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TRADEOFFS OF SITUATEDNESS: ICONICITY CONSTRAINS THE DEVELOPMENT OF CONTENT-ORIENTED SENSORIMOTOR SCHEMES

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Mathematics education practitioners and researchers have long debated best pedagogical practices for introducing new concepts. Our design-based research project evaluated a heuristic framework, whereby students first develop acontextual sensorimotor schemes and only then extend these schemes to incorporate both concrete narratives (grounding) and formal mathematical rules (generalizing). We compared student performance under conditions of working with stark (acontextual) vs. iconic (situated) manipulatives. We summarize findings from analyzing 20 individually administered task-based semi-structured clinical interviews with Grade 4 – 6 participant students. We found tradeoffs of situatedness: Whereas iconic objects elicit richer narratives than stark objects, these narratives may detrimentally constrain the scope of potential sensorimotor schemes students develop in attempt to solve manipulation problems.

Keywords: Cognition, Learning Theory, Number Concepts and Operations, Technology

Introduction: Forging an Embodiment Middle Ground Between Formalisms-First and Progressive Formalization

Scholars of mathematics education tend to hold two diametrically opposed positions on best pedagogical practices for introducing new mathematical concepts (Nathan, 2012). The formalisms-first approach (e.g., Baird, 2004; Kaminski et al., 2008; Sloutsky et al, 2005; Stokes, 1997; Uttal, Scudder, & DeLoache, 1997) posits that students should first learn symbolical representations of a new concept and only then apply their formal strategies to situated contexts. The progressive-formalization approach (e.g., Goldstone et al., 2005, 2008; Harnad, 1990) posits that students should begin from concrete situations and then progressively formalize their understandings of the situations towards normative abstract representations

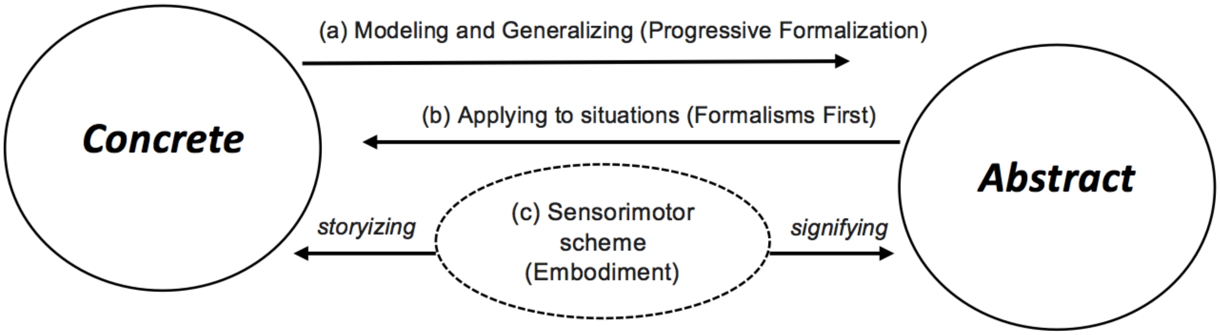

Figure 1. Positioning the (c) Embodiment approach with respect to the (a) Progressive-Formalization approach and the (b) Formalisms-First approach.

And yet in between these two oppositional stances there may be a third position (see Figure 1). Inspired by the embodiment approach (Campbell, 2003; Chemero, 2009; Clark, 2013; Nemirovsky, 2003; Varela, Thompson, &, Rosch, 1991), this position implicates sensorimotor schemes as the epistemological core of mathematical learning and knowing. We conjectured that students could encounter new mathematical concepts by first developing sensorimotor schemes and then both grounding these schemes in concrete situations (storyizing) and signifying the whereas we embrace the proposal to ground mathematical meaning in "our direct physical and perceptual experiences" (Nathan, 2012, p. 139), we decompose this idea by foregrounding and differentiating its two inherent phenomenological dimensions, sensorimotor schemes and situatedness (contextuality). We argue that these two dimensions have been conflated in historical debates (e.g., Barab et al., 2007; Bruner, 1986; Burton, 1999). That is, we maintain that learning activities can be created such that sensorimotor schemes are fostered either in contextual or acontextual situations. We hypothesized that different levels of contextuality would have different effects on learning, and we assumed that sensorimotor schemes would mediate this effect. We believed that students would develop different sensorimotor schemes in low- vs. high-context activities, and that the low-context condition would prove advantageous.

To evaluate this hypothesis, we designed and implemented a learning activity complete with materials, tasks, and facilitation techniques based on the embodied-design framework (Abrahamson, 2006, 2009, 2014). In the empirical study reported herein, we varied the contextuality of a manipulation problem by either incorporating or not incorporating iconic information that would cue narrative framings of the situation, and we measured for effects of this experimental variation on qualities of students' behaviors as they engaged in solving the problem. Our study thus aimed to empirically evaluate this in-between embodiment position with respect to the ongoing debate of formalisms-first vs. progressive formalization.

Theoretical Background

Nathan (2012) has characterized two opposing approaches to mathematics education as follows:

- *progressive formalization* proposes that students should encounter new concepts in the context of meaningful concrete situations and then abstract toward formal models of these situations by progressively adopting mathematical forms and nomenclature (Goldstone, Landy, & Son, 2008; Harnad, 1990); and
- *formalism first* proposes that students should encounter concepts through abstract procedures and then map formalisms to concrete situations via application problems (Baird, 2004; Kaminski et al., 2008; Sloutsky et al., 2005; Stokes, 1997).

The *embodiment approach* put forth in this article borrows the progressive-formalization epistemological position that abstract notions are grounded in concrete situations yet also partially subscribes to the formalism-first ontological position that mathematical concepts should be grounded in generic images. On the one hand, as per *progressive formalization*, *embodiment* learning materials are pre-symbolic and informal. On the other hand, per *formalisms first*, the materials are generic or stark, that is, highly economical on any situated or narrative content.

Affordances and Constraints of Stark (Acontextual) vs. Rich (Situated) Manipulatives

Pedagogical approaches inspired by embodiment theory champion the principle of fostering opportunities for students to build new sensorimotor schemes prior to signifying the schemes in a discipline's semiotic register (Abrahamson, 2006; Nemirovsky, 2003). Our study considered from an embodiment perspective the effect of situatedness on the development of sensorimotor schemes. We thus sought a theory of situated perception and action that would enable us to model, anticipate, and analyze for effects of experimentally varying an activity's situatedness.

Our focus on the relationship between the properties of objects that students manipulate and their actions on these objects led us to consider the theoretical notions of affordances and constraints as relevant to the goals of this study. *Ecological psychology* (Gibson, 1977) theorizes an agent's potential actions on the environment as contingent on the agent–environment relations. An agent (e.g., a mathematics student) perceives opportunities for acting on objects in the environment (e.g., classroom manipulatives) in accord to these objects' subjective cues; the agent tacitly perceives the object as *affording*, that is, privileging certain forms of goal-oriented engagement. Importing Gibson's interactionist views into educational research, Greeno (1994) modeled student learning as the process of attuning to constraints and affordances in recurring situations. Araújo and Davids (2004) further offer that an instructor can "channel" students' engagement in goal-oriented activity by controlling environmental constraints.

Still, to the extent that one subscribes to the constructivist thesis underlying this research, namely that sensorimotor learning mediates conceptual learning, *why might different degrees of the learning materials' contextuality afford different sensorimotor learning?* The answer, we believe, lies in the nature of these sensorimotor schemes vis-à-vis the particular features of the learning materials that the students mentally construct in the course of developing the materials' new perceived affordances. That is, a given situation may lend itself to different goal-oriented sensorimotor schemes. And whereas a variety of schemes may accomplish the prescribed task, some of these schemes may be more important than others for the pedagogical purposes of the activity. We hypothesize that *the situatedness (contextuality) of learning materials constrains which sensorimotor schemes the materials might come to afford*. Where particular contextual cues unwittingly constrain student development of pedagogically desirable affordances, the students' conceptual learning will thus be delimited.

In evaluating this hypothesis pertaining to the nature and quality of situated learning, we needed a theoretical construct that would both cohere with the embodiment perspective and enable us to implicate in our data which sensorimotor schemes students were developing. We realized we were searching for a means of determining how the students are mentally constructing the materials; what specifically they were looking at that mediated their successful manipulation. Such a theoretical construct already existed: an attentional anchor (see below).

An *attentional anchor* is a dynamical structure or pattern of real and/or projected features that an agent perceives in the environment as their means of facilitating the enactment of motor-action coordination (Hutto & Sánchez–García, 2015). Abrahamson and Sánchez–García (2016) demonstrated the utility of the construct, which originated in sports science, in the context of mathematics educational research. Abrahamson et al. (2016) studied the role that visual attention plays in the emergence of new sensorimotor schemes underlying the concept of proportion. They overlaid data of participants' eye-movement patterns onto concurrent data of their hand-movements. They found that the participants' enactment of a new bimanual coordination coincided with a shift from unstructured gazing at salient figural contours to structured gazing at new *non*-salient figural features. The participants' speech and gesture confirmed that they had just constructed a new attentional anchor as mediating their control of the environment.

For this study, we adopted the construct of an attentional anchor as a key component of our methods. We sought to characterize what attentional anchors students developed during their attempts to solve a motor-action manipulation task. By so doing we hoped to gauge for effects of varying the contextuality of learning materials (iconic vs. stark) on student development of the sensorimotor scheme mediating an activity's learning goal. We hypothesized that richer manipulatives would constrain the scope of attentional anchors students develop.

Methods: Designing Constraints on Sensorimotor Engagement of a Technological System

The Mathematics Imagery Trainer for Proportion (MIT-P; see Figure 2) sets the empirical context for this study. Students working with the MIT-P are asked to move two cursors along vertical axes so as to make the screen green and keep it green. Unknown to the students, the screen will become green only if the cursors' respective heights along the screen relate by a particular ratio, otherwise the screen will be red. For instance, for a ratio of 1:2, the screen will be green only when the right hand is twice as high along the monitor as the left hand. Students develop a variety of motor-action strategies to satisfy the task demand (Howison et al., 2011).

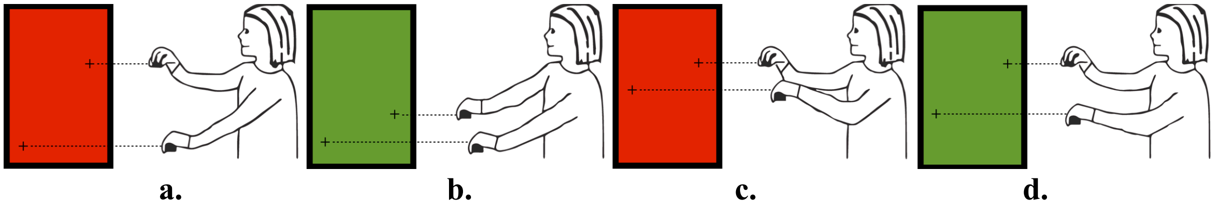

Figure 2. The Mathematical Imagery Trainer for Proportion (MIT-P) set at a 1:2 Ratio. Compare 2b and 2d to note the different vertical intervals between the hands and, correspondingly, the different vertical (or diagonal) intervals between the virtual objects. Noticing this difference is crucial to experiencing, then resolving a key cognitive conflict.

In the current study, images appear at students' fingertips when they touch the screen. These images are either stark crosshair targets (see Figure 3a) or iconic images (e.g., hot air balloons; Figure 3b).

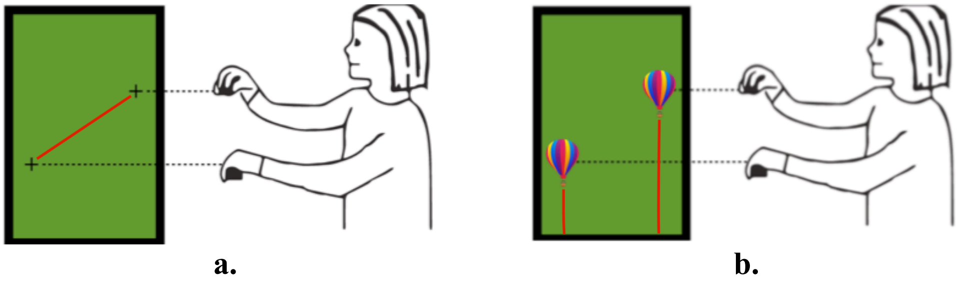

Figure 3: Experimental conditions and hypothesized attentional anchors: (a) stark crosshair targets cue the vertical or diagonal interval between the hands; and (b) iconic images (hot-air balloons) that cue the interval from each object to the bottom of the screen. In the actual experiments we used large touchscreens where the hands are on the interface.

In both experimental conditions (stark and iconic) students are led through a task-based semi-structured clinical interview. Following an unstructured orientation phase, in which the participants find several green locations, they are asked to maintain green while moving both hands from the bottom of the screen to the top. The interviewer and student then engage in a coordination challenge, where the interviewer manipulates the left image and the student manipulates the right image. The student is asked to predict the green locations. All along, the students are prompted to articulate rules for making the screen green.

We wished to investigate for attentional anchors that emerge during children's interactions with the technology. We reasoned that the attentional anchors would indicate what sensorimotor schemes the students developed. More specifically, we explored for an effect of experimental condition (stark vs. iconic cursors) on the types of attentional anchors students construct and articulate (via speech and/or gesture). We also looked at the effect of condition sequence.

Twenty-five Grade 4 – 6 students participated individually in the interviews, 14 in the "stark-then-iconic" condition and 11 in the "iconic-then-stark" condition. We exclusively interviewed students around the numerical item of a 1:2 ratio. These sessions were audio–video recorded for subsequent analysis. As our primary methodological approach, the laboratory researchers engaged in a micro-genetic analysis of selected episodes from the data corpus, focusing on the study participants' range of physical actions and multimodal utterance around the available media (Ferrara, 2014). Our working hypothesis, to iterate, was that the virtual objects' figural elements may cue (afford) particular sensorimotor orientations and thus "filter" the child's potential scope of interactions with the device. Namely, we analysed for effects of the manipulatives' perceived affordances on participants' scope of interaction.

Results: Perceived Affordances of Stark vs. Iconic Situations Mediate Student Strategies

A main effect was found. Below, we report our findings in each experimental condition by first describing participants' typical strategies and then illustrating these behaviors through brief vignettes. The section ends with comparing observed student strategies under the two conditions.

Stark Targets Afford the "Distance Between the Hands" Attentional Anchor

In the trials where participants interacted with stark targets first, they began the activity by placing their left-hand- and right-hand fingertips on a blank touchscreen. Immediately they noticed crosshairs appear at the locations of their fingertips. In an attempt to make the screen green, the participants began moving their hands all over the screen with no apparent strategy, "freezing" their fingers as soon as the screen turned green. Eventually, participants oriented toward the spatial interval between their fingers, soon discovering that their fingers have to be a certain distance from each other at different heights along the screen. Finally they determined a dynamical covariation between the interval's size and height: the higher the hands, the bigger the interval must be (and vice versa). We turn to several vignettes (all names are pseudonyms). As we shall see, both participants will refer to an imaginary diagonal line connecting the cursors.

Luke (age 10). As he found various green-generating screen location, Luke commented about the space between his hands at these various locations: "It's the same angle. Well, I mean the line connecting them is the same direction" [4:53]. Later, he noted that the "[angle] is changing because my right hand is getting faster, so when this goes up that much (moves left hand approximately 2 in. on the screen) this one goes up at this much (moves right hand approximately 4 in. on the screen)" [11:10].

Amy (age 9). Amy reported her observation: "The diagonal [between the hands] at the top is different than [at] the bottom" [7:15]. Then later during the iconic challenge, she said: "You have to make them different diagonally from each other to make it change color" [7:42].

Thus during the stark-target trials the participants not only noticed that the interval between their hands was changing in size, they came to see this interval as an imaginary line between their hands. In turn, this imaginary line—its size, angularity, and elevation along the screen—apparently served the participants in finding and keeping green, ultimately enabling them to articulate a strategy for doing so. This imaginary line along with attributed properties is an attentional anchor: it is crafted out of negative space to mediate the situated coordination of motor intentionality; subsequently this mentally constructed object serves to craft proto-proportional logico–mathematical propositions. This spontaneous appearance of a self-constraint that facilitated the enactment of a challenging motor-action coordination is in line with dynamical-systems theory (Kelso & Engstrøm, 2006).

Of the 14 students in this stark-then-iconic experimental condition, 10 spoke about the interval between the hands still within the "stark" phase of their interview, with eight of them referring explicitly to its magnitude. Then during the "iconic" phase of the interview, 2 of these 10 students began to speak about the icons as separate entities, focusing on the speed of each respective icon, or reverting to a focus on the color feedback of the screen to determine where to place the hands. The remaining 8 of these 10 students continued to use the interval line between their hands as a guide for making the screen green. These students' attention to the diagonal line was consistent, suggesting that this imaginary "steering wheel" had become perceptually stable in their sensorimotor engagement with this technological system.

Iconic Images Afford the "Distance From the Bottom" Attentional Anchor

Similar to the stark-then-iconic condition, in the trials where students interacted with rich icons first, they began the activity by placing their left-hand- and right-hand fingers on a blank touchscreen. However in this condition they immediately saw hot-air balloon icons (not stark targets) appear on the screen. Thus, the virtual manipulatives in this condition are situational and not stark, even as the tasks are otherwise identical. Recall that these students worked first with the iconic images and then with the stark images. As we will now explain, beginning with the iconic images cued a narrative-based strategy that was based on a frame of reference that did not attend to the interval between the images but instead to each of these hot-air balloons vertical distance above the "earth" (the bottom of the screen). As we will see, this alternative sensorimotor orientation was so strong that it carried over to the stark condition, so that by-and-large these participants were less likely to attend to the interval and thus were less likely to benefit from its potential contribution to their problem-solving strategy.

Leah (age 11). Having generated green for the first time, Leah noticed that when she moves one hand, the greenness dulls out toward red. Later, she described her strategy for making the screen green referring gesturally to the hand's distance from bottom of the screen: "I would say what I said before, where one hand chooses a place and the other hand chooses a color based on where the hand is, and you can adjust it to keep it green. Once you find that, you just need to keep it the same height [from the bottom]" [8:40]. Then in the next task, she maintains her strategy, saying: "When you move one hand up you need to move the other hand up so it's the same distance [from the bottom], but higher" [12:22].

Jake (age 11). Jake described his initial strategy: "Try putting your hands together in the middle and then try moving one down or the other one up. One of the balloons should stay in the middle while the other moves" [4:47]. Note how "middle" refers to that balloon's location along a vertical axis irrespective of the other balloon. Jake perseverates with this strategy throughout the set of challenges, moving his hands up along the screen sequentially rather than simultaneously. When later tasked to make the screen green with the stark targets, he appeared disoriented, noting, "This is harder because I don't have a starting point" [24:12]. Jake refers to the absence of an "earth" as a grounding frame of reference for the cursors' vertical motion.

Of the 11 students who encountered the rich images first, 4 began to speak about the interval between the hands still during the rich condition, however these students did not elaborate about the line between the hands, and rather focused on each hand as a separate entity (e.g., stating that one hand controls color and the other controls brightness). During the second phase, in which they encountered the stark images, 2 of these 4 students as well as 3 of the 7 who had not attended to the interval demonstrated the emergence of this attentional anchor. The remaining students treated each of the two icons as separate entities throughout the entire interview, and hardly spoke about the interval between the hands. Collectively, these students were more inclined to treat the two objects on the screen as separate entities, focusing on the changing height of each object and the differing speeds of the two objects as they move upward.

In summary, participants who began in the stark condition oriented toward the distance between their hands as their attentional anchor, whereas participants who began in the iconic condition tended to treat the manipulatives as independent, untethered entities. It would appear that participants who began in the stark condition generated the interval as their attentional anchor because no other frame of reference was cued. Participants who began in the iconic condition, on the other hand, followed the cued narrative implicit to the familiar images and tended rather to visualize the two balloons as launching up from the ground.

It thus appears that objects bearing rich associative content introduce a new layer of baggage onto an interaction task, including forms, dynamics, hierarchies, and social conventions that guide the students' perception of the action space (on framing, see Fillmore, 1968; Fillmore & Atkins, 1992). For instance, we typically think of hot-air balloons as "starting" at a point, such as the ground at takeoff, and these evoked frames implicitly constrain the scope of possible attentional orientations to a situation by privileging the interval from each object down to the bottom of the screen, at the expense of the interval between the hands. In contrast, when manipulating stark cursors, there is no "starting point" as such, enabling the possibility for students to attend to the interval between the hands.

Supporting our study's hypothesis, the findings suggest an effect of situatedness on the construction of sensorimotor schemes. The finding is relevant to mathematics pedagogy, because sensorimotor schemes mediate conceptual learning. It follows that situatedness of instructional materials is liable to impede mathematical learning by precluding the emergence of sensorimotor schemes pertinent to a cognitive sequence toward the generalization of rules. Future iterations of this intervention would avail of eye-tracking (e.g., Abrahamson et al, 2016) to corroborate students' verbal and deictic report of attentional anchors.

Conclusion

Mathematics education researchers have long debated the question of whether concepts best develop from rich or stark learning materials. We contributed to the debate by offering that the focus of such research should be not on the learning materials per se but on the sensorimotor schemes they may afford (cf. Day, Motz, & Goldstone, 2015, for a competing position). Richer materials, we demonstrated, constrain the scope of sensorimotor schemes students may develop through engaging with the materials. In particular, richer materials may diminish opportunities for conceptual development, because they draw students' attention toward less mathematically relevant ways of thinking about the situations. Students might even miss out on opportunities to think about the situation in ways that are critical for an instructional sequence.

Students, that is to say children, are highly imaginative. They readily engage in pretense with stark objects, visualizing them one way and then another way. It is the *low* situativity of stark manipulatives that lends them to a greater variety of narratives and consequently a greater variety of sensorimotor orientations. And so we agree with Uttal, Scudder, and DeLoache (1997) that sensory richness of manipulatives may derail certain forms of mathematics learning. But we stress that the issue here is not so much about sensory overload distracting from intended forms of engaging the objects. It is not about manipulatives but about *manipulation*—it is about task-oriented sensorimotor schemes students should develop in solving challenging bimanual motor-action problems. So the issue at hand is the hands' motion. Learning is moving in new ways, and we should ensure that the tasks we create facilitate this motion. Sometimes the objects children manipulate might be so perceptually stark that there are no objects at all—just imagined objects. One might speak of mathematics students' right to bare arms.

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