

Using Mathematics and Computational Thinking

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A group of high school students is conducting a study of air quality in their city. They are interested in mapping where air quality is better and worse, finding out how it is changing over time, and identifying potential causes for poor air quality.

Before they start collecting data, the students work with their instructor and science experts to decide what information they need to answer their questions. They choose to focus on areas around the public transit system, and carry GPS units and particulate matter sensors in their backpacks as they travel to key locations. During their visits, they take observational field notes and photographs. As they begin to notice patterns and factors that might influence air quality, the students note features of each location: whether it is indoor or outdoor, coastal or inland, suburban or urban. They begin to more systematically take air quality measurements, ensuring that sensors are positioned both inside train cars and on train platforms for pre-defined periods of time. They begin to develop more systematic ways to keep track of observations they make in the field, developing systems to describe how crowded subway trains were, or how intensely train cars and stations smelled, at different points during the day.



Figure 1. Students collecting observational data (left). Notes can be scanned, coded, and analyzed using a software kit called *Local Ground* (right).

During these investigations, the students use a mobile mapping technology called *Local Ground* to organize their measurements, notes, and observations. The tool allows users to make sketches and notes on paper maps, scan, and compile them to create a collection of annotations on a virtual, shared map. Students also take photos, and make audio recordings using their own mobile devices, which they can then link to the air quality measurements they have collected with the sensors in their backpacks. They look for patterns in data collected for different parts of the city and times of day, and begin to develop codes measurements for qualitative observations, such as notes about train car smell and crowdedness, so that those observations become comparable and replicable. The students also begin to hypothesize about causal relationships: for example, whether air quality near a power plant is worse than in other areas.

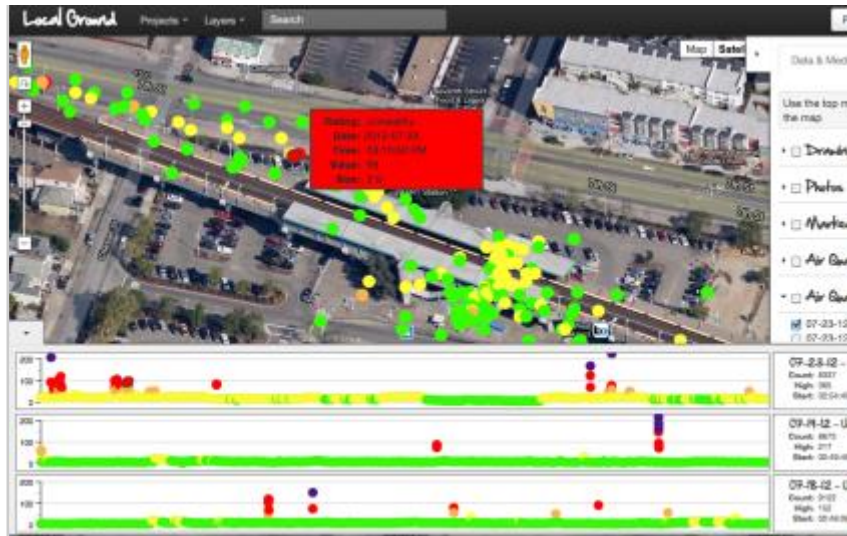


Figure 2. Using computational tools and GIS technology to visualize data across time and space.

By finding ways to organize their data and turn observations into measurable quantities, the students then conduct a more systematic inquiry. They compare air quality measurements, features of the trains, platforms, and traffic patterns at stations they explored, and their observations in the field. Across these data, the students notice a dramatic difference in air quality between underground and above-ground stations. Even when taking other factors into account such as time of day, number of cars on the trains, and other sources of pollution such as freeways and power plants, underground stations consistently produced more hazardous levels of air quality than their outdoor counterparts.

Students also conducted more focused investigations to learn about the source of air particulate in train cars and stations. One thing they found was that air particulate levels fluctuated dramatically *inside* trains each time they stopped. Noticing this pattern, students began to hold sensors near seats that they hit deliberately. They used this evidence to argue that air quality fluctuations inside trains were in part caused by particulate trapped in padded cloth seats, released by passengers as they sat and rose. The students presented their findings to the local port authority and transit authority, and presented their work at a national research conference. They made recommendations for addressing the problems they found, including for providing better ventilation for underground stations, and cleaning or changing the seating materials inside of subway cars.

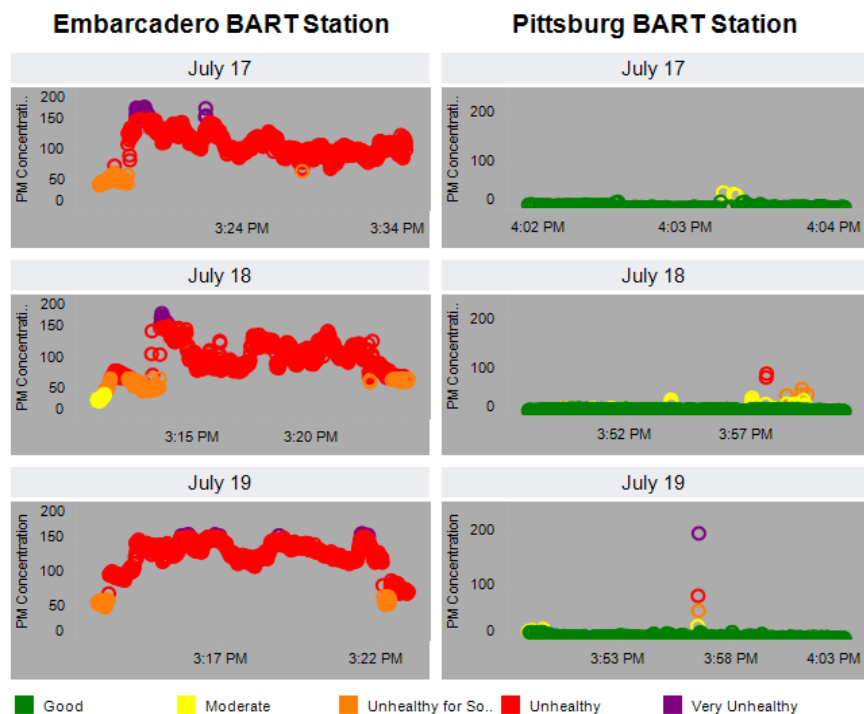


Figure 3. Using data plots across study sites to support a scientific argument.

This vignette is based on an actual investigation done by high school students. It is connected to performance expectations where students use mathematics and computational tools to explore the influence of human activity on Earth systems (HS-ESS3-6), and to propose and test solutions to complex problems that involve many factors (HS-ETS1-4). It would be hard to imagine how these students would have been able to do such an investigation *without* using mathematics and computational thinking. Mobile technology helped them collect qualitative and quantitative data that were distributed, across space, time, and multiple investigators. It helped them consolidate and visualize dramatically different types of information (transit, air quality, cleanliness, behavior on train cars). Mathematical tools and representations allowed the students to systematize and quantify their observations, and find and compare the patterns they noticed. More recently, a 4th grade class has been using *Local Ground* to organize their investigation of soil-based ecosystems by developing measures and mapping relationships between biotic (insects, plants, worms), and abiotic (moisture, compactness) indicators around their school.

1. Why is using mathematics and computational thinking important?

Often when we begin to explore a natural event or think about solutions to a problem, we are not just interested in describing *what* is happening. We are also interested in *how much*, *how fast*, or *how frequently* something has happened, and how it may happen in the future or in a different circumstance. Mathematics and computational thinking offer ways to do this by allowing scientists and engineers to build precise, predictive models, to conduct investigations, and to analyze data in new ways.

Using mathematics and computational thinking is all about finding precise ways to describe the patterns and processes that make up scientific and engineered systems. Calculus was originally

developed to study and better understand the motion of planets. Biologists have been able to explore new population models because advances in statistics and computation allow them to work with larger and more complex types of data. In the environmental sciences, researchers use GIS (geographic information systems) and sophisticated sensors to model global climate trends and ecosystem dynamics.

In the same way that becoming proficient in a language involves learning how to both *read* and *write*, using mathematics and computational thinking is about learning how to both *apply* formulae and tools that already exist, and to *create* new ones. In this chapter, we'll show how building, revising, and testing mathematical and computational representations can help students express and refine their models of what relationships and mechanisms are important in a system. Mathematics and computation can provide students with special ways to propose and test scientific relationships, and to make predictions. By exploring how mathematics and computation represent models and help them become more precise, students can begin to understand how even the most complex mathematical formulae or computer simulations are fundamentally connected to observations, experiences, and ideas about the world around us.

Building Testable, Predictive Representations of Models

Both mathematics and computational thinking are ways to represent models using precise, shared languages that can be interpreted without having the original creator around. Mathematics is sometimes described as a “universal language” that focuses on quantities and relationships. Similarly, computational thinking is about decomposing and describing processes using specific rules, to program or “teach” the model to a computer so that we can explore what happens over time or in new conditions.

For example, with just a few mathematical symbols, the equation $p=mv$ (momentum = mass * velocity) can say a lot about how physical objects behave. It describes things even young children intuitively know: It is much harder to stop a heavy object from moving than a light one. This is represented by the direct relationship between p and m ; if one side of the equation increases, so does the other. It also represents a model that can explain surprising or unexpected outcomes. If a heavy ball is rolling very slowly toward an object and a light ball is rolling very quickly, the light ball may still create more of an effect on the object than the heavier ball. This is because if velocity v gets smaller as mass m stays the same, momentum p decreases. If velocity gets bigger as mass gets smaller, the total momentum may increase, remain the same, or even get smaller depending on *how much* each of these parts of the system change relative to each other.

Using mathematics to describe relationships precisely is useful both for understanding scientific systems, and for making engineering decisions. If we want to design a machine that can pitch balls of *different* mass so that they have the same momentum, we can find out exactly how much to change the pitch velocity to balance the change in mass. Depending on the mathematical tools students have at their disposal, students can express the very same mathematical relationships in different ways. Elementary students who have not yet been formally taught algebra can create data tables or plot the outcomes of experiments using marbles of different mass to explore mathematical relationships. High school students able to work with vectors can use the same equation $\mathbf{p}=m\mathbf{v}$ to explore the motion and interaction of objects in 2D and 3D space.

Even with all of this power, mathematical models like $p=mv$ always only tell part of the story. In a given situation, friction, air resistance, or the elasticity of materials might affect how a system or machine works. In this way, every mathematical model – like scientific models in

general – is incomplete. Depending on the questions someone has about a system, mathematical models may need to be extended, or new ones may need to be developed.

Similarly, computational representations like simulations or data visualizations can allow scientists and engineers to express and test models in powerful and precise ways. These descriptions might include rules for actions like movement, cause-effect relationships between parts of a system, or provide specific calculations or organizations of data.

Figure 4 shows two simulations of a simple predator-prey system. Both can be used to explore ecosystem dynamics, but each emphasizes different aspects of the system. The simulation on the left focuses on relationships between various rates and quantities in the system. Using that simulation, someone can quickly explore how combinations of different birth, death, and predation rates affect overall population patterns. The screenshot on the right focuses on how individual predators and prey interact over time: when a predator comes into contact with a prey, the prey is eaten and the predator gains energy and reproduces. This emphasizes the specific conditions (predation, energy transfer, density) that create a scientific pattern. It also reflects how stable or unstable a system might be, and illustrates how small changes to behavior (such as whether and how a predator might chase prey) can change what happens to the system as a whole.

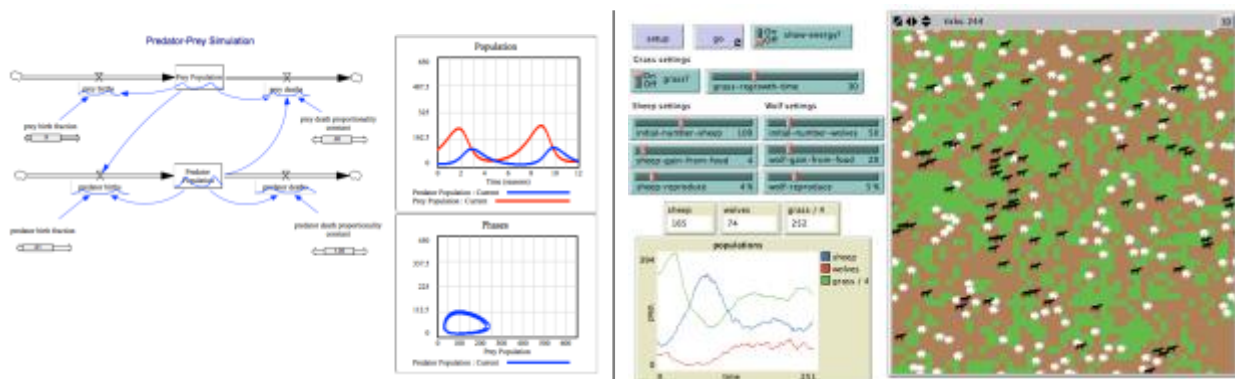


Figure 4. Computational models of predator-prey dynamics. Left, a systems dynamics model in Vensim. Right, an agent-based model of predator-prey dynamics in NetLogo.

Although computational models can exist in many different forms, one thing they have in common is that they can free people from repetitive, computationally intensive parts of scientific and engineering tasks. For example, representing some models of systems requires people to do many difficult calculations, or to imagine every possible interaction between many objects such as molecules or cells. Computational models do this *for* us, so that we can spend more time making sense of the results of these relationships and interactions. In fact, scientists’ understandings of large systems such as insect colonies, or engineers’ understandings of the stability and failure of power grids would be almost impossible without computational models.

Conducting Investigations

Mathematics and computational thinking also provide new ways to collect and analyze data. They allow people to explore and test otherwise unobservable aspects of scientific systems. For example, epidemiologists have found ways to predict the spread of diseases such as the common cold by measuring and analyzing how frequently people use certain words in their social media, and investigating how those words spread through a network. Computer simulations allow

scientists and engineers to explore “what if” scenarios and conduct virtual experiments that may be impossible in real life, and compare solutions to large-scale and even global problems.

Similarly, mathematics and computational thinking can help students collect, organize, and make sense of data. Simple tools like spreadsheets allow students to organize and visualize data easily, and publicly available data is increasingly accessible on the internet and through educational software tools. Digital probes and sensors can be connected to computers so that students can easily collect detailed data, and GIS allow students to explore large-scale or global environmental patterns. More advanced data analysis and visualization tools allow people to uncover hidden and unexpected patterns and relationships deserving of further study.

2. What is using mathematics and computational thinking all about?

Many people remember using mathematics in science class by plugging numbers into an equation, or using particular formulae to solve science-related word problems. Similarly, they might think about using computer simulations as a way to view and interact with scientific processes that are too small, complicated, fast or slow, or otherwise difficult to observe or interact with in real life. These applications are useful, but using math and computational thinking is not just about getting answers or seeing what can’t be seen. Instead, it is a collection of ways that learners can theorize, test, and refine their own scientific understandings about the patterns, relationships, and processes they notice in the world.

When we focus on mathematics and computational thinking as ways to describe, make comparisons between, and test predictions about systems in the world, then it is easy to see how students are naturally inclined to think mathematically and computationally. As soon as a child says something like “the heavier car will hit the wall harder than the light car”, or “the smaller ice cube will melt faster than the big one”, then they are on their way to thinking mathematically—in terms of relationships—about how specific quantities (weight, impact; size, time) might be related. In fact, much of Piaget’s work explored exactly this—how young children’s intuitive senses of quantity developed into more robust mathematical understandings (Piaget, 1946/1970). Similarly, as soon as a child begins to describe instructions like “every time a predator catches a prey they gain energy to live for a few more months”, play Simon says, or makes plans like “my robot’s motor should turn faster when there is more light”, they are thinking computationally about rules and ways that components of a system interact with each other.

Using mathematics and computational thinking provides tools and techniques to turn these early noticings into powerful ways of thinking about, representing, and testing scientific and engineered systems. Different branches of mathematics provide tools for thinking about different things, such as shape (geometry) or change (calculus). The same is true for computation—each programming language is “...like a natural, human language in that it favors certain metaphors, images, and ways of thinking” (Papert, 1980). Together, they offer a myriad of ways to organize students’ thinking about:

- What are a system’s **key components**: the matter, energy, organisms, materials, or other parts that together make up a system or design? Examples: atoms and molecules in chemical interactions; mass, force, and energy in a physical system; predator and prey levels, birth rates, death rates, and predation rates in a population dynamic system.

- What are the important **features or properties** of those components that might influence how the system works? Examples: their size, patterns of movement, population levels, or density.
- What are **relationships** between different parts and properties of a system? Examples: Do changes in one component always cause changes in another? Do components change over time? What is the “shape” of change: linear, exponential or “S” shaped?
- How can properties and relationships be **mathematically or computationally described** in a way that allows for meaningful comparisons across situations or object types?
- How do components and relationships **interact and combine** to create patterns? Examples: What happens to components over time? Are there chains of cause and effect that might explain surprising or unexpected results?

There are some specific mathematical formulae and computer tools that are especially important in science and engineering — such as recognizing and using $F=ma$, or using spreadsheets to organize and visualize data. However, the practice of using mathematics and computational thinking is about having access to a toolkit of resources that help students make sense and make progress. Arithmetic, algebra, geometry, calculus and statistics are all parts of this toolkit. So are simulations, data analysis tools, programming environments, spreadsheets, and probes and sensors. Later in this chapter, we review a few strategies and resources for building an accessible math and computing toolkit in your classroom.

Even though here we present mathematics and computational thinking separately, they are often interconnected. Computational tools can be used to collect, organize, visualize, and analyze data, and computational simulations often generate quantitative data and graphs as part of their output. Similarly, some computational simulations are programmed using mathematical expressions, and many mathematical analysis tools and statistical analysis apps are special kinds of computer programming languages.

Mathematics

Using mathematics is about describing a system quantitatively. This requires students to search for and take note of patterns and trends that might be important for better understanding or building a system. It also involves finding ways to measure those patterns, and express measurements as quantities in units that different people can use to compare their results to one another consistently. Sometimes these measurements are straightforward, like measuring the height of plants using a ruler or using a scale to find mass. Other times they can be more complex, like calculating the density of a substance using measurements of mass and volume, or developing a new measure that students decide is important (like the ‘smelliness’ of trains or ‘dryness’ of soil in the opening vignette).

Once patterns and quantities are identified, students need to describe the relationships between them using the language and symbols of mathematics. Arithmetic, algebra, and geometry are all ways to describe and predict how quantities are connected through physical or causal relationships. Combining objects or substances of like units can be expressed using arithmetic addition. Algebra or graphs can be used to describe a proportional relationship in which an increase in one variable corresponds to an increase or decrease in another. As more data are collected, students can test or adjust the mathematical relationships they describe. For

example, students may notice that a relationship they first believed was linear – such as the growth of a population – is actually speeding up and might be better described as exponential.

These aspects of using mathematics do not necessarily happen in order. For example, while there are many long-established ways of taking measurements, new measuring devices or techniques are developed all the time even for well-known systems. Or, a mathematical model might be developed based on theoretical conjectures about physical or causal relationships rather than measures from data, to be validated against empirical data later. Using mathematics is a process of working within and constantly building connections across these aspects, rather than completing each as a linear process.

Computational Thinking

While mathematics focuses on quantities, computational thinking focuses on processes. Students engaged in the practice of computational thinking break a complex problem or process up into smaller steps in order to better understand, describe, or explain it. It involves thinking about how computer tools and algorithms – specific instructions for how something should be done – can be used to make jobs like data collection and analysis or theory testing easier, more manageable, or more powerful.

One way that students can engage in computational thinking is by using existing computer models and simulations to explore, test, and better understand scientific and engineered systems. There are many freely available simulations and computer tools that can help students conduct explorations and collect and analyze data (we list a few in Section 7). Some allow students to conduct virtual experiments, while others allow students to visualize specific parts of systems or models of scientific phenomena. Figure 5 shows three different types of computer simulations designed for classroom use.

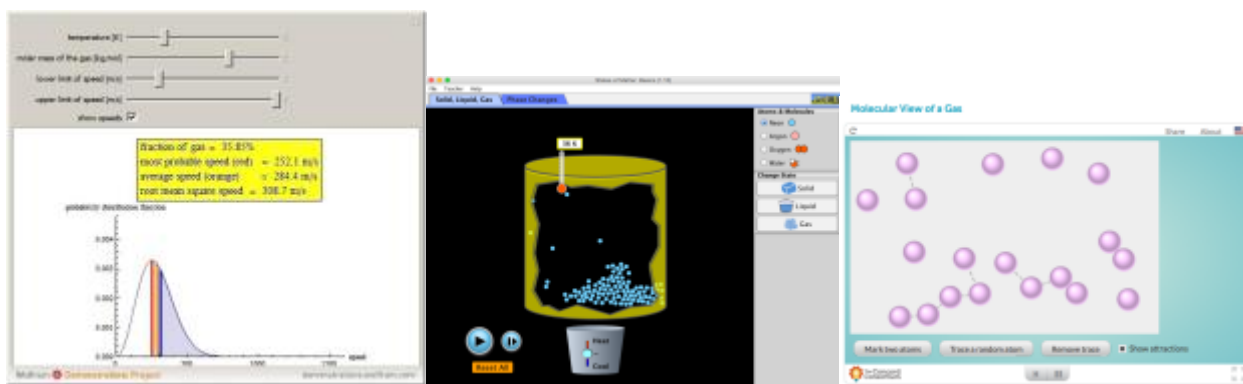


Figure 5. Computational models of gasses. From top left: Wolfram Demonstrations model of Maxwell-Boltzmann speed distribution. PhET Phase Change Simulator. Molecular Workbench gas simulation.

Like mathematics, computational thinking is not just about using tools that already exist, but also about building your own. Many computer environments exist that allow students at even the elementary level to create their own models of scientific systems. Usually, students do this by thinking about what objects in the system are important, and defining how those objects should behave using text computer code or visual instructions. As they add more objects to their system, they can think about the relationships and interactions between them, they can run their model to see how each piece interacts, and to test whether their models behave the way they expect. Figure 6 shows three different programs designed for students to use to create computational

models, each with a student-produced or student-modified model of molecules acting together as a gas. NetLogo (left) allows students to write models with text code. StageCast (center) allows students to physically “teach” objects how to move by dragging them on the screen (middle). Scratch (right) allows students to write programs by putting together puzzle blocks. NetLogo is often used at the middle and high school level; StageCast and Scratch were designed to be accessible to elementary students.

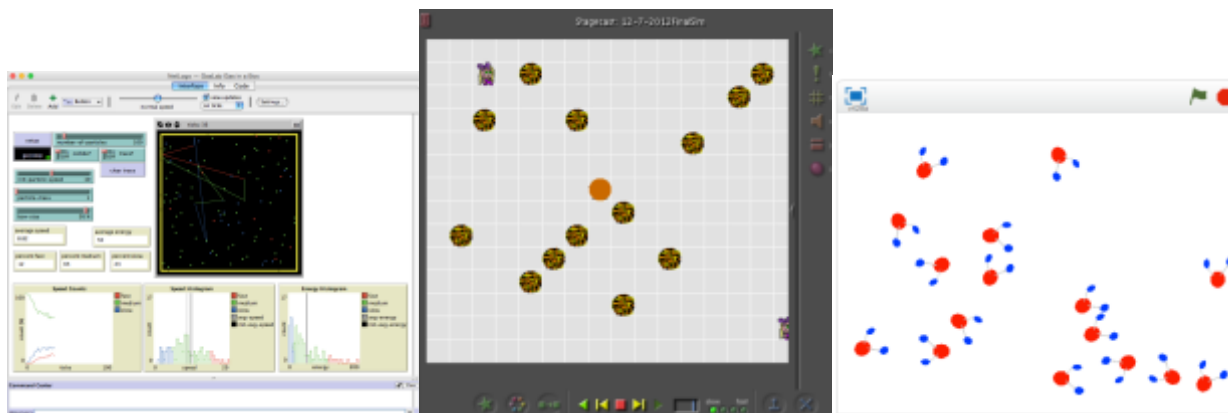


Figure 6. NetLogo simulation of Maxwell-Boltzmann distribution modified by a high school student. Stagecast Creator simulation of diffusion created by a group of middle school students. Scratch simulation of water created by a Scratch community member.

Although computational thinking, like mathematics, involves learning about and using formal symbolic languages, there are a few important differences. Computational models are often used to explore problems that are too complex to describe mathematically, or systems that have random or probabilistic elements. For example, the predator-prey system described in the last section might have very different outcomes depending on how predators and prey are distributed in space. Simulations allow this uncertainty to be represented and explored. This can help students understand the fragile and at times unpredictable nature of some complex systems.

3. How does using mathematics and computational thinking relate to the other practices?

Like all of the NGSS practices, using mathematics and computational thinking is likely to interconnect with other practices in many ways, depending on the topics students are exploring and the problems they are working to solve. They are specialized ways to develop and use models that are powerful and predictive. Because of this, the practice shares many features with the modeling practice, and we refer readers to Chapter 6 for more information.

Using mathematics and computational thinking also provides learners with a powerful way to engage in argument with evidence. Students can use mathematical representations or simulations to generate theoretical data that they can use to support their own scientific claims, or challenge those of others. Computational models can generate theoretical data about systems that might be otherwise difficult to interact with, like geological or population dynamic systems. Mathematics and computation also offer new ways to connect real-world evidence to theoretical models: Some tools even allow learners to link computer models with experiments and measurements conducted in the real world, to test the validity of their models and compare how idealized models do and do not reflect what happens in the “messiness” of real life.

Finally, using mathematics and computational thinking is closely related to analyzing and interpreting data, and communicating information. Mathematical methods, data collection and analysis tools, and visualizations all provide ways that students can work with large quantities of data. And, students can use mathematical representations, data displays, simulations, and graphical representations to communicate their findings in precise ways that might be difficult to do with text or spoken language alone.

4. What does using mathematics and computational thinking look like when it happens in classrooms?

Mathematics and computational thinking should be used in the science and engineering classroom when they can help students to understand, describe, predict, and change the world around them. Here, we provide an example from a late elementary school classroom where students used computational thinking to make sense of a scientific system. At the end of this section, we talk more generally about what teachers can expect across K-12.

An Example from Elementary School.

A 5th grade science class has been working in small groups to model evaporation and condensation using animation and computer simulation as representational tools. The students had already explored condensation by creating models of why they think droplets form on a cold bottle of soda when it is taken out of the fridge. During that part of the activity, they described how particles of water (what they called water vapor) are always in the air around us, and these particles stick to the surface of the bottle. Next, the students were asked to show their ideas about what happens to puddles of water outside during a sunny day. The students had used a modeling toolkit called SiMSAM (Simulation, Measurement, and Stop Action Moviemaking) and craft materials to create different stop-motion animations of their evaporation models.

After a gallery walk during which they are able to inspect their peers' work, the students had many questions and comments for one another. Some of these questions dealt with what exactly was happening in the animation: What were the cotton balls meant to represent? Why did things seem to move more slowly at some times, and faster at others? Should only one water particle leave a puddle at a time? Many of these questions were posed because of the way students' models were represented: While animation is good at showing how things change over time, it is difficult to create animations that are consistent or that involve many moving pieces. These sorts of questions and needs motivate computer simulation, which allows students to write automatic rules for when and how interactions between objects take place.

Students selected and cropped objects from their animations they thought were most important for modeling the evaporation process. They could then visually demonstrate how those objects should behave, by double-clicking and moving objects on the screen. Objects could change position, turn a certain amount or randomly, duplicate, change size, or create other objects. The students gave each object in their simulation a rule for how it should behave when it is alone, and how it should behave when it bumps into other objects. For example, in the simulation below, "water vapor" (represented by pipe cleaners) is emitted by the puddle, and water droplets (blue puff balls) are released by clouds. As water vapor objects come in contact with clouds, more clouds appear in the simulation.



Figure 7. An example of an animation (left) and simulation (center, right) constructed by one fifth-grade group.

After student groups each created their own simulations, one (Figure 7) was projected to the front of the room as a basis for further discussion. The students agreed with several of the features included in the simulation. They agreed that clouds should increase as water evaporates from the puddles, and that puddles should emit vapor. However, they were dissatisfied with how “clean” the process was: they felt that the vapor should “move everywhere” because there is always water in the air around us. Some commented that the simulation did not show enough of the *process* of evaporation: that is, when, why, or how vapor comes out of the puddle. Several wanted to introduce more precise quantitative descriptions to the simulation: for example, they argued that one “vapor” object should not be enough to create a new cloud, and some wanted more vapor objects to come from larger puddles than from smaller ones. Their conversation was much more focused on objects, interactions, and qualitative and quantitative differences than it was before.

Toward the end of the class discussion, one student Emily suggested that the puddles should be made up of many vapors placed tightly together. She wanted to show that the puddle itself was made up of many water vapor objects (molecules), and that evaporation was the result of these objects breaking off into the air. She further noted that this way, the puddle would not have to be given a separate instruction to get smaller—it would do so automatically as vapor pieces break off from the group. Emily’s proposal synthesized many of the ideas the students were exploring: the connections between matter of different states, evaporation, and the water cycle. Her proposal also represented a shift in how she was thinking of scientific models – from simply showing a physical situation, to proposing ways for explaining *how* the situation works.

On the last day of the activity, we asked the class to watch a time-lapse video of a new evaporation situation. The video showed two beakers full of water, one covered and one uncovered, heated over several days so that water level of the uncovered beaker reduced over time and the covered beaker appeared to “sweat” from the inside. This time, we asked students to create paper simulations to show what was happening using index cards instead of computers (Figure 8). Building on their experiences building simulations on computers, many students created paper objects representing water molecules that would either leave or be trapped inside of the beaker.



Figure 8. Omar’s group builds a paper simulation that combines their exploration of evaporation in beakers with condensation on cold drinks to show that the same idea explains both.

One group completed their paper simulation quickly, and was encouraged by their teacher to include a different object in their new “simulation”: the soda bottle from their very first animation task. One student Omar quickly noted that water particles are everywhere in the air, and they all act the same way. So he decided to show that while water particles in the covered container are trapped, the water particles in the uncovered container “go up” and leave. However, he notes, they don’t just go up but *everywhere*, and may even stick to the soda bottle. Here, using computational thinking—specifically, thinking about how rules and interactions might work in new and different arrangements—led Omar to connect and generalize his model across different contexts.

This example is related to the performance expectation that students “develop a model to describe that matter is made of particles too small to be seen” (5-PS1). It illustrates how, by articulating the specific rules needed to build computer simulations, students started to think in detailed ways about how molecules behave during the process of evaporation and condensation, and about what matter is made of. Wilkerson-Jerde, Gravel, and Macrander (2015) explores in detail how these ideas develop as students model and re-model scientific systems using tools like animation and simulation.

Thinking About Your Own Classroom

The NGSS provides some insight into how the practice of using mathematics and computational thinking might unfold over students’ K-12 experiences in Appendix F. In early elementary grades, students should be able to record information based on qualitative and quantitative amounts, and use counting numbers to find patterns in scientific phenomena. By upper elementary, they should be able to determine if certain data is relevant to understanding a given system, become comfortable organizing data graphically and describing measurements. At the middle level, students are expected to begin to use digital tools to collect data. They are also expected to use simple mathematical concepts such as ratios, percent, averages and basic operations to prepare scientific arguments and construct tests for engineering designs.

By the time students leave high school, they are expected to be able to use mathematical and computational thinking to predict and describe the world around them. This includes creating and revising simulations to accurately reflect the scientific phenomenon, using algebra to solve and scientific and engineering problems, and applying ratios, percentages and unit conversions to complicated mathematical scientific problems. However, given the wide applicability and flexibility of this practice and the collection of mathematical and computational tools available, we encourage you to think about ways you can use mathematics and computation to support and build on the sensemaking you see in your own classroom.

5. How can we work towards equity in using mathematics and computational thinking?

There are a number of ways to make sure that using mathematics and computational thinking is done equitably and inclusively in the science classroom. Perhaps most importantly, this practice – like all of the practices – is about giving students *ownership* over science ideas and explorations. The practices are ways that students can make and evaluate their own claims and evidence and develop their own theories and solutions to problems, rather than always looking to their teacher or other authorities for all the answers.

Mathematics and computation are popularly known languages of science, and offer students a visible and respected way to convince others of their findings. As students learn how to speak these languages, they can use them to explore and argue about issues they care for. In our introductory vignette, high school students used research software, computer sensing devices, and mathematical representations to explore air quality in their own community. They found ways to report their observations and evidence in a way that is verifiable and replicable, so that anyone can use the same tools, measures, and methods to challenge or support their claims. They were also able to use these tools to make convincing recommendations about how those issues might be improved.

Mathematical and computational representations, simulations, data visualizations and mathematical formulae all provide students with *more* ways to show and share their ideas. This can be especially important for engaging English Language Learners. Focus on students' arguments, gestures, questions, ideas, and claims rather than vocabulary. If ideas can be expressed more precisely, try revoicing ideas in a way that introduces mathematical language subtly. Rather than making sure students use technical terms or phrases, focus on having them explain what those terms mean in their own words.

Finally, understanding how mathematics and computational thinking are used in science and engineering can help students to become critical consumers of popular science and media. More than ever, students are exposed to conflicting information about controversial topics – climate change, vaccination, nutrition – that can affect their future decision-making. Being able to critically analyze statistics, recognize and explore mathematical trends, and recognize the benefits and limitations of what computer tools and simulations can do can help students making informed decisions for themselves and their communities.

6. How can I support and assess using mathematics and computational thinking in my classroom?

Supporting your students in this practice is all about helping them structure their thinking, observations and arguments in ways that can be described and tested using mathematical and computational tools. Be on the lookout for student observations, comparisons, and descriptions

about qualitative patterns that lend themselves to quantitative specification. Think about how you can challenge or ask them to elaborate their ideas about patterns. For example, if a student notes that something in an exploration you have conducted happens faster than something else, encourage them to think about measuring and quantifying that: “So you say it was faster, can we think about ways to find out how much?”.

Once you have found these ways to connect to students’ observations and questions, give them ways to organize and formalize their observations. If students are given a prefabricated chart that tells them what information they need to collect, they may take a “fill in the blank” approach instead of a mathematical thinking mode. Similarly, if they are simply given a formula, they may apply it without understanding what the results mean. Instead, have a conversation about what representations make sense for a given investigation or scientific question. If you are making a data table, what values should be recorded and how should they be measured? What do the variables in an equation represent, and what do results mean in terms of the physical, biological, chemical, or engineered system being studied? Have students interpret formulae and predictions based on qualitative trends – “so if my mass gets larger but velocity stays the same, this formula tells us that momentum should do what?”.

When supporting mathematical thinking, there is a fine line between allowing flexibility and supporting students as they gain proficiency. Remember that sometimes, a wrong turn can be a learning experience. If students conduct measurements or include variables in a mathematical model that prove extraneous, this helps them hone their skills in deciding what to include and what not to in the future. Your main goal should be to make sure students are able to justify their answers and make connections to the science and engineering concepts.

Think of your one-on-one conversations with students and student groups, classroom discussions, and the written work students produce as opportunities for formative assessment. Some things you may want to check for include:

- Are students connecting what they are doing with mathematics and computation to the scientific ideas they are exploring?
- Can they identify appropriate mathematical tools and computational techniques to tackle a given problem?
- Are they checking their work using common sense and reason?
- Can they recognize and diagnose errors in others’ mathematical solutions or reasoning?
- Are they using mathematics or computational information as evidence to support arguments during discussion?
- Are they considering how they might extend or apply mathematical and computational ideas in new situations to make predictions or comparisons?

These questions show that using mathematics and computational thinking is difficult to assess on its own, without thinking about the concepts and ideas it is linked to. Because of this, the performance expectations of NGSS are nice examples of summative assessment. Each performance task is worded so that it includes a practice, disciplinary core idea and cross cutting concept. There are many performance tasks that require explicit use of mathematical and computational thinking in order to conduct particular kinds of inquiries, provide evidence for arguments, develop solutions or communicate results. Use these as an example of how you can build these types of performance expectations into your own curriculum.

7. How do I get started?

Getting started using mathematics and computational thinking in your own classroom involves three interrelated issues: promoting student sensemaking, fostering a classroom culture, and finding tools and resources that work for your classroom and curriculum.

An Example: Using “Science Stories” to Make Mathematical Connections

A 7th grade science class is about to start a unit on line graphs as a way to display and analyze quantitative data. Before they start, they do a short activity called “Showing Science Stories”, where they invent their own representations of scientific systems. They were given three stories involving scientific systems that exhibit patterns of change over time: one that involves the position and velocity of a traveling car, one about a population of animals responding to changes in birth and death rates, and a third about a collection of plants grown by children over the summer. (These stories were adapted from research studies in mathematics and science education.) Their task during the class period was to find a way to show the scientific system described, so that someone who did not know the story still could understand how the car’s speed, total population of animals, and plants’ heights changed over time.

Even though these students hadn’t yet used graphs in their science class, they came up with a number of useful ways to represent quantities that changed over time. Some students were already familiar with graphs, and quickly recognized how they could convert the patterns described in the stories to graphical form. Others developed innovative ways to show how different quantities in the story changed over time.

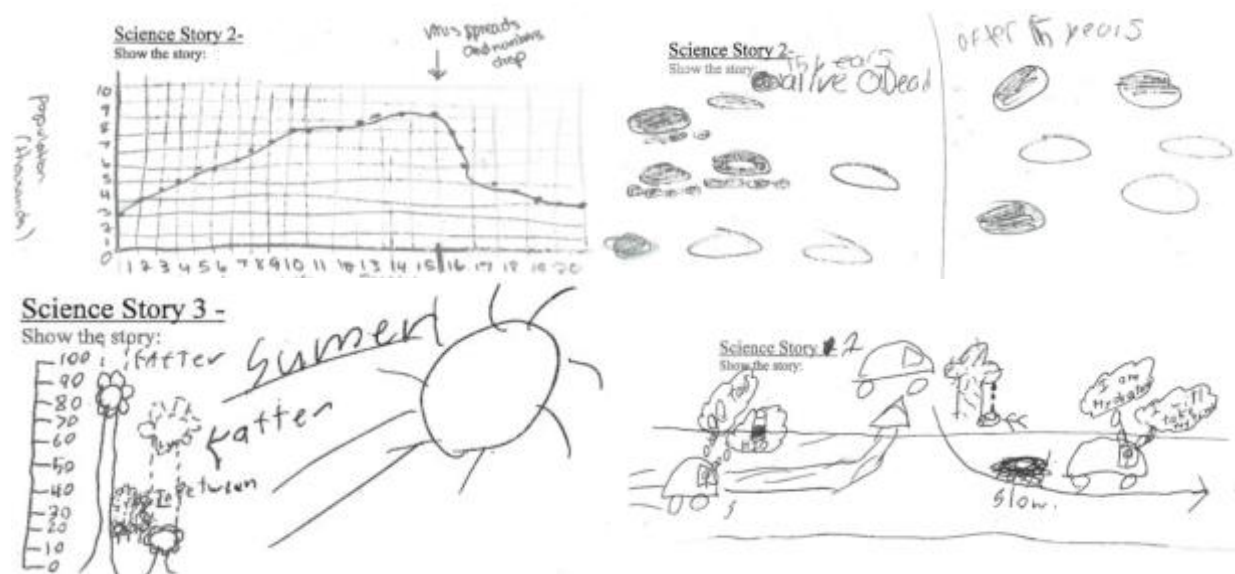


Figure 9. Invented representations of qualitative and quantitative change in different “science stories” created by 7th grade students during an introduction to graphing activity.

Many of the displays students came up with were not conventional. Some might even at first glance seem outright wrong or confusing. However if we look closely, they tell us a lot about how students were thinking about the patterns and quantities in each systems. Some students, such as the one that produced the conventional line graph of population growth in the upper left corner, already had a good understanding of how graphs could be used to show trends in very different systems. Others, like the student that produced display in the lower hand corner,

made less scientifically conventional representations that nonetheless tell us a lot about what they are thinking. In particular, this student's representation is already organizing the plants into a maximum, minimum, and some sort of measure of center (indicated by the small cluster of plants labeled "In Between")—ideas that connect naturally to statistics. The representation also contains information about what the student thinks might be *causing* patterns of change over time, sunlight.

Even representations that looked less like what we would expect to see in a science textbook still tell us a lot about what mathematical ideas students might be ready to explore, and what qualitative and quantitative measures they believe are important to communicate. The student who created the display on the upper right used circles of different colors to represent the number of animals who lived vs died during different periods of the story (which connects well to ideas of ratio), as well as the number of offspring they expected from living animals (which connects to exponential models). The comic-like display in the lower right uses the thickness of the arrow in different parts of the image to represent differences in the car's speed, and uses the convention of left-to-right to represent the passage of time. Even if students don't use the *right* visual features to show quantitative differences, there are ways to build on what they do know. Thinking of the height of a point as representing the same thing as the thickness of an arrow might give this student one way to start to make sense of line graphs to represent velocity.

Unlike the extended explorations we presented at the beginning of this chapter and in this section, this activity took only one day in a classroom. In that short time, the classroom teacher was able to learn a lot about what her students thought was happening mathematically in these systems: what qualitative and quantitative patterns to pay attention to, how that data can be represented visually, and even how they might want to organize and explore the data more systematically using statistical analysis. They did this even for systems that involved complex, interrelated patterns that all unfolded together over time. Research suggests that this classroom is not unique – other, more developed examples can be found in diSessa, 2004 or Lehrer and Schauble, 2004.

Short activities like these can form the basis for more extended and rigorous discussions of how mathematics can be used in scientific exploration. Students can do a gallery walk to see and critique one another's displays, or have a class discussion about which ones are clearest to understand. For example, while the image in the lower left of Figure 9 includes many interesting ideas, it also features an axis with numbers that are not connected to a particular unit of measurement. This omission can become the focus of a discussion about why units are important. Similarly, the two displays on the right-hand side of Figure 9 only show "snapshots" of the system over time, rather than describing patterns of change. This sort of activity can also serve as a brainstorming session that helps a whole class think together about how they might organize and display data during a more in-depth scientific investigation. Because these representations start with students' drawings, they are more likely to recognize and understand the links between the mathematical representation that is used, and events in the scientific story.

This example shows how in just one day, students were able to find ways to model the important quantities and relationships in a variety of situations studied in the science classroom. Because of the richness and familiarity of the situations used, students' models then reveal a number of connections that can be made: to mathematics and computation (through the use of visualization or spreadsheet tools to conduct data analysis), but also to the relevant disciplinary

core ideas the underlie each situation. For example, many of the students' representations of the plant and population situations revealed some foundational ideas that can lead to explorations and discussions about flow of energy and patterns of interactions in ecosystems (MS-LS2).

More generally, we suggest a few principles to get started on introducing the Using Mathematics and Computational Thinking practice in your own classroom. First, it is important to motivate and recognize students' interest in the patterns and trends that characterize scientific and engineered systems. Do they notice patterns or have questions that mathematics or computational tools can help answer? Can you present demos or ask questions in a way that move beyond *what* happens to *how much* or *how intensely* it happens, to make qualitative and quantitative differences clear and open for debate and exploration?

Second, work to establish a classroom culture where students are expected to use mathematics and computation as ways to tell scientific stories, test ideas, make predictions. Have students practice making and evaluating one another's qualitative and quantitative observations and mathematical claims. Do not just focus on data or the results of mathematical procedures, but also on how data are obtained through measurement, and how results can be interpreted in terms of scientific predictions. Offer students opportunities to use simulations or data visualizations as part of scientific discussion. Build a shared toolkit of mathematical and computational tools - software, sensors, measures, data displays, models, and techniques - that students have available to refer to during discussions and investigations.

Third and related, find out what resources work best for your classroom and curricular plans. There are many modeling tools, data analysis tools, and simulations that have been designed especially for K-12 students. Many of these tools also have dedicated educator communities that can help you get started. Below we list just a few tools that are free for educational use, and include a wide range of subjects and student levels. But as the example above shows, even paper and pencil can get you quite far – what is important is how well the tools allow students to share and refine their models, and make connections to the disciplinary core ideas under study.

Concord Consortium Simulations (concord.org) – Online simulations and curriculum resources.

Google Spreadsheets – Online spreadsheet tool to organize, analyze, and visualize data.

NetLogo (ccl.northwestern.edu/netlogo) – Programmable agent-based modeling environment. The download includes many pre-built simulations that users can use or modify, and users can start fresh to build their own.

PhET Simulations (phet.colorado.edu) – Educational computer simulations in physics, biology, chemistry, and earth science.

ScratchEd (scratched.gse.harvard.edu) – Curricular resources and how-to guides for the Scratch drag and drop programming environment.

VenSim (vensim.com/vensim-software) – A systems dynamics modeling environment.

In doing all this, you shouldn't feel like you are alone. More than for many of the other practices, teaching the practice of using mathematics and computational thinking is about working in partnership. Talk to your fellow mathematics and technology teachers, coaches, and specialists about what is reasonable to expect from your students at different grade levels. Consult the Common Core State Standards in Mathematics to get a sense of what your students

might have already learned. Find out what technology experiences your students might already have, and think about how they can work in your own classrooms. Try developing a shared language across classes that helps students recognize connections.

What is NOT included in this practice.

- Using simulations or data visualizations to illustrate a target scientific phenomenon or pattern, without allowing students to pursue their own questions or theories or explore how the phenomenon works or why the pattern holds.
- Using spreadsheets to input data and perform calculations, without having students reason about what those calculations mean scientifically.
- Having students complete simple word problems or fill out pre-defined data tables to reinforce a given formula.
- Using computer-based flashcards, quizzes, wikis, or videos to introduce science concepts.

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Snapshot image of "The Maxwell Speed Distribution" from the Wolfram Demonstrations Project (<http://demonstrations.wolfram.com/TheMaxwellSpeedDistribution>). Authored by Jan Fiala.

Screenshot image of "GasLab Gas in a Box" NetLogo model used with permission from Uri Wilensky. (<http://ccl.northwestern.edu/netlogo/models/GasLabGasinaBox>). NetLogo (ccl.northwestern.edu/netlogo) and "GasLab Gas in a Box" are produced by the Center for Connected Learning and Computer-Based Modeling at Northwestern University (ccl.northwestern.edu).

Screenshot image of "Molecular View of a Gas" Molecular Workbench interactive (concord.org/stem-resources/molecular-view-gas) used with permission from the Concord Consortium. Molecular Workbench (mw.concord.org) and "Molecular View of a Gas" are produced by the Concord Consortium (www.concord.org).

Screenshot image of “States of Matter: Basics” PhET simulation (<http://phet.colorado.edu/en/simulation/states-of-matter-basics>). Authored by Wendy Adams, Paul Beale, John Blanco, Sara McKagan, Emily Moore, Kathy Perkins, Noah Podolefsky, Trish Loeblien. CC BY 3.0.

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