The SAGE Encyclopedia of Out-of-School Learning

Embodiment and Mathematics Learning

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Developed in intellectual disciplines as diverse as philosophy, linguistics, robotics, kinesiology, and cognitive psychology, embodiment is a relatively new paradigm for the field of learning sciences. This entry discusses the theory of embodiment, focusing on how the theory is informing new directions of research and pedagogy in the particular domain of mathematics education. More specifically, the entry addresses an enduring research problem in the learning sciences pertaining to the role of embodied action in the learning and teaching of mathematical concepts.

Theory

Brains are material organs. Minds are another matter. The mind is grounded in the brain but extends beyond it to the body, including our hands, and beyond the body to encompass anything we work and think with—media such as pencil and paper, computational devices such as a calculator, tools and instruments such as lathe or clarinet, procedural forms such as a recipe for bran muffins or an algorithm for quadratic equations, and even other people who collaborate with us in getting things done, whether or not these people are copresent in location or time. Language itself extends the mind, equipping and shaping it with civilization's legacy. All these physical, cultural, and human resources collectively participate in facilitating the enactment of complex social activities, such as the mundane cultural practices of design, manufacture, and logistics. In a sense, any human thought or action is distributed beyond our corporeal self and situated in the world, even if we close our eyes and do not move, because we are then simulating our skilled performances with numerous forms we have internalized so as to "relive" our experiences, reflect on them, and plan future actions.

Take counting, for example. We can count sheep with our eyes shut, but then again the vocabulary of counting, the procedure of counting, and even the very idea of counting originated from action in the world with other people. In fact, scanning our brain as we count sheep would show the same areas lighting up as when we see real sheep, voice the counting words, and perhaps gesture toward the sheep. The very same cerebral faculties operate whether we are seeing or imagining, and this neural overlap is near complete under hypnosis. Whether we perceive real or imaginary objects, all of these are the mind's constructions—in either case, we can perceive only what we know.

From an evolutionary perspective, imagination sprouted from sensorimotor cerebral faculties—short of enacting external motion, the cognitive activity of imagining coopts the sensorimotor neural system. Language, or more generally multimodal communication that includes gesture, expands imagination into the social sphere, enabling multiple agents to confer by imagining together with sufficient overlaps of reference.

Still, one might object that imagination is all in the brain—imagining is perhaps simulating worldly experience, but nevertheless, it is only in the head. But say we are counting objects on our fingers. We are using our own bodily material—a set of 10 discrete extremities—to facilitate the execution of a task. The fingers serve as a medium for encoding quantitative information gleaned from observation (of actual or imagined objects). As we unfurl (or furl) one finger per object, following a familiar order, we do not need to think about numbers. We could do that later when we have tagged all the relevant objects in the perceptual field and are ready to read off the "how many" information from our hands. Meanwhile, the fingers off-load and store this content and thus free up cognitive resources. This is why we say that the fingers extend our mind—it is as though we have an extra canvas or buffer—only that this particular embodied form happens to be not in our working memory but a physical material serving as an epistemic instrument.

As we count, we do not experience a partition between in and out of the head. More generally, we operate systemically, optimizing the distribution of cognitive functions by availing suitable tools within our reach in accord with our skills. When we run out of fingers, we might jot down some hash marks on paper, now freeing up our fingers to help in monitoring the count. We might now even temporarily suspend the count, say, to make a phone call, without having to restart later. Unlike fingers, hash marks are more stable and permanent, and these inscribed notations readily extend beyond 10 units. We can use marks on paper or another medium to expand our quantification activity into the base-10 place value system. Broadly speaking, sign systems have different qualities, and so we use them selectively to support our epistemic activity. But even symbolic notations present themselves to our sensorimotor system as objects in the world. We perceive and act on symbols, such as when we manipulate algebraic variables, moving them across the two sides of an equation.

And so we say that the mind extends beyond the body. When we think of artifacts that extend our mind, one useful classification is instruments that enhance perception (the various artifacts whose names end with “scope”) and others that enhance action (from syringes to monster trucks). However, whereas this perception/action classification bears certain technological validity, it may not quite capture how humans engage these artifacts and may not bear functional validity.

We usually think of seeing and doing as different actions. Seeing—or perceiving, more generally—seems to be passive, as though the world impresses itself on us via our sensory organs and then the brain interprets those electric signals to make sense of the world. Doing—or acting, more generally—seems active, as though our limbs externalize our intentionality into the world. And so we usually think of perception and action as, respectively, input and output, with the brain sandwiched between them, churning information from the sensors and then issuing commands to the actuators.

This facile conceptualization of the human mind as a computer, with the brain processing symbolic propositions in between perceptual input and action output, was the dominant model of cognition for the past half century. But it appears to be too facile. First, neuroimaging has never found traces for symbolic propositions but instead has discovered that both perception and action faculties are active during reading, writing, speaking, and thinking. Second, a paradigm shift to systemic views of human behavior, including the philosophies of phenomenology and enactivism, now models action and perception as functionally irreducible, with each serving the other—we move our body so as to bring it to positions that optimize perception, even as, complementarily, we perceive the world so as to guide our actions. These mutually supportive perception–action behaviors are coordinated, complexly interdependent, and interadjusting in highly frequent iterated loops. When artifacts extend the mind, it is via new perception–action integrations between body, artifacts, and media on which they operate. These integrations coevolve throughout the life span into a level of dexterity concordant with the demands of survival and practice.

At this point, the reader might concede that human reasoning is embodied, situated, distributed, and extended. Still, the reader may hold onto the last bastion of reasoning, which is drawing inferences from thought processes. Surely inference is a higher order cognitive operation! Well, perhaps it is higher, but the action is lower.

We decide with our bodies. As we mull over choices for what to do, eat, or buy, our central neural system receives somatic feedback from simulating the choice. Ask yourself if you would rather listen right now to jazz, hip-hop, or classical music. Have you made your decision? Well, you might believe that you made a rational choice given your knowledge about your current psychological state. But if you are at least vaguely partial to all three musical options, in fact you probably simulated each of the experiences, felt your embodied responses, and leaned toward that which felt best, at which point your conscious ego arrogated to itself the whole decision-making process. Later today, you might feel like something else. And your choice of accompanying beverage will probably vary accordingly, again via somatic feedback.

But how do we know what someone else is thinking? By embodying them. Neurotypical humans are equipped with a mimetic sensorimotor system, the mirror neurons, by which we tacitly empathize with fellow humans and primates. This intersubjective mechanism is ever more effective when we know the others' goals—what they are trying to do, how they are trying to do it, and what they are attending to. This other knowing is helpful in supporting those we care for, predicting people's behaviors, and learning by imitation.

Application

Educational researchers who have been inspired by embodiment theories are interested in creating empirical contexts in which they can apply, evaluate, and elaborate on these theories, also bearing in mind traditional pedagogical acumen. The idea is to design interactive learning activities that are geared to recruit children's natural ways of engaging the world—that is, by actively perceiving and acting on it. The rationale is that children can learn curricular content by first solving a hands-on, or perhaps "body-on," problem and only later formalizing their solution using vocabulary, diagrams, symbols, and other forms of reference that shift their understanding into professional ways of talking and thinking. So first students struggle to solve a problem in a specially designed learning environment, and then they model their natural solution according to the target content domain. For example, students in an astronomy lesson might learn about gravitational pull by acting out the motion of a meteor caught in a planet's gravitational field. By literally running after an image of a meteor projected on the floor, the students learn to anticipate the curvature of the meteor's trajectory as it passes by a planet, and the larger the planet, the greater the curvature. Later, the students can reinterpret the bent orbit they have acted out in terms of curricular notions such as force, acceleration, velocity, and distance as well as in symbolic notation such as algebraic propositions. The students thus learn what these formal expressions are about because they have experienced these notions firsthand ("firstbody"). The notions and symbols will be not abstract tokens of schoolwork but references to actual personal experiences shared within the classroom forum. Thus, the science will make sense to students or, rather, students will make sense of the science. Moreover, they will appreciate the veracity of the formulaic propositions, because their personal physical experience will provide compelling validity.

Whereas embodied learning activities extend science inquiry, what would it mean to engage in mathematics inquiry? After all, science is about actual concrete phenomena in the world, and so it makes sense to create real or virtual laboratories for studying these phenomena. But what would possibly be mathematical phenomena that one could study? Is not mathematics about abstract ideas, explicitly disembodied? What would embodied mathematics activities be about? What would it mean to embody a mathematical idea?

From the perspective of embodiment theories, all reasoning is about objects, even if these objects are intangible, imaginary, complex, and even impossible. What counts is that the student experiences the ideas as objects. Ethnographic studies of professional mathematicians have revealed that most of them consciously perceive the most complex ideas as things. And they use their hands to make these things present to themselves and to others. One prominent mathematician, Terence Tao, has even described rolling on the floor as he worked on a problem involving waves rotating on top of one another.

Consider the idea of addition, the most basic arithmetic operation. If you ask people to explain the meaning of this word, for example, to an adult who speaks another language, they will often gesture a bringing together of two groups of stuff. These gestures are thought to reveal the tacit dynamical image schemes that underlie people's understanding of the mathematical idea in question. Some researchers are concerned that mainstream mathematics education does not create opportunities for students to develop compatible embodied meanings for more advanced concepts, beginning with fractions. Other researchers are seeking to respond to this problem of "disembodied mathematics" by designing learning environments for students to develop dynamical image schemes for challenging concepts. This form of pedagogical research-and-development work has been called "embodied design," because it is both informed by and contributes to embodiment literature.

Embodied design is in dialogue with embodied interaction, a philosophy of human–computer interactions that foregrounds the user's physical activity as the designer's focus of attention. Embodied design was formulated with the vision of enabling teachers to make present in the classroom their own dynamical image schemes for complex mathematical concepts, such as ratio and proportion. The rationale is that teachers' conceptualizations of mathematical notions often derive from rich experiential resources that are difficult to import, concretize, simulate, and quantify, and therefore, teachers convey mathematical concepts to students using only "low-tech" resources and naturalistic multimodal discourse.

For example, one meaning of the mathematical concept of proportion foregrounds specifically a relation of equivalence—that is, recognizing that two expressions (e.g., 2:3 = 4:6) denote different pairs of quantities that nevertheless are somehow "the same." This notion of proportionality as sameness is rooted in mundane cultural practices as the identity of two perceptual sensations experienced in the mixing or matching of two different quantity pairs, such as the identical green color we get from mixing either two cups of blue paint and three cups of yellow paint or four cups of blue paint and six cups of yellow paint, or the identical flavor from mixing two cups of lime cordial and three cups of water as compared with mixing four cups of lime cordial and six cups of water. In these examples, we think of proportion as a sensuous identity of two situated, equivalent quotients, for example, 2/3 and 4/6.

Note how this experiential notion of proportion as equal sensations is different from a procedural conceptualization of proportion, prevalent in school curriculum, as the paired running totals of two rates, as in the situation of the increasing fortunes of Robin (earning $2/day) and Tim (earning $3/day)—that is, 2:3 = 4:6 = 6:9 = …. In this Robin–Tim case, the paired accumulations do not foreground the constant multiplicative relation between the two numbers within each ratio, such as the quotient (2/3), and therefore cannot offer a sense of equivalence between two ratios. Students might learn how to generate and evaluate a string of number pairs that, they are told, are "proportionally equivalent," and yet not experience what is equivalent to what or how these things are equivalent. They do not develop an embodied dynamical image scheme of proportionality as a sensation that remains constant even as quantities change.

To help students develop an image scheme of proportion, Dor Abrahamson and researchers with the UC Berkeley Embodied Design Research Laboratory developed the Mathematical

Imagery Trainer for Proportion. This educational resource is a technologically enabled interactive learning environment designed to create for students opportunities to discover, practice, and formalize dynamical image schemes pertaining to the mathematical concept of proportion. The Mathematical Imagery Trainer sets interaction problems between students and a system they initially do not understand. This activity creates opportunities for students to learn how to move in a new way so as to operate the system. This new way of moving is oriented toward operating elements or aspects of the system called attentional anchors. An attentional anchor, a construct coming from sports science, is a concrete or imaginary aspect of the perceptual field that you invent and focus on while you are learning a sensorimotor skill—it is your emergent psychological interface with the world. But doing so demands a new coordination of actions that are focused on the new attentional anchor. So students working with the Mathematical Imagery Trainer must discover appropriate attentional anchors even as they develop appropriate schemes for operating these attentional anchors as a solution to the problem. The attentional anchor then evolves into a mathematical concept when a mathematical frame of reference is interpolated into the perceptual field.

Children working with the Mathematical Imagery Trainer for Proportion first learn to operate the system with their hands by inventing the interval between their hands as an attentional anchor. They begin noticing that coordinated variation in both the interval's size and the elevation in space are correlated with effecting the system's desirable state (making the monitor screen green). The interval then evolves into proportionality when a Cartesian grid is overlaid onto the screen. The students can now quantify the interval between their hands and revisualize variations in its size and elevation in terms of two hands each moving at its own constant pace. They next come to realize the constant multiplicative relation between the two hands, such as the right hand always being double as high along the screen as the left hand. It is so by discovering and cultivating an attentional anchor that sensorimotor schemes are extended into cultural concepts.

Embodied design has been applied to additional mathematical concepts. For example, a Mathematical Imagery Trainer has been developed in which young students "reinvent" the Cartesian space. Here, too, the task involves the simultaneous manipulation of two virtual objects. A tablet application enables users to move one cursor up and down along a vertical axis on the left side of the screen (the *y* axis to be) while moving the other cursor right and left along a horizontal axis on the bottom of the screen (the *x* axis to be). Once again, the task objective is to make the screen green and keep it green while moving both hands. In empirical studies, using eye-gaze tracking technology, an attentional anchor was repeatedly observed evolving across multiple students at the Cartesian point that carries information about the location of both the left and the right hands. That is, the attentional anchor emerged at a moving point on the screen that was across from the left hand and above the right hand. For example, if a grid were to be laid over the screen and the left hand was at 3 units up along the *y* axis, while the right hand was 5 units along the *x* axis, then the attentional anchor would be at [5, 3]. As in the earlier case of parallel motion, in this case of orthogonal motion, the attentional anchor sprouted into being where no explicit visual stimuli were available. The attentional anchor sprouted to serve as an embodied solution to a bimanual motor–action coordination problem within the student–task–environment dynamical system.

Further embodied design is currently under way for geometrical transformation functions. In one activity, the student traces a shape on the screen (left hand) while marking its reflection across a line of symmetry (right hand). It is conjectured that mathematical functions can be grounded in bimanual motor–action coordination by revisualizing the hands' respective locations as embodying domain–codomain relations—that is, with the left hand embodying *x*, while the right hand embodies *f*(*x*).

One advantage of embodied-design technological learning environments is that the teacher can literally see how students are reasoning, because the students' physical actions externalize their reasoning process. The teacher guiding this work acts like a sports coach by giving the students feedback on how they are moving, molding the students' physical movements hands on and suggesting productive ways of operating the technology. At the same time, the technological system can be set up to log the students' physical actions, and these logs can be processed automatically to assess the students' learning trajectory and achievement. These algorithms come from fields of research called educational data mining, learning analytics, and machine learning. In turn, the automatization of assessment means that a virtual agent (avatar) could respond to the students' actions with formative feedback that supports the students in accomplishing the task and making sense of their solution. These activities can be either informal games the child elects to play or part of the child's homework assignments. The technology can also be set up so that the activity logs are sent in real time via the Internet/cloud to a central processing unit that interfaces with various stakeholders, such as the teacher, district, researchers, and game developers. As such, advances in technology are creating hybrid practices that are at once out of yet in school.

Even as interactive learning environments are creating opportunities for students to learn curricular content, they are creating opportunities for researchers to rethink science, technology, engineering, and mathematics (STEM) education. In particular, learning mathematical concepts begins with learning to move in new ways, and yet most educational researchers neither know much about how people learn to move in new ways nor know how teachers would coach students to move in new ways. For this reason, some researchers have become increasingly interested in out-of-school pedagogical traditions related both to vocational and recreational practices centered on physical action—for example, carpentry, dance, and martial arts. In particular, emerging dialogue between STEM and sports education researchers is fostering intellectual innovation at the intersection of cognitive and kinesiological perspectives on embodiment theory.

Embodiment theory is a young yet fast growing area of research in the learning sciences. Its application to the design of learning environments should both benefit STEM students and, reciprocally, create empirical contexts in which to further develop the theory. In particular, embodied design is enabling researchers to formulate new insights on the emergence of STEM concepts from embodied interaction.

See alsoConstructivist Learning; Learning Sciences; STEM Learning; Tangibles and Tangible Learning; Technology-Mediated Learning Environments

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Further Readings

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