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# Cascading Failure in the Maximum Entropy Based Dense Weighted Directed Network: An Agent-based Computational Experiment

Liang Zhang<sup>1,2\*</sup>

<sup>1</sup>*School of Economics and Management, Hebei University of Technology, Tianjin 300401, China*

<sup>2</sup>*School of economics, Tianjin Polytechnic University, Tianjin 300387, China*

*\*Corresponding author(E-mail: seanzhang@126.com)*

Zhenxiang Zeng

*School of Economics and Management, Hebei University of Technology, Tianjin 300401, China*

## Abstract

Cascading failures would have serious impact on a variety of important networks, and simulation in the accurate network model is the mainstream method to study the cascading failure. Maximum Entropy (ME) approach is widely applied to construct the network model with partial information available, but its complete network assumption is criticized for leading to impractical result. In this paper, ME approach is proved to be capable of building the accurate network model for cascading failure simulation in the dense weighted directed network. The China interbank network of listed banks is selected as research object, and maximum entropy approach is used to calculate the weight of each link in the interbank network according to the sum data. Then the agent-based computational experiments of cascading failure are launched on this model. Results show that the 5 largest banks can cause cascading failure, especially ICBC. When the asset loss rate threshold is reached, any one of the 5 banks failure could trigger a cascading failure to ruin the entire network. Above the threshold, loss rate is positively correlated with intensity of cascading failure and negatively correlated with duration. Simulation results coincide with the conclusion of financial supervision and regulation departments, and it proves that the maximum entropy approach applied to dense weighted directed networks practicably.

**Key words:** Cascading Failure, Maximum Entropy Approach, Dense Weighted Directed Network, Agent-based Computational Experiment

## 1. INTRODUCTION

A cascading failure is a kind of failure in a network structured system in which the failure of one part can trigger the failure of successive parts and contagion along the network. Cascading failure may happen in many types of systems, such as power grid (Bompard et al., 2016), computer networking, infrastructures systems, financial systems, human bodily systems, and cause huge losses. For example, the close of hub airport on the air routes will lead to widespread flight delays, one fiber optic cable damage may cause large areas of Internet congestion, tripping of a few transformers and power stations touch off the power grid blackout, a handful of financial institutions failures trigger the global financial crisis. The study of cascading failures is particularly important in today's networked world.

Along with the complex network theory rising, cascading failure draws constant attention and a lot of research results have appeared. The scale-free(SF) network is fragile when facing with the cascading failure, removing of hub nodes cause the whole network collapse (Motter and Lai, 2002).The comparison of cascading failure in small-world(SW) and SF networks showed that deliberate vertex attacks can result in larger cascading failures than deliberate edge attacks in the SF network, however, in the SW network the situation is opposite (Bao et al., 2009). Many theoretical researches focus on the correlation of cascading failure and network topology, and ways to control or defense the cascade(Motter, 2004). The real-world network components usually have loads, such as the traffic on the road and liability of the bank. So the research of cascading failure in weighted networks make more sense. The load of the links is considered to be the product of the betweenness centrality of the end nodes is favored for the robustness of the network against cascaded failures(Mirzasoleiman et al., 2011).In finance, the risk of cascading failures of financial institutions is referred to as "systemic risk": default of one financial institution may cause other financial institutions (counterparties) to fail, cascading throughout the system(Roukny et al., 2013). The data of real financial flows is hard to obtain, so related information are used to estimate the financial networks with Maximum Entropy(ME) approach(Mistrulli, 2011).

The ME approach delivers a fully connected network structure, which considered to be unrealistic. Such network architecture is atypical for interbank markets, and when such a network is used in the research of

cascading failure, it tends to introduce a bias that underestimates the true extent of cascading failure (Markose, Giansante and Shaghghi, 2012). Simulations on more realistic networks show that greater diversification through interlinkages indeed lowers the probability of cascading failure, but may also raise its damage when cascading failure occurs.

In reality, interbank activity occurs through relationships are sparse. In the United States interbank market (from 1997 to 2006), there are less than 1% of potential bilateral linkages in active use (Bech and Atalay, 2010). The similar results are drawn from the Germany interbank market (from 1999 to 2007). At the same time, less-connected banks are more likely to connect with well-connected banks than with other less-connected banks. This reflects the economic rationale that smaller banks are more likely to transact with each other (Craig and von Peter 2014). Similar observations hold for dealer networks in financial markets and in international trade (Helpman, Melitz and Rubinstein, 2008; Ahn, Khandelwal and Wei, 2011).

The remainder of this paper is organized as follows. China interbank network is dense network and close to the complete network assumption. So ME approach can be applied to estimate the architecture of the dense weighted directed network. According to the data of China listed banks in 2015, the interbank network model is figured out and be put into an agent-based computational experiment to simulate the cascading failure. The results prove the ME approach applied to dense weighted directed networks practicably.

## 2. METHODOLOGY

In actual network, the weights of the edges cannot be directly measured, and the probabilistic estimation method is usually used to estimate the other incomplete information from the known partial information. It is often that the sum of the weights of all the edges on the node can be determined and the weights of the edges are estimated. The most typical is the interbank network, in which the bank as a node, the interbank lending relationship as link. On the interbank market, transaction data is strictly confidential, the bank's regular public financial statements in accordance with regulatory include the total amount of borrowed from and lend to other banks. In those cases, the leading method is for researchers to fill in the blanks as evenly as possible, using the available information on each bank's total interbank lending (Upper, 2011).

### 2.1. Maximum Entropy Approach

The problem is described as follows. To suppose there are  $n$  banks in closed financial system  $X$ , the claims(assets) of bank  $i$  to bank  $j$  are denoted as  $x_{ij}, i, j \in [1, n]$ , and it is also the debts(liabilities) of bank  $j$  to bank  $i$ . Total interbank assets of Bank  $i$  is  $a_i = \sum_{j=1}^n x_{ij}$ , total interbank liabilities is  $l_j = \sum_{i=1}^n x_{ij}$ . The objective is  $n \times n$  matrix  $X$  consist of  $x_{ij}$  under permission of known  $a_{1...n}$  and  $l_{1...n}$ .

$$X = \begin{matrix} \begin{bmatrix} x_{11} & \cdots & x_{1i} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{j1} & \cdots & x_{ij} & \cdots & x_{jn} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{ni} & \cdots & x_{nn} \end{bmatrix} & \left\| \begin{matrix} a_1 \\ \vdots \\ a_j \\ \vdots \\ a_n \end{matrix} \right. \\ \hline \begin{matrix} l_1 & \cdots & l_j & \cdots & l_n \end{matrix} & \left\| \begin{matrix} a_1 \\ \vdots \\ a_j \\ \vdots \\ a_n \end{matrix} \right. \end{matrix} \quad (1)$$

$$a_i = \sum_{j=1}^n x_{ij}, l_j = \sum_{i=1}^n x_{ij}$$

There are two constraint conditions. Firstly, due to the closure of the financial system  $X$ , the total interbank assets should be equal to the total interbank liabilities and not negative,  $\sum a \equiv \sum l, a, l \geq 0$ . Secondly, Since banks cannot borrow from themselves, the diagonal elements of square  $X$  are zero,  $diagonal(X) = 0$ .

The concept of information entropy needs to apply in the process of solving the object matrix  $X$ . Information entropy is a statistic that represents the probability distribution of the probability system. It represents the uncertainty of the system. The greater the entropy, the harder it is to predict which event will occur in the system. The principle of maximum entropy states that, subject to precisely stated prior data, the probability distribution which best represents the current state of knowledge is the one with largest entropy. Take precisely stated prior data or testable information about a probability distribution function. Consider the set of all trial probability distributions that would encode the prior data. According to this principle, the distribution with maximal information entropy is the proper one. So the approach to solve this problem is called Maximum Entropy approach.

In the absence of complete information on each of the parties in the interbank transaction, assuming that the probability distribution of interbank lending is dispersed as much as possible before solving  $X$ , the interbank assets and interbank liabilities are independent.

The joint entropy of the probability matrix  $(X, Y)$  is defined as  $H(x, y) = -\sum_j \sum_i p_{ij} \log_2 p_{ij}$ , where  $p_{ij}$  is the joint probability of  $X$  and  $Y$ . The Shannon entropy of  $X$  is  $-\sum_j \sum_i x_{ij} \log_2 x_{ij}$ . According to the principle of maximum entropy, the problems transform to solve  $\min \sum_j \sum_i x_{ij} \ln x_{ij}$ , constraints are  $\sum_{i=1}^n x_{ij} = a_i, \sum_{j=1}^n x_{ij} = l_j, x_{ij} \geq 0$ , and the solution is  $x_{ij} = a_i l_j$ .

A transition matrix  $X^0$  is introduced as:

$$X = \begin{bmatrix} 0 & \dots & x_{1i}^0 & \dots & x_{1n}^0 \\ \vdots & & \ddots & & \vdots \\ x_{1j}^0 & \dots & 0 & \dots & x_{jn}^0 \\ \vdots & & \vdots & & \vdots \\ x_{1n}^0 & \dots & x_{ni}^0 & \dots & 0 \end{bmatrix} \begin{matrix} a_1 \\ \vdots \\ a_j \\ \vdots \\ a_n \end{matrix} \quad x_{ij}^0 = \begin{cases} 0, & i = j \\ a_i l_j, & i \neq j \end{cases} \quad (2)$$

$l_1 \quad \dots \quad l_j \quad \dots \quad l_n$

The original problem is transformed in to optimization problem:

$$\begin{aligned} & \text{Min}_x \sum_{i=1}^n \sum_{j=1}^n x_{ij}^0 \ln \frac{x_{ij}^0}{x_{ij}} \\ & \text{s. t. } \sum_{i=1}^n x_{ij} = a_i, \sum_{j=1}^n x_{ij} = l_j, x_{ij} \geq 0 \end{aligned} \quad (3)$$

Using RAS algorithm can solve this optimization problem.

### 2.2. The Applicability of ME Approach

The criticism of ME approach is the complete network assumption, and the ways to build the incomplete network artificially in order to solve this problem, such as deleting some links with very light weights. For sparse matrix this is really an unsolvable problem. However, for some special practical networks, dense and weighted and directed, ME approach still has applicability.

The traditional theory is that banks can't connect all other banks in the interbank market as counterparty because interbank lending for liquidity requirements is usually achieved through as few counterparties as possible to reduce transaction costs and complexity. But the current interbank market in China transactions are very frequent, making the assumption that the complete network closer to the facts.

In June 2013 the Shanghai Interbank Offered Rate (Shibor) raised sharply, and the overnight Shibor reached an unprecedented 13.44%. Then the banks released 2013 annual report showed that, in addition to the four major state-owned banks, many joint-stock commercial banks also held a large number of interbank positions. Take CIB as an example, in June 2013, CIB's ratio of interbank assets and interbank liabilities close to 1:1. The annual report of 2013 showed that CIB's interbank assets accounted for 33.7%, ranking first in all China listed banks, and interbank net assets gross margin was over 50%. The interbank market has become an important source of profit for CIB. Since then, a number of China joint-stock banks have used the interbank market as a source of profits, rather than just as a short-term financing channel to address liquidity needs. Just like investors in the financial market in order to profit and risk aversion, will hold a series of financial products as a portfolio and frequent operation, rather than holding several long-term financial products, once the interbank market is viewed as a source of profit, the bank will make frequent transactions and will not deliberately reduce the number of counterparties for the purpose of saving transaction costs. The result is China interbank market goes hot, and because the main players in the interbank market are only 24 listed banks, the connection becomes very dense and close to complete network.

In many researches, ways are introduced to make the network model generated by ME approach sparse. Most of these ways are deleting some links according to certain rules, such as removing the links with significantly lower than the average weight. Furthermore, the modified network models are binarized to be the weight-free network models. In the weight-free networks, the networks' properties will be largely determined by its topology. So both binarization and deleting some light-weight links will cause more serious distortion than the complete network assumption.

The easiest way to solve it is keep all the links with weight. Because the properties of the weighted directed networks are greatly affected by the weight and direction of the links, not only by the topology. For weighted directed networks, light weighted links have little effect on the network properties, and it is not necessary to deal with the deleting rules setting. In the cascading failure simulation in the interbank network, a very small amount obligatory relationships erodes the capital of bank very slightly even the loss rate is 100%. The simulation is focused on the default of the banks rather than operating performance, so the light-weight deletion or not has little effect on the results.

In summary, the current interbank network of China is denser than the traditional sparse interbank networks. This makes it possible to use the fully connected weighted directed network structure obtained by the ME approach to study the cascading failure in the interbank network of China.

### 3. MAXIMUM-ENTROPY BASED NETWORK MODEL

#### 3.1. Data

The data from the balance sheets of listed banks of China will be used to build the interbank network by ME approach. The consolidated balance sheet in the annual report is the most comprehensive, and the financial reports of all the 24 joint stock commercial banks listed in China can be downloaded in Shanghai Stock Exchange and Shenzhen Stock Exchange. According to the computational requirements of the ME method, it is necessary to obtain the interbank assets and interbank liabilities held by banks at the end of 2015. In the balance sheet, the interbank assets include three items: Due From Banks, Loans To Other Banks and Buying Back the Sale of Financial Assets. The interbank liabilities include three items: Due To Banks, Loans From Other Banks and Financial Assets Sold for Repurchase. Moreover, the simulation of banks default needs the data of capital, it includes Net Amount of Core Tier 1 Capital and Net Amount of Weighted Venture Capital. Data are collated and calculated in Table 1.

**Table 1.** The values of time in the mill drum first section (Unit: Million Yuan)

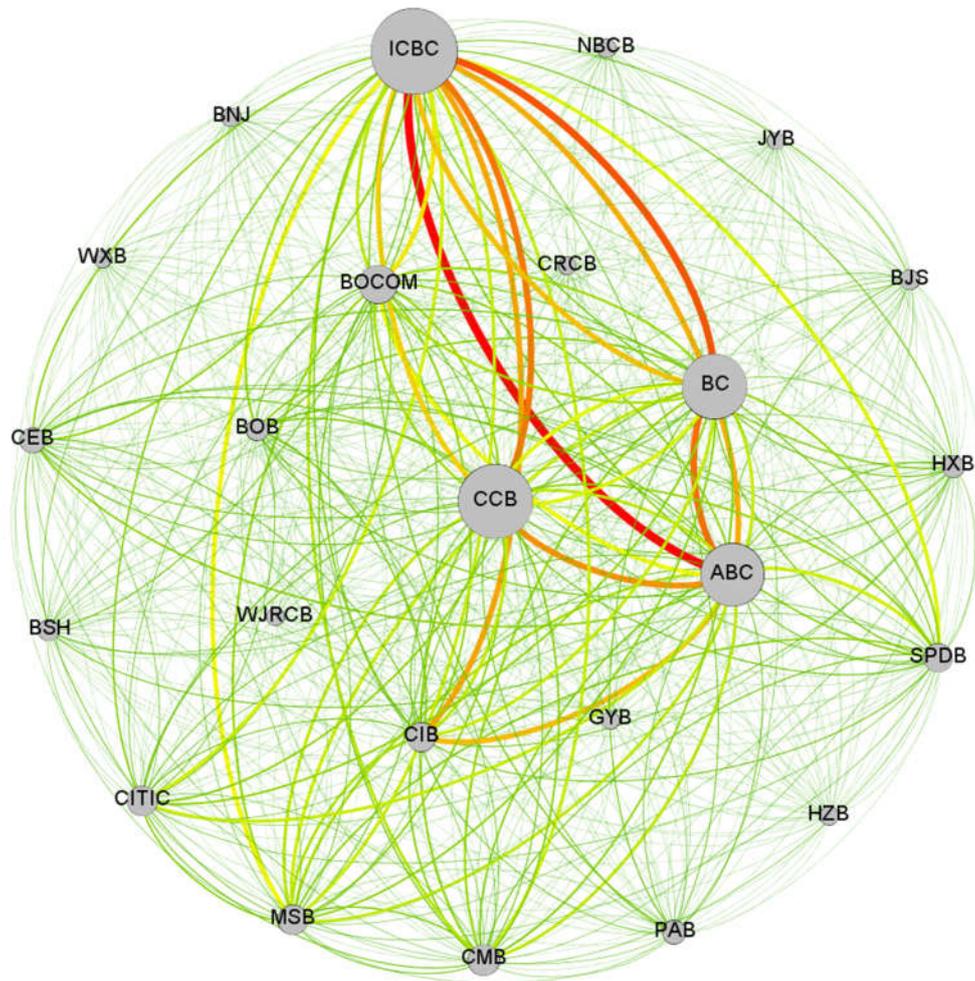
Bank name (abbreviation)	Interbank liabilities	Interbank assets	Net amount of core tier 1 capital	Net amount of weighted venture capital
SPDB	12617420	3594120	2881950	33678340
HXB	4148320	3257630	1182480	13303910
MSB	10399040	9013020	3068730	33462320
CMB	10759840	5933960	3474340	32081520
WXB	119935	132951	70434	663324
BJS	3617793	1359293	653545	7605061
HZB	799960	723316	317572	3359943
BNJ	1320769	405586	472484	5036431
CRCB	113434	42588	75590	693965
CIB	19174010	3246070	2881750	34276490
BOB	4747860	4936280	1118200	12760340
BSH	4090335	1693881	922694	8941191
ABC	16264640	16739840	11246900	109863020
BOCOM	15059190	6111910	5184870	46537230
ICBC	26030510	16801260	17014950	132166870
CEB	6602440	3717170	2018350	21855160
CCB	20291190	9744720	14081270	107220820
BC	22122640	10078550	11823000	106540810
GYB	107022	103389	140957	1319975
CITIC	11889600	3381400	3161590	34681350
WJRCB	55887	45948	64132	515653
PAB	3342490	3029730	1500700	16617470
NBCB	1353308	265617	399703	4425626
JYB	116907	12211	74813	581311

#### 3.2. Weighted directed network of China listed banks

According to the data in 3.1, ME approach can be used to calculate the amount of debt between each pair of banks as the weight of the links in the interbank network. The result forms a matrix and the interbank network model can be built by this adjacent matrix. The table of adjacent matrix of china listed banks interbank network is available in <http://pan.baidu.com/s/1kULAuvP>. Then let net amount of core tier 1 capital be the weight of the nodes. The interbank network is built by these data and be visualized in Figure 1.

In Figure 1, the size of the node represents the net amount of core tier 1 capital, the top 4 are China's 4 largest state-owned banks, Industrial & Commercial Bank of China(ICBC), China Construction Bank(CCB), Agricultural Bank of China(ABC), Bank of China(BC). The thickness and color of the link represents the debt amount among banks. All links are directed, but the arrows are too small to see and are obscured by the nodes.

The backbone network is obviously in the graph. Even the network is complete, the significant backbone network ensures the basic property of the whole system from being confused by the numerous light-weighted links.



**Figure 1.** Interbank Network of China Listed Banks

#### 4. AGENT-BASED COMPUTATIONAL EXPERIMENT DESIGN

Agent-based computational experiment is a kind of computer simulation experiment using the agent-based model. An agent-based model (ABM) is one of a class of computational models for simulating the actions and interactions of autonomous agents with a view to assessing their effects on the system level. We introduce ABM to simulate the banks' behavior when a sudden failure hit on one banks and the process of cascading failure that may occur later.

##### 4.1. Agent properties

There are 2 breeds of agent in the experiment: bank agents and credit link agents. The credit link agent's direction is from the creditor bank to the debt bank. Its only property is the amount of the debts as the weight. The properties of bank agents are more. The balance sheet of the bank is abstracted to the left side of the asset and the right side of the liability + capital. The assets on the left is the amount that the bank lends to other banks and are the start of the links point out (out-links), equal to the sum of all out-links weights. The liabilities on the right is the amount that the bank borrows from other banks and are the end of the links point in (in-links), equal to the sum of all in-links weights. The relation of the properties of two breeds of agents is shown in Figure 2.

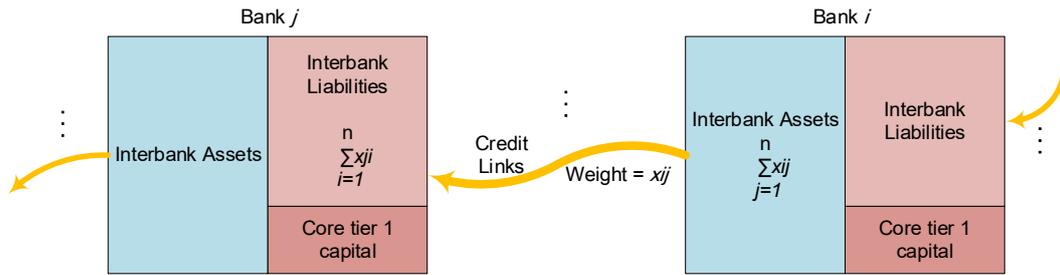


Figure2. Bank agents, credit link agents and the properties

4.2. Agent behavior

The experiment begins with the “sudden death” of a bank agent, the trigger. Then iterate the four steps.

Step 1: The trigger agent’s only behavior is debt default, which means all the credits links between the “dead” agent and its “alive” creditors are removed.

Step 2: The impaired creditor bank agents reduce the weight of the removed out-links from its interbank assets.

Step 3: The bank agents rebalance the balance sheet. The same amount will be reduced from the Core tier 1 capital. If a bank’s Core tier 1 capital is eroded gone or lower than the bank’s capital adequacy regulation, it goes bankruptcy and become the next trigger.

Step 4: Iteration, until no bank fails.

It should be noted that, in theory, the bank will default after the core tier 1 capital all eroded away. But in fact China's financial regulatory authorities have strict restrictions on the capital adequacy ratio of commercial banks. Referring to the Basel III, China's financial regulatory authorities developed a series of requirements for the Capital to Risk Weighted Assets Ratio (CRAR) of the Commercial Banks. The main requirement of this series is the ratio of net amount of core tier 1 capital to net amount of weighted venture capital is no less than 5%. According to this regulation, if the core tier 1 capital adequacy ratio is eroded to below the regulatory requirements, the bank will not be able to operate. So in the experiments, the standard of bankruptcy is set with the core tier 1 capital adequacy ratio lower than 5%.

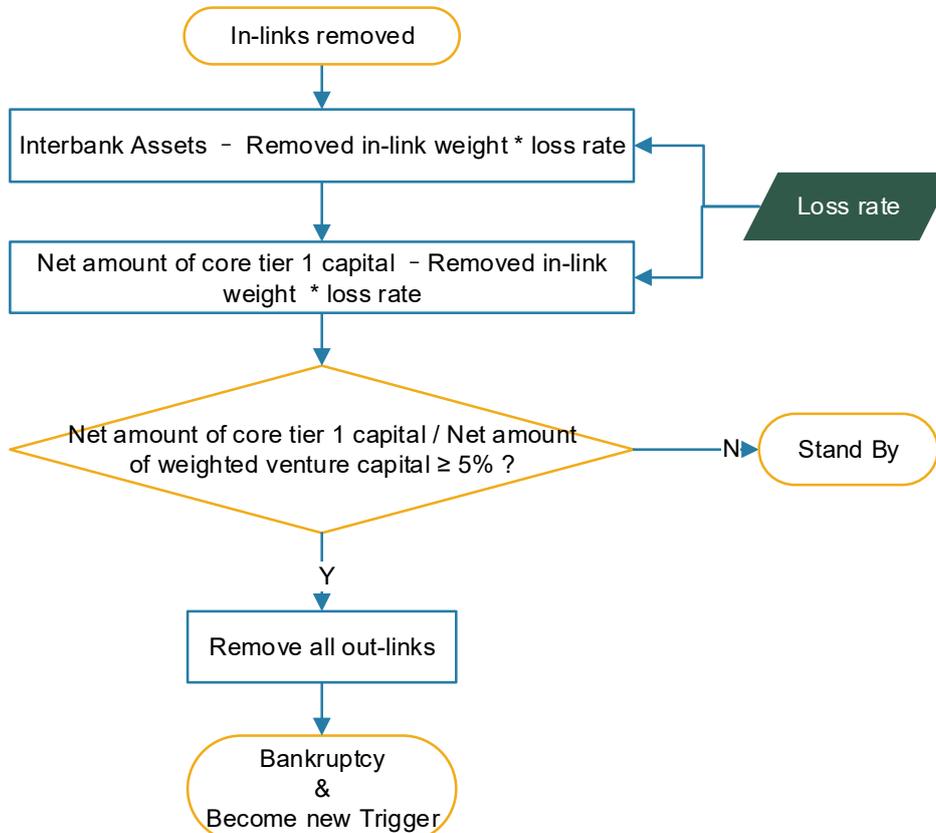


Figure 3. Bank agents behavior flowchart

### 4.3. Experiment design

For debt default, the actual loss is determined by the loss rate  $\in (0, 100\%]$ . So the loss rate is set to be the experiment factor. The experiment is divided into two stage: experiment stage 1 and experiment stage 2.

Experiment stage 1: To investigate the occurrence of cascading failure in the interbank network with the change of loss rate. The experiment factor is loss rate, the dependent variable is number of banks remaining after network stabilization. In fact, the loss rate rarely below 50%, and such a low loss rate hardly to trigger the cascade. Therefore the lossrate is set to [50%, 100%], and the setup is set to 10%.

Experiment stage 2: The banks who triggered the cascading failure in the stage 1 will be put into this stage. Each of them will be used to trigger the cascading failure, and the simulation will be run in small steps of loss rate. The experiment factor is loss rate  $\in [50\%, 100\%]$ , and the setup is set to 5%. The dependent variable is the rounds of cascading failure and the number of bank failures per round.

### 4.4. Computational experiment platform

There are several kinds of agent-based modeling and computational experiment platform, such as Swarm, Repast, Mason, Netlogo and so on. Netlogo is simple to program and provides sufficient experiment tools, and it also have friendly HMI and is operating system independent. In this paper, Netlogo(Version 5.3.1) is used to build the agent-based model and preform the experiment. The computational experiment interface is showed in figure 4. The experiment tools are working in "headless mode" of Netlogo for speeding up, it not appears in this interface.

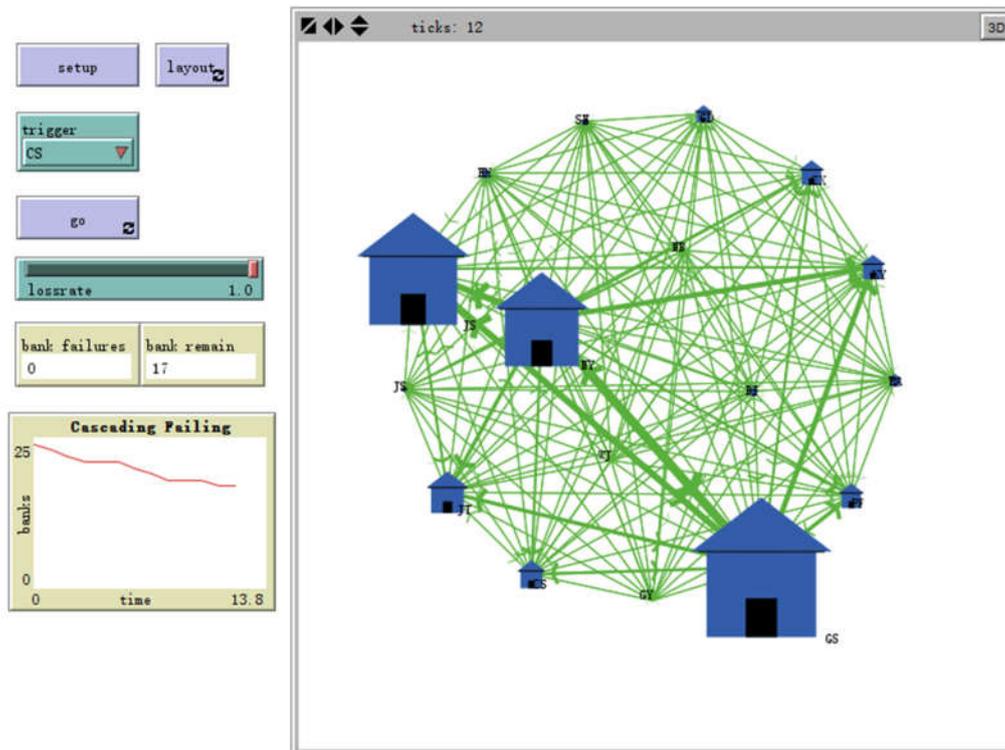


Figure4. Computational experiment interface in Netlogo

## 5. Results and discussion

### 5.1. Results of experiment stage 1

The result of experiment stage 1 shows that default of 6 banks can cause other banks default in the interbank network of China listed banks. These banks are Industrial & Commercial Bank of China (ICBC), China Construction Bank (CCB), Bank of China (BC), Agricultural Bank of China (ABC), Bank of Communications of China (BOCOM), and China Industrial Bank (CIB). The first 4 of them are largest 4 state-owned banks of China, and all of them are on the Global Systemic Important Banks (G-SIBs) list of 2015 made by Financial Stability Board (FSB). BOCOM is the 5th largest bank of China. CIB is the most active participants in the interbank market of China. So the results of experiment stage 1 is credible, and the model can be used in the next stage.

BOCOM dragged only 1 counterparty when lossrate are at 90% and 100%. Technically, the cascading failure did not occur by the default of BOCOM, so it will not enter the experiment stage 2. The 5 banks participate experiment stage 2 are ICBC, CCB, BC, ABC and CIB.

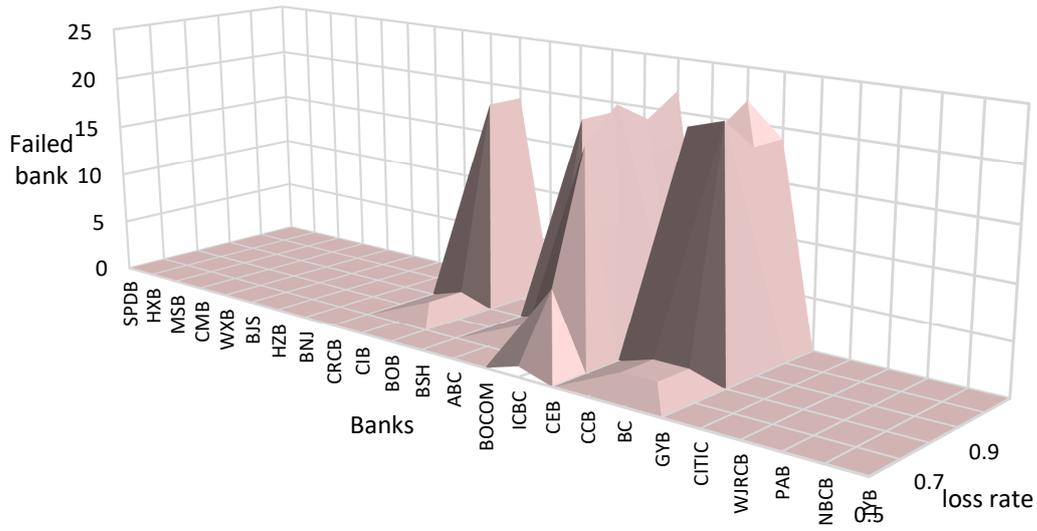


Figure 5. Result of experiment Stage 1

5.2. Results of experiment stage 2:

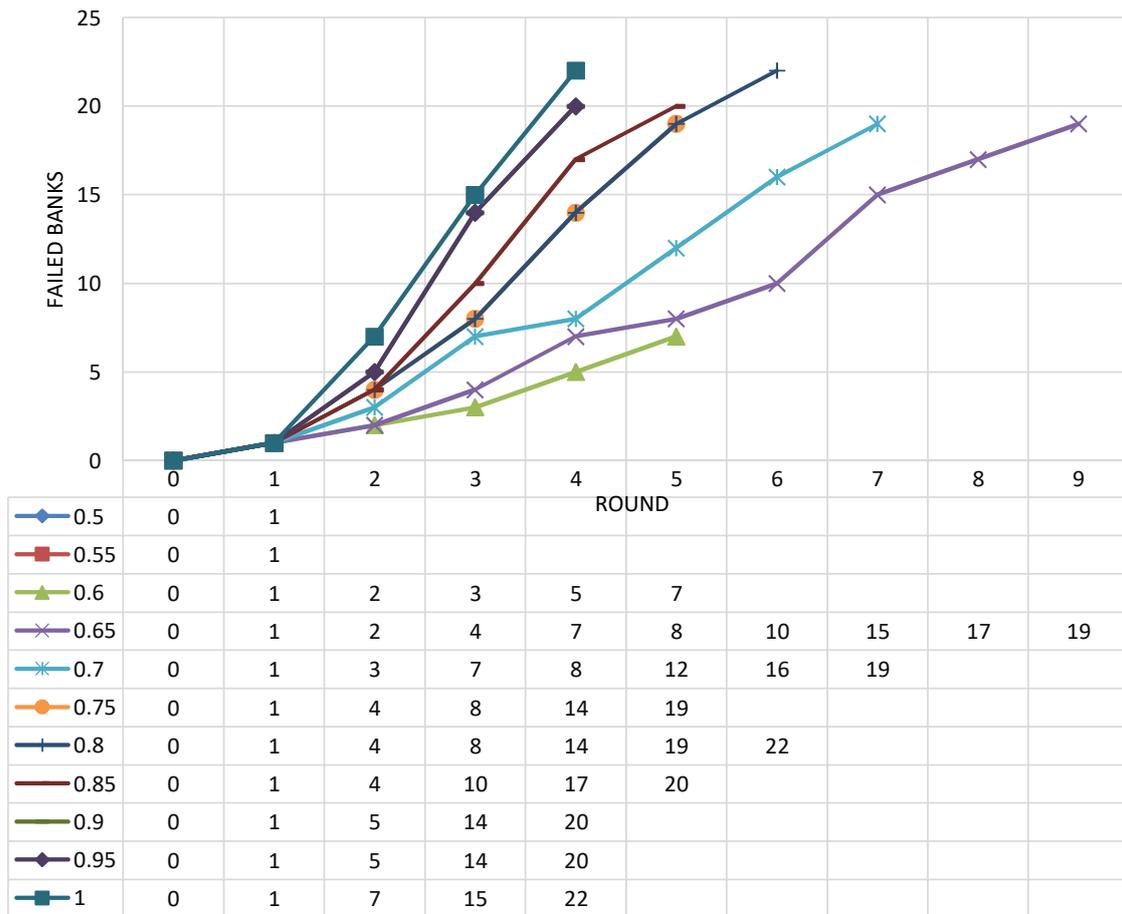


Figure 7. The cascading failure in interbank network of China listed banks, ICBC as trigger.

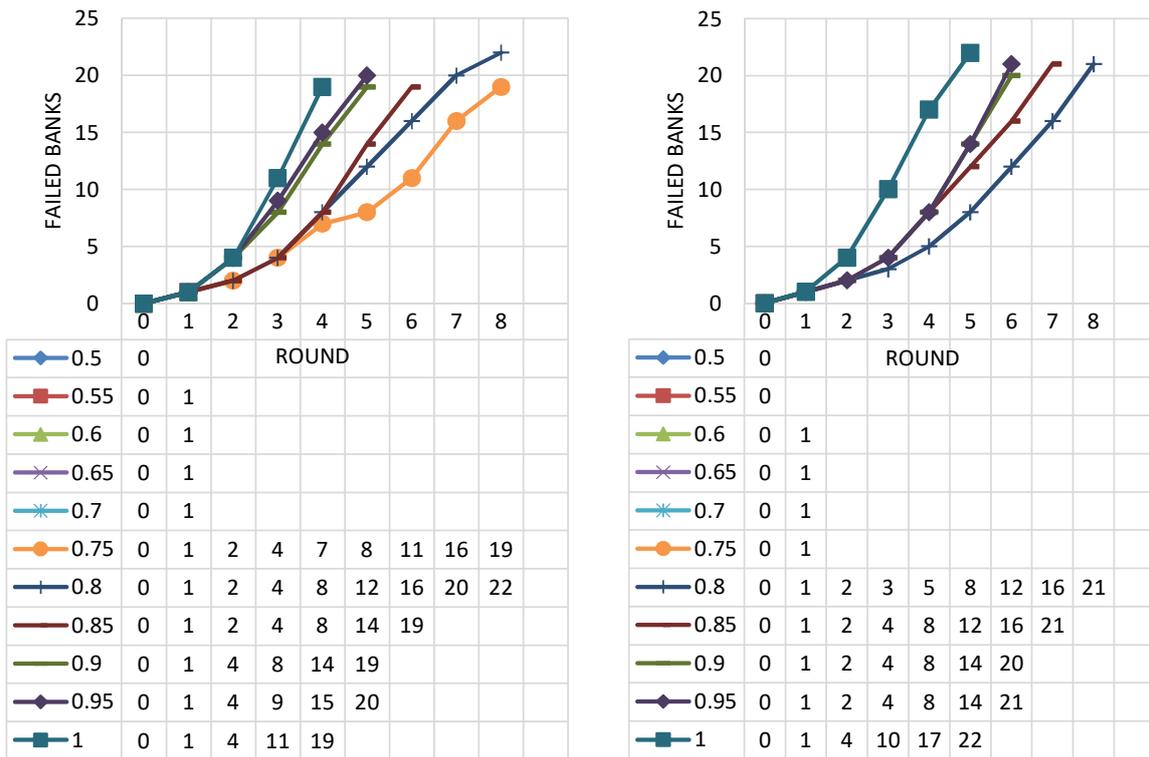


Figure 8. BC triggered cascading failure(left) and CCB triggered cascading failure(right)

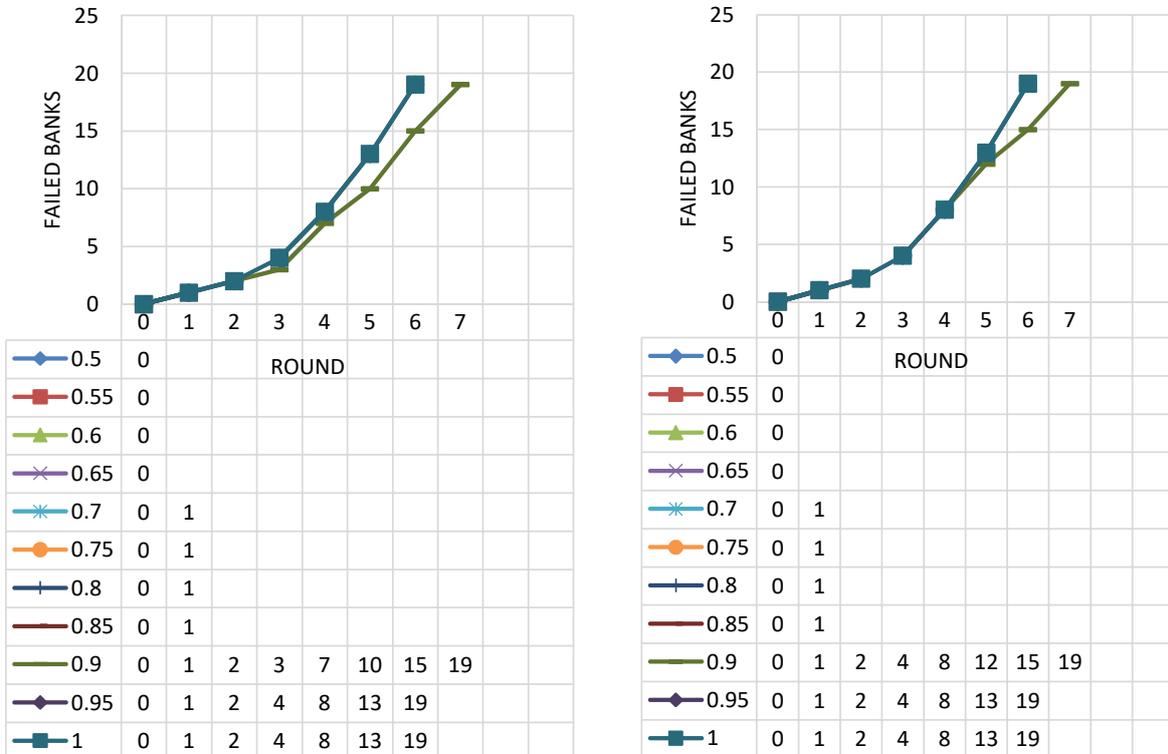


Figure 9. ABC triggered cascading failure(left) and CIB triggered cascading failure(right).

The results of experiment stage 2 show that all the 5 banks can trigger the cascading failure in the interbank network. The cascading failure will crush most of the banks in the network. The activation of the cascading failure has a threshold of loss rate, which is about 60% to 90% depending on the trigger. In general, when the cascading failure occurred, the lower the loss rate, the longer the process lasted.

The default of ICBC can cause the most drastic cascading failure in network, even the loss rate is as low as 60%. When the loss rate is 60%, the cascading failure appears in a small scope, only lasted for 5 rounds and destroyed 7 banks. When the loss rate is 65%, the cascading failure triggered by ICBC will lasted as long as 9 rounds and destroy 19 banks. When the loss rate reaches 100%, the cascading failure can wipe out almost whole system in only 4 rounds. Along with the rising of loss rate from 65% to 100%, the duration of cascading failure shortened and the intensity increased. BC and CCB are similar. BC triggered the cascading failure when the loss rate is 75% and CCB triggered in 80%. But the cascading failure triggered by CCB are more violent than CB in both duration and intensity. ABC and CIB are more inactive. The cascading failure only appears when the loss rate reaches 90% and lasted 7 rounds at most. The number of banks destroyed in the cascading failure are 19, a little less than ICBC, BC and CCB.

## 6.CONCLUSIONS

In summary, 3 conclusions can be drawn from the works above. Firstly, the Maximum Entropy approach can be competent for figuring out the adjacent matrix of the network when the dense network hypothesis is realistic. Secondly, the cascading failure will be triggered by the fail of certain banks with the threshold of loss rate, ICBC is the most dangerous one and the BC, CCB, ABC, CIB following closely. When the cascading failure happen, most banks in the interbank market will be destroyed no matter who triggered and what the loss rate is. Thirdly, the agent-based computational experiment can carry out simulation as good as the numerical simulation in this case.

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