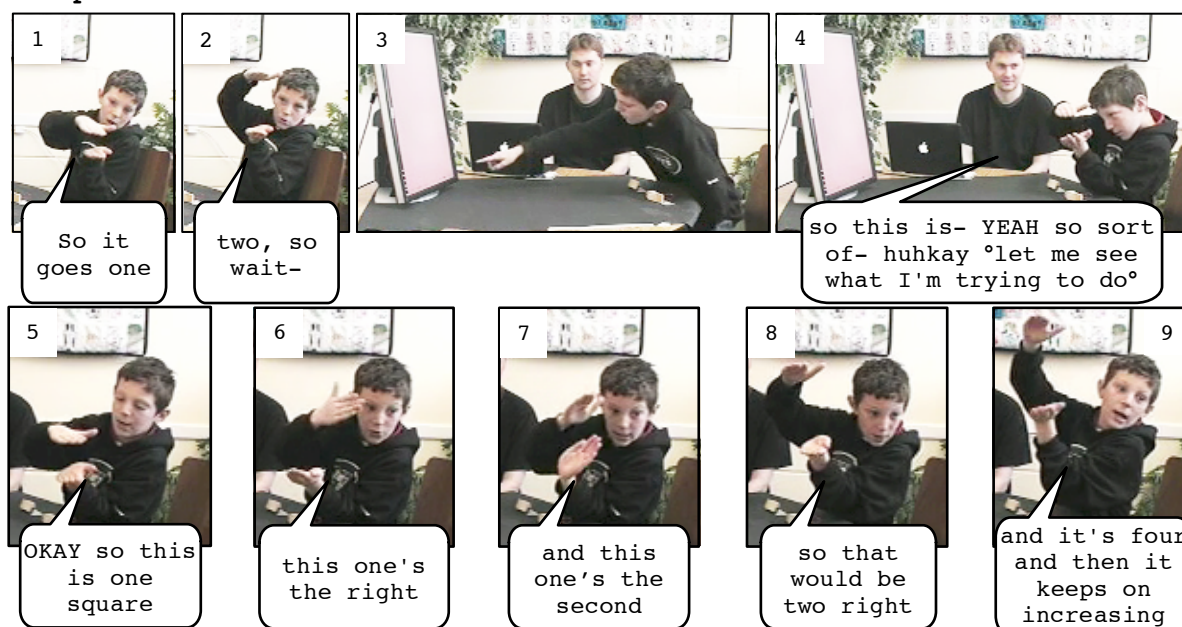
In Excerpt 2, Devon had both *claimed* and *demonstrated* an understanding of Boaz's initial proposal (Excerpt 1) that Boaz could monitor (Sacks, 1992; p. 252). Boaz had not challenged this demonstration and seemed to treat it as evidence that Devon had attended to the same features Boaz was instructing him to perceive. Now, in 3.8–3.9, Devon's turn provides another key demonstration of his understanding for Boaz to assess. Devon demonstrates that he is orienting to Boaz's "plus" (from 3.7) as a reference to each individual cursor by proposing the alternate, pluralized label, "crosshairs" (3.8). Then, he carries the plural into the indexical description "they're increasing" (3.9). With this turn, Devon creates a public display of what features

he is attending to: the two cursors. However, now, Boaz's challenge (3.10–3.12) suggests that he treats Devon's latest display as evidence that Devon *has not* appropriately oriented to what Boaz wishes to highlight for him.

The trouble in Excerpt 3 leads to productive work for the negotiation of the object-in-progress. Devon's display in 3.8–3.9 creates a new priority for Boaz to re-design his approach. In 3.10–3.12, Boaz repairs his description by elaborating it significantly in response to Devon's displayed misapprehension. In Excerpt 4, Boaz further instructs Devon (and Dean who has remained silent but still participates as an on-looking audience) in how to see the phenomenon: He uses a new approach for assembling its features, re-designing his description through a rich performance of gestures and speech to orient his audience.

Boaz starts with his right hand (palm-down) above his left hand (palm-up) in 4.1. Then, he moves both hands farther apart as he lifts both of them higher in the air (4.2). In 4.3, he leans forward to point at the screen. However, unable to reach, he moves back into his seat and creates a space between his hands, sighting through it at the monitor (4.4). In 4.5, with this bounded space assembled, he turns his body toward Devon. He instructs Devon how to see each of his hands: The right hand stands in as the right cursor (4.6) and the left hand stands in as the second, left cursor (4.7). Boaz then increases the spacing between his hands and moves them both upward (4.8). Then he raises both hands even further up and makes the space between each hand even larger (4.9). Each time he lifts his hands and increases the spacing between them, Boaz calls out increasing numbers. He concludes with the retrospective indexical, "*it keeps increasing*," pointing back at his performance.

Excerpt 4



Through this multimodal performance, the *material nothingness* surrounding Boaz's two hands is transformed into something significant: Boaz generates a perceptually available *empty, vertical space* (Nemirovsky et al., 2012) for Devon and Dean to attend to. To create this empty space from nothingness, Boaz configures the outline of the space with his hands, lifting it as a figure from the ground, much like bounding the contour of the vase in Figure 2 lifts it out of a formless background. The empty space is given quasi-presence through bodily activity; it is an imaginary feature that Boaz must instruct Devon and Dean to experience as part of the unfolding object. This now quasi-present imaginary feature allows Boaz to juxtapose several cases of the increasing, covariant quantity: The empty space is getting bigger *as* it moves upward.

Excerpt 5

1 **Devon:** Should we come up with some kind of word=



In Excerpt 5, Devon displays his understanding by recreating Boaz's gestures. He elaborates the original gestures by explicitly tracing the space between his two hands (5.2–5.3). As he says “thing” (5.4) he collapses and re-opens his hands to emphasize his attention to the space between his own hands. Co-timed with the word “growing,” Devon increases the space between his hands as he moves them upward (5.5–5.6). This multimodal revoicing of Boaz's idea, with its explicit effort to highlight the empty space, publicly demonstrates that Devon has organized the same features in the same way as Boaz.

Boaz does not challenge any part of Devon's interpretation, suggesting that he now finds Devon's orientation appropriate: Devon's multimodally displayed understanding of the indicated empty space as *a thing that grows as it is elevated* is visibly in alignment with Boaz's displayed understanding. Boaz demonstrates his satisfaction with the currently intersubjectively-achieved configuration of real and imaginary features—i.e., the presently accomplished organization and mutual availability of a perceptual pattern—by now going along with the proposed project of naming it. He asks if Devon has any suggestions: “Do *you* have a name for it?” Devon offers the label “distance” that Boaz promptly affiliates with: “Oh, ye::ah, the distance.” When Devon further elaborates, “the distance between the hands” while repeating his gestures from 5.2–5.4, Boaz smiles, laughs, and accepts the name: “yeah.” Devon finally proposes a candidate consensus conclusion, “so the distance is growing?” and Boaz confirms it: “The distance is growing.”

This name—“distance”—now comes to stand in for a reified assemblage of an ordered collection of real and imaginary features. With this, a *mathematical object* has been jointly established: It is a covariant variable that increases while the hands—and therefore the cursors on the screen—move upward. This mathematical object, now called *distance*, itself is an imaginary object that has no material presence. Rather, it is a *property* of the configuration of the cursors that was made perceivable through Boaz's and Devon's figuring of the height-dependent empty space between their hands.

Conclusions and Implications

In this article, we demonstrated how a young learner and an adult tutor working with a technology-enabled embodied learning device for mathematics jointly establish a mathematical object—a covariant variable. Both active participants engage in nuanced forms of multimodal interactional work to achieve the mutual availability of the object as a coherent assembly of physical and imaginary features. The process of reifying this mathematical object is distributed temporally across the interaction, across embodied activity (gestures), across material artifacts (the MIT-P), and notably, across individuals. In this case, the objectification that occurred is irreducible to any individual mind. In particular, we identified two key interactional mechanisms occurring between adult and learner as vital to the intersubjective achievement of the ratified object: (1) Participants' attention and propagation of prospective indexicals to occasion their elaboration; and (2) participants' attention to and mimicking of gestures (multimodal revoicing) to demonstrate mutual understanding. Our findings elaborate current frameworks that trace objectification as a form of ontogenesis (Radford, 2003).

While specificity and preciseness are considered hallmarks of sophisticated scientific and mathematical practice, our findings suggest that indexicality and vagueness play a beneficial role in configuring and establishing mutually available mathematical objects. In our case, Boaz's initially vague, prospective indexical description provides the key resources for interpreting his subsequent performance as the proposal of a significant pattern to be attended to. Devon's responsiveness to this prospective indexical generates an environment that allows for the continuing negotiation and specification of the object-in-progress. It also affords an opportunity to engage in the STEM disciplinary practice of developing context-independent representations of phenomena. We therefore add the use of prospective indexicals as a productive form of “systematic

vagueness" that enables discourse and negotiations between teachers and learners about disciplinary content (Newman et al., 1989; p. 62). An implication of this study is that explicit responsiveness to indexicality and multimodality in learners' explanations can foster productive engagement in STEM disciplinary practices within TEELEs.

Our episode also challenges interpretations of zones of proximal development where learning is seen as the result of agentive adults or experts structuring a situation for a child or novice, and regulating their participation. What unfolds in our episode is not a simple case of an adult knowing the cultural future and bringing it to bear on a child's proposed interpretation of events (cf. prolepsis, Stone, 1993). Conceptualizations of scaffolding as an adult-initiated, asymmetrical process cannot capture the reciprocal instruction necessary for jointly establishing the covariant variable in our episode. Boaz must instruct his adult tutors in how to experience what he is experiencing in order to make the subsequent negotiation of meaning possible. He must also carefully monitor his tutor's response for evidence as to whether they have attended to and interpreted what he has highlighted, and he must be responsive to their interpretations. Our case supports recent efforts to recapture and highlight the symmetry of participation in interactional zones of proximal development by analyzing instruction as a *bi-directional* process between experts and newcomers (Roth & Thom, 2009).

With respect to this year's conference theme, we suggest that empowering learners involves appreciating their symmetrical role in the negotiations of meaning in pedagogical situations. We emphasize the importance of finding ways to support learners' use of indexicality and multimodality. As demonstrated in our episode, both are productive strategies for building mutual understandings, and they should be considered valuable and relevant ways of practicing and knowing in scientific and technical disciplines.

Endnotes

- (1) Jeffersonian Transcript Conventions (Jefferson, 2004): (.) for a micro-pause <1 sec; (2.5) for timed pause in seconds; CAPITALS for loud speech; = for latching; :: colons for elongated speech; underline for emphasis; ° for quiet speech.

References

- Abrahamson, D., Gutiérrez, J., Charoenying, T., Negrete, A., & Bumbacher, E. (2012). Fostering hooks and shifts: Tutorial tactics for guided mathematical discovery. *Tech, Knowledge and Learning*, 17(1-2), 61–86.
- Becvar, L. A., Hollan, J., & Hutchins, E. (2005). Hands as molecules: Representational gestures used for developing theory in a scientific laboratory. *Semiotica*, 4(156), 89–112.
- Enyedy, N., Danish, J., Delacruz, G., & Kumar, M. (2012). Learning physics through play in an augmented reality environment. *Intl Journal of Computer-Supported Collaborative Learning*, 7(3), 347–378.
- Garfinkel, H. (2008). *Toward a sociological theory of information*. (A. Rawls, Ed.). Boulder: Paradigm.
- Garfinkel, H., Lynch, M., & Livingston, E. (1981). The work of a discovering science construed with materials from the optically discovered pulsar. *Philosophy of the Social Sciences*, 11, 131–158.
- Garfinkel, H., & Sacks, H. (1970). On formal structures of practical actions. In J. C. McKinney & E. Tiryakian (Eds.), *Theoretical sociology: Perspectives and developments* (pp. 337–366). NY: A-P-C.
- Goodwin, C. (1996). Transparent vision. In E. Ochs, E. A. Schegloff, & S. A. Thompson (Eds.), *Interaction and grammar* (pp. 370–404). Cambridge: Cambridge University Press.
- Gurwitsch, A. (1964). *The field of consciousness*. In R. M. Zaner & L. Embree, (Eds.), New York: Springer.
- Hindmarsh, J., & Heath, C. (2000). Sharing the tools of the trade: The interactional constitution of workplace objects. *Journal of Contemporary Ethnography*, 29(5), 523–562.
- Jefferson, G. Glossary of transcript symbols with an introduction. In G.H. Lerner, (Ed.) *Conversation analysis: Studies from the first generation*. John Benjamins, Amsterdam, 2004, 13–31.
- Koschmann, T., & Zemel, A. (2009). Optical pulsars and black arrows: Discoveries as occasioned productions. *Journal of the Learning Sciences*, 18(2), 200–246.
- Latour, B. (1999). *Pandora's hope: Essays on the reality of science studies*. Cambridge: Harvard Univ. Press.
- Lindgren, R., & Moshell, J. M. (2011). Supporting children's learning with body-based metaphors in a mixed reality environment. In *Proc. of the IDC Conf.* (pp. 177–180). NYC: ACM.
- Lyons, L., Slattery, B., Jimenez, P., Lopez, B., & Moher, T. (2012). Don't forget about the sweat: Effortful embodied interaction in support of learning. In *Proc. of TEI Conf.* (pp. 77–84). New York: ACM.
- Murphy, K. M. (2009). Imagination as joint activity: The case of architectural interaction. *MCA*, 11(4), 37–41.
- Nemirovsky, R., Kelton, M. L., & Rhodehamel, B. (2012). Gesture and imagination: On the constitution and uses of phantasms. *Gesture*, 12(2), 130–165.

Flood, V. J., Harrer, B. W., & Abrahamson, D. (In press). The interactional work of configuring a mathematical object in a technology-enabled embodied learning environment. In C.-K. Looi, J. Polman, U. Cress, & P. Reimann (Eds.), "Transforming learning, empowering learners," Proceedings of the International Conference of the Learning Sciences (ICLS 2016). Singapore: National Institute of Education.

Newman, D., Griffin, P., & Cole, M. (1989). *The construction zone: Working for cognitive change in school*. NY: Cambridge University Press.

Radford, L. (2003). Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. *Mathematical Thinking and Learning*, 5(1), 37–70.

Roth, W.-M., & Thom, J. S. (2009). The emergence of 3D geometry from children's (teacher-guided) classification tasks. *Journal of the Learning Sciences*, 18(1), 37–41.

Sacks, H. (1992). *Lectures on conversation: Volume II*. (G. Jefferson, Ed.). Cambridge: Blackwell Publishing.

Schegloff, E. (1991). Conversation analysis and socially shared cognition. In L. B. Resnick, J. M. Levine, & S. D. Teasley (Eds.), *Perspectives on socially shared cognition* (pp. 150–170). D.C.: APA.

Stevens, R., & Hall, R. (1998). Disciplined perception: Learning to see in technoscience. In *Talking mathematics in school: Studies of teaching and learning* (pp. 107–149). NY: Cambridge University Press.

Stone, C. A. (1993). What is missing in the metaphor of scaffolding? In E. A. Forman, N. Minnick, & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's learning*. NY: Oxford University Press.

Acknowledgments

This material is based on work supported in part by the National Science Foundation under Award No. 1321042 and in part by the Institute of Education Sciences under Award No. R305B090026.