Applying Contemplative Practices to the Educational Design of Mathematics Content: Report from a Pioneering Workshop

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Researchers in the field of mathematics education are beginning to appreciate the potential of contemplative practices such as mindfulness to alleviate students’ stress and increase their focus. What researchers do not yet know is whether, and if so how, bringing focused attention to somatic experience through a wide variety of contemplative–somatic practices (i.e., yoga, Feldenkrais, body–mind centering, and attending to bodily sensations in meditation) may support student learning of specific mathematical content. As a first step toward conceptualizing and ideating the pedagogical design and facilitation of content-oriented contemplative exercises, we convened a workshop to explore these ideas. Here we report on findings from this pioneering workshop, which brought together international scholars and practitioners interested in the relations between contemplative–somatic practice and mathematical reasoning and learning. This report elaborates on participants’ experiences and derived pedagogical insights to offer the field new horizons in the development of the theory and practice of contemplative mathematics.


Contemplative–somatic practice (CSP) covers a wide range of practices in which bodily movement and awareness is used to draw the practitioner’s focus inwards to help them become more reflective or contemplative. Practices commonly used are: yoga, qi gong, tai chi, Feldenkrais, body–mind centering, ecstatic dance (i.e., 5Rhythms), walking meditation, and breath awareness practices used in Buddhist and Hindu meditation. In fact, any repetitive movement, such as running, rowing, or swimming, can lead to contemplative or meditative states.

Mathematics education theorists have begun to engage with ideas concerning the potential role of somatic–contemplative contributions in supporting enhanced pedagogical practice. These efforts include developing mathematics pedagogy (Brady, 2018).
2007), designing contemplative practice for students suffering from high math anxiety (Brunyé et al. 2013), examining the effect of "meditational mathematics"\(^2\) on the affect of special-needs students (Rodd, 2006), and investigating intentional structured reflection for mathematics researchers (Wolcott, 2013). Notably, all this important prior research has focused on contemplative practices’ remedial qualities: that is, for reducing stress and anxiety. However, as Morgan and Abrahamson (2016) have proposed:

As long as contemplative practice is applied only to mathematics anxiety or business-as-usual number crunching rather than to deep embodied and pre-conceptual meaning making [of specific mathematical content], we submit, the field is only scratching the surface of contemplative practices and losing out on their very essence and gift. (p. 36)

Hence our quest, reported herein, is to go beyond general affective effects of contemplative practice\(^3\) on mathematical pedagogy and gain insight into the potential targeted-content effects of these practices—in this case, with an eye on learning specific mathematical ideas. We offer here not the results of an empirical study—we’re not there yet—but rather a dawning overture to potential horizons for this line of inquiry. We have arrived at articulating these horizons by way of collaborating with experts from both mathematics education and disciplines that have historically placed contemplative movement at the core of their scholarship and practice; we invited them to think with us.

On March 17, 2016, fifteen scholars gathered in a quiet hall in Berkeley, CA, to participate in a visioning retreat investigating potential relations between contemplative practices and mathematics education. The retreat was initiated and run by Abrahamson of University of California, Berkeley and Morgan of The Contemplative Academy and funded by Abrahamson’s laboratory. Abrahamson and Morgan sought to bring together contemplative somatic practitioners—in this case a Feldenkrais practitioner and a qi gong practitioner—and researchers from philosophy, mathematics, cognitive science, and educational design interested in experiencing firsthand and sharing connections between physical movement, sensory perception, contemplation, reflection, and mathematics learning.\(^4\) The workshop ran from 9:00 a.m. to 8:00

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\(^2\) In her examination of affect training for students with special needs, Rodd (2006) suggests that meditation may assist these students to “educate the emotional mind.” Furthermore, she links meditation with the Kumon Method, in which students practice easy, silent, daily mathematics, which she describes as “meditational mathematics.”

\(^3\) Contemplative scientists are increasingly suggesting that meditation and other contemplative practices can result in positive affect and affect regulation (see Desbordes et al., 2015).

\(^4\) The educational contingent of the workshop leadership is a group of design-based researchers. That is, they conduct their investigations (many in the field of mathematics design) into cognition, teaching, and learning in the context of developing and evaluating experimental educational interventions. When these investigations suggest that the designed products and activities bear promise, researchers may then scale up their efforts so as to include educational practitioners and conduct further evaluations in schools.
p.m., starting with a gathering and introductory words followed by three contemplative–somatic creative sessions—Feldenkrais, qi gong, and contemplative art—each including an activity period followed by group reflection and then a summary discussion.

The sessions were linked conceptually by the presenters’ shared concern with drawing participants into a somatic–preconceptual space of learning, a space understood by the presenters to be a “liminal” place of learning residing between contemplative consciousness and cognitive assimilation of knowledge (Stelter, 2000; Morgan, 2012, 2013). In the field of contemplative inquiry, the term “preconceptual” is used to describe first-person consciousness, which is subjective, pre-verbal or preconceptual. In turn, second-person, intersubjective consciousness can result from contemplation. And third-person consciousness is objective/public, cognitive, cerebral, and conceptual; it can be framed by contemplative experience, but resides outside of it. While these definitions are used to support theoretical discussion, they are not meant to suggest that these states of consciousness are separate from each other; rather, they are understood as constituting parts of a continuum of consciousness.

INTELLECTUAL RESOURCES

The very idea of infusing mathematics education with contemplative–somatic practice (CSP) occurred to us through engaging in contemplative inquiry theory and practice and design-based research on enactivist mathematics education. Enactivism is a philosophy of cognitive science which holds that: (a) sensory perception is irreducibly intertwined with the action it guides, because the evolution of biological organisms selected for the capacity for sensory perception as a means of regulating their actions, and thus perception and action are inherently connected; and (b) cognitive structures emerge from recurrent patterns in perceptually guided action, so that what we come to know as ontological entities is determined by our goal-oriented sensorimotor experiences (Varela, Thompson, & Rosch, 1991). In this approach, knowledge—that is, the lived phenomenology of understanding, knowing, and reasoning—emerges through ecologically situated, goal-oriented multimodal sensorimotor activity.

Building on this foundational theoretical work, Hutto, Kirchhoff, and Abrahamson (2015) put forth an enactivist approach to conceptualizing mathematics learning, knowing, and reasoning. Their work lent credence to the notion that sensorimotor activity constitutes an early grip on mathematical concepts, a grip that becomes both consolidated and accessible through activities of discourse, quantification, and symbolization. Yet just how cognitive structures emerge from sensorimotor activity to conscious reflection—what Piaget called reflecting abstraction (Abrahamson, Shayan, Bakker, & Van der Schaaf, 2016)—is not quite understood. Similarly, earlier approaches linking subjective experience and mathematics—John Mason’s (2003) discipline of noticing and Wolff-Michael Roth’s (2012) state of pure being—identify the presence of the preconceptual in the learning of mathematics but do not detail the mechanisms by which this happens.
Advancing this work, Morgan and Abrahamson (2016) have referred to an “epistemic bottleneck” between pre-symbolic notions and articulated expression. As it turns out, philosophers and psychologists have been concerned with this very issue of minding the gap between preconscious and conscious phenomenology in contemplative philosophy, psychology, and mathematics theory (e.g., Morgan, 2017). In particular, we have drawn on literature investigating the mechanisms of CSP to propose insight gained from CSP as a means to open epistemic bottlenecks between enactive and reflective modes of mathematical knowing (Gendlin, 1997; Mason, 2003; Petitmengin, 2007; Rothfield, 2009; Roth, 2012; Stelter, 2000).

MODES OF INQUIRY: COLLECTIVE DISCURSIVE REFLECTION ON INDIVIDUAL FIRSTHAND EXPERIENCES

The pioneering workshop we report on herein could be described as “experimental” in the sense of innovation, because we are not aware of any similar meeting in the past. That said, the workshop was not experimental or even quasi-experimental in the sense of scientific research design: there were no well-defined manipulations, measurements, or controls as such. Still, in our evaluation, this phenomenological experiment (Giorgi, 2012), grounded in first-person experience, provides embryonic perspectives and insights worth sharing among the mathematics and greater educational communities; we maintain that the new felt-ideas gained from the workshop could bring about further, more concerted scientific pursuits in years to come. The objective of this report is to bring forth some of these ideas and give them form in light of current and emerging efforts in the contemplative, enactivist, phenomenological, and educational realms of intellectual and practical endeavors.

The entire workshop was audio–video recorded for subsequent qualitative analysis, which generally followed principles of grounded theory (Glaser & Strauss, 1967; 1995; Strauss & Corbin, 1990) and interaction analysis (Jordan & Henderson, 1995) as applied to our entire body of data, including transcriptions of (a) participant discussions after each of the three practice sessions, (b) the workshop’s concluding conversation, and (c) oral and textual dialogue among workshop facilitators before and after the workshop. Even as we searched for emerging themes in our data regarding participants’ insights as they related to the articulation of pre-symbolic notions, we continuously interrogated the implications of these themes for a heuristic design framework on contemplative mathematics (CM) education.

RESULTS

Analyses of the data corpus gave rise to several themes with respect to participants’ experiences in the workshop. Here we present these themes as contextualized in the workshop activities and participants’ contributions to the discussions. In discussing these themes, we will draw implications for future design research in CM pedagogy.
Heightened Somatic Awareness

A central finding relates to the importance of heightened somatic awareness gained through the CSPs of qi gong, Feldenkrais, and contemplative art used in the workshop. Many participants’ experience of this heightened somatic awareness brought new meanings to the surface: these included emergent understandings of their conscious reasoning as anchored in their corporeality as well as attempts to conceptualize their workshop experiences in the context of developing a contemplative mathematics. They commented most often on the expanded bodily awareness the exercises furnished. However, some participants’ limited prior personal experience with these practices appeared to compromise their ability to access contemplative embodied experience, remain in it for an extended period, or explain contemplative embodied experience.

We thus suggest that the best results will come from extended and ongoing CSP such as soft gazing from qi gong. This CSP draws awareness into the body by reversing the usual experience of sight as “looking outwards” (in the case of the workshop, toward a vase of flowers) to one of “looking inwards” (where one may sense the flowers looking at you). After experiencing soft gazing, one participant reflected that the challenge posed to his usual experience of viewing an object had resulted in his awareness of a more nuanced understanding of vision. He said, “There is seeing, and there is seeing; there is hard looking-at, and there is soft receiving of images.”

Results of this and similar exercises offer designers of math pedagogy insight into the development of sense-based exercises (i.e., looking practices) that provide an opportunity for the embodied metacognition described by workshop participants. This increased awareness relates to both learning content and the self-awareness/realization of individual learning processes. Learning through the senses in this way arises from engaging the subtleties of somatic awareness and is strengthened through practice, honing new felt meanings. For example, one participant described finding muscular tension in his neck as he searched through his body for the new way of seeing in soft gazing. This was important, for in reaching for the part of himself that held the reversed vision, this participant stumbled across and released the tension in his neck that he felt had blocked his interoceptive path to sensing the flowers looking at him and thus sensing a new way of knowing (see also Cole, 2004).

Becoming conscious of one’s own way of knowing can be a key experience in shifting to new mathematical understandings. For example, if you are gazing at a rectangular array of 12 dots and are seeing them as 3-by-4, becoming conscious of this implicit perceptual pattern is a first step to seeing the array alternatively as 4-by-3. Toggling between these constructions is a perceptual instantiation of the commutative property of multiplication (viz., $a \times b = b \times a$); thus doing so creates an opportunity to ground this foundational concept.
In the *imagined movement* exercise of the Feldenkrais Method (FM) session, participants were asked to imagine physical maneuvers without actually enacting them in practice. Results from this exercise highlight how FM (and other CSPs) can heighten somatic awareness. Here a participant describes his discovery of phases in the continuity of the “imagined” and the “doing,” realizing that the boundary between them is ill-defined and perhaps merely an illusory rationalization or category error:

I felt I could move back and forth between the imagined and the actual, and that there’s actually continuity there, because I can imagine that I’m lifting my hand, I can lift my hand, but there’s that very subtle in-between, where I’m just ready.

Participants’ descriptions suggest that enhanced somatic or bodily consciousness is in effect a form of knowing, as it appears to link directly to new meaning-making. For designers, a central aspect of enhanced bodily awareness and learning relates to students’ improved ability to access subtleties of movement by shifting between mental and physical foci, between the imagined and the felt.

Again, the need for regular practice is important, as it is required to strengthen refinement of focus. Designing for the subtle intertwinings of imagined and actual movement that FM and other CSPs reveal can expand the repertoire of pedagogical resources for designers. In particular, individuals’ sub-motor experiences of movement suggest that efferent activity may be sufficient for grounding mathematical meanings; for example, intending to sweep a hand across a spatial expanse that one means to measure may constitute a sensorimotor experience that is closer to actually executing that action than educators had hitherto assumed.

Participants’ sense of moving between corporeal (bodily) and incorporeal (abstract) realms, which resulted from heightened somatic awareness, also helped strengthen the learning passage between movement and reflection. Shifting between the contemplative–somatic–preconceptual and the conceptual was a recurring theme throughout the workshop and therefore could potentially constitute an important element for designers. Significant here is the way participants, through heightened somatic awareness, drew their sensed knowing of the preconceptual into conceptualization. The interspersion of the preconceptual, somatic, and cognitive was signaled by participants’ use of somatic metaphor. In this shift between domains, one participant (a mathematics educator and theorist) realized through the practices and his reflection on design for mathematics that “[you] *do* and *become* the procedure”; this was likened to learning to ride a bicycle, where the learning was in the “feel” of riding, and for him it resulted in a feeling of “wholeness.” Thus becoming and doing, the somatic–intuitive and the cognitive as realized through CSP, led to an experience of corporeal/incorporeal unity. This unity must contain both the preconceptual and the conceptual to instill what this participant described as a “feeling of balance in an answer.” Linking his experience with mathematics problem solving, he emphasized the affective importance of his felt sense of balance in bringing forth insight:
You’re noticing the whole center. Although you can’t describe all the parts, you feel balanced [as in riding a bicycle], and so when you’re solving math problems…there’s chaos, there’s chaos, there’s chaos, and then you see the whole thing in balance.

In sum, participants’ experiences in the CSP problem-solving exercises resulted in their expanded and integrated sense of preconceptual-corporeal-cognitive space, thus supplying them—and, in turn, mathematics designers—with new resources.

**Describing the Preconceptual**

A primary issue in CM is understanding and describing, to oneself and others, one’s fragile sensations and notions that are often characterized as ineffable. Thus when designing CM it is important to supply students with the means to translate their felt knowing from the contemplative interior into verbal realization: that is, to help them shift from the contemplative and somatic modes of awareness to the cognitive and verbal. We suggest this can be made possible by scaffolding students’ deeper awareness through the use of contemplative creative practices and specific forms of discourse. Importantly, these reorientation techniques need to encompass the somatic, contemplative, preconceptual, conceptual, and idiosyncratic ways in which students may engage with such processes.

In the interest of providing participants with firsthand experiences of these ineffable qualia as well as techniques for rendering them effable, Morgan ran exercises from contemplative-art practices that have been described by Morgan and Abrahamson (2016) as *bridging practices*: exercises that enable students to firm up preconceptual somatic meanings in color, shape, rhythm, and form before shaping them into words. These techniques center on the role of formulating language “inside-out”—that is, bridging the preconceptual–somatic with the semantic–semiotic realms of consciousness via contemplative creative practices. After being asked to contemplate a problem and then express it through contemplative art, one participant described the experience:

> The forms I drew were patterns; some rounded and spiraling.…The problem, shown this way, included or seemed to represent itself in an organic context that did not comprise a negative…and it is easy to experience again in my memory.

As such, the art experience enabled this participant to contemplate a problem within a stratum of embodied cognition that was at once effective in promoting a solution and untethered by the restrictive facets of symbolic notation. Similarly, when contemplative art is offered after contemplative engagement with personal or content (mathematical) problems, it acts as a bridge between preconceptual meaning-making and cognitive translation of that new meaning; in other words, it brings preverbal meaning forth into words.
Contemplative art also offers the opportunity to work with students’ idiosyncratic symbol systems, thus offering a nonverbal means, sited between the somatic and cognitive, to knead meaning-making. In this way, contemplative art can be used in mathematical problem solving. Participants experienced this form of problem solving in the workshop, where they were invited to bring a problem into their somatic–preconceptual consciousness and engage with it through guided visualization. They were asked about the problem’s size, shape, and color, and after unstructured time with the problem they were invited to draw their experience, which then supported their ability to describe it verbally.

Morgan (2018) hypothesizes that the act of contemplative drawing grounds the contemplative experience, as it provides an intermediary space between preconceptual-somatic and discursive experience. The use of contemplative creative practice offers a much needed space of integration, half in and half out of the liminal, for the sense of felt knowing can easily dissolve by the time the practitioner or student returns to get what Gendlin (2010) calls another “feel” of it. This dissolution, in turn, can lead to a frustrating sense of “losing it,” a “slipping away” of the “feel” of problem resolution. This highlights two relevant points in contemplative mathematics: firstly, the importance of providing creative bridging practices, as they support students’ intuitive grasp of mathematics; and secondly, the vitality of emphasizing the need to let go of self-judgment, for those students who believe that in losing the “feel” of problem resolution they have “not done the contemplative exercises properly.”

Making Meaning, Meaning-Making as Learning

A significant insight from our findings for designers of mathematics-learning activities was the two-fold nature of meaning-making arising from CSP: we found that participants’ meaning-making was both focused on their somatic contemplative experiences and facilitated by these experiences. One participant, reflecting on her qi gong experience, reported that through heightened somatic awareness (focusing on her experiences) she could sense her actions shifting from discrete to continuous movements, and that this sensation in turn facilitated an improved performance of the entire qi gong form. The passages between discrete and continuous negotiations of spatial intervals which she described are in turn pertinent to complementary mathematical conceptualizations of motion. This aspect of meaning-making, in which heightened somatic awareness becomes a mode of learning, occurred in different ways throughout the workshop. Frequently, exercises offered access to an increasingly refined ability to sense the process of change or learning as it occurred in participants’ contemplative–somatic interior; thus, tracing the steps of their learning enhanced participants’ ability to make cognitive meaning of their learning, which in turn supported the learning. This aspect of the meaning-making that can result from CSP is important for educational

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5 Interested readers are referred to the following URL to watch an 11-minute video presentation on this study: https://youtu.be/h_qim48FRw. The video was presented at the 2017 annual meeting of the Jean Piaget Society.
designers to consider; bearing it in mind, they might design mathematics pedagogy that includes prompts to elicit this meta-awareness in contemplative–somatic learning.

Participants also attempted to make meaning conceptually, either by reflecting on past experience or via conceptual models. Some participants applied the concept of culture to their experience, generating interesting connections between felt and cognitive learning. As an example of the development of personal meaning-making narratives, one participant conceived through his contemplative–somatic experience the notion of a “cultural body,” linking it to ideas of generative faculties and sensory reality to form what he described as a “bodily algorithm,” which he understood to be a cultural form. This significant meaning-making aspect of CM needs to be encouraged through pedagogical devices. Both types of new meaning-making occurring in the workshop—the contemplative–somatic and the cognitive—provide pedagogically rich approaches for contemplative mathematics.

**Pedagogy**

Drawing from our workshop findings, we believe a central tenet of CM is the idea that conceptual meanings come forth through planning, visualizing, and enacting movement. Though these three activities may be phenomenologically disparate, they lead to similar outcomes, by way of enhancing awareness of one’s sensory pathways. As a participant said, “I’m just sort of struck by attention, the attention being held through the body so precisely, and then the second thing is attention about attention.” As discussed earlier, the theory of enactivism conceptualizes subjective cognition of mathematical concepts as grounded in reflective physical action on/with cultural forms, which may be material, virtual, or imaginary. Designers of mathematics-learning activities build environments, activities, and facilitation protocols that create conditions enabling instructors to intervene in students’ skill-learning processes by way of correcting their movement and/or orienting their awareness to their proto-movement. Applying these methods of heightening students’ somatic awareness can aid instructors in formatively assessing “where” each student is in bringing forth a new notion: for example, students’ awareness of pre-action can indicate whether they are or have been in the pre-cognitive ground of learning.

With regard to “enacting,” the workshop evinced occasions of FM and qi gong facilitation methods carrying students beyond the immediate context to consider and enact movement forms, particularly via qi gong’s cultural–historical use of somatic metaphor and FM’s use of instructions for imagined movement. Focusing on different forms of instruction, media, and representation practices is important when designing CM. Instructional resources can be used to document and communicate target sets of actions (such as in offering students visual feedback on their movements) and then furnish designers with crucial information (e.g., signifying the ways students have read and understood traces of their own movement interaction). Multimodal expressive activity, particularly verbal and gestural utterance, creates for instructors a linguistic manifold for guiding students in a nuanced, indirect manner.
Finally, two common outcomes of the workshop’s CSPs have implications for mathematics design: first, students’ development of enhanced ability to focus and precise awareness of somatic, preconceptual, and cognitive resources, which in turn support heightened focus; second, the enhancement, through greater awareness of these resources, of students’ ability to assemble them for problem solving.

**SUMMARY AND FURTHER REFLECTIONS**

Each of the practice sessions provided useful insight for the design of CM. The qi gong session emphasized a need to encourage heightened somatic focus. Qi gong’s movement forms and use of metaphor were shown to heighten the somatic focus that supports entry to an internal ground of somatic–preconceptual learning: the movement forms, which encourage inner focus, allowed participants to shed discursive and habitual thinking, while instructional metaphor encouraged the development of non-habituated, preconceptual, and heightened somatic awareness. Movement, unsettling habituated perception, and metaphor enable entry into the somatic–preconceptual, understood in qi gong as a space evenly balanced between receptivity and engagement. This linking of the passive and active in an internal space mirrors the linking of brain and body through imagined and actual movement in FM. Lastly, the use of metaphor in qi gong is understood to work both ways; that is, it can both cast practitioners into inner or preconceptual space and support their return to cognitive awareness (see Abrahamson, Sánchez-García, & Smyth, 2016, for an interdisciplinary ecological-dynamics discussion of metaphor).

Several considerations in FM instruction are significant to the design of CM. Perhaps most important is ensuring that, while there are instructions and these are at times quite precise, students’ felt outcomes are not prescribed but rather supported in their emergence. The affective dimension of heightened somatic awareness is another important element. Also, as contemplative practice heightens somatic, affective, and psychological sensitivity and can thus lead to negative emotional, psychological, or physiological outcomes such as a sense of disorientation or disembodiment (though these are not common), designers must build into contemplative pedagogy an ethic of care; for this reason we suggest that contemplative pedagogues should also be contemplative practitioners, with experience and training in this and all other relevant aspects of CM. Finally, the contemplative-art approach to mathematical problem solving offers a means to bridge the preconceptual and cognitive through movement, color, form, and symbol-making; creative responses can then act as points of reflection that students can use to stabilize and cognize felt meaning arising from the preconceptual interior.

One participant, in reflecting on her FM session, outlined eight additional key points for CM designers, points that are relevant for each of the practices:

- Ensure the presence of students’ attention in the body.
- The development of meta-awareness is directly related to the development of attention and learning through the body.
• Comparison—across felt experience and reflection (e.g., FM’s actual and imagined movement)—can be useful in pedagogy, as it maps well onto analogous exercises in mathematics.

• It is important to disentangle all components or resources, such as ideas, imagination, bodily movements, or postures, to understand their separate and combined impacts and applications.

• Pay attention to the preconceptual–somatic–cognitive resources that students and educators draw on regularly.

• Due to the subtlety of contemplative-preconceptual-somatic awareness, it is important in the educational relationship to maintain awareness of student and educator responses and exchanges.

• Ensure non-judgmental awareness and compassion for self and others in educational practice.

• Experiencing all the above in person through CSP and creative practice provides the opportunity for educators to reflect on these experiences before designing them for others.

We hope these suggestions regarding the qualities and pedagogical potential of contemplative–somatic mathematics contribute information to the scholarship and practice of our field that is both useful and relevant to the philosophy, theory, research, design, and facilitation of mathematics learning.

REFERENCES


