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ORIGINAL ARTICLE

Enactivism and ethnomethodological conversation analysis as tools for expanding Universal Design for Learning: the case of visually impaired mathematics students

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Abstract

Blind and visually impaired mathematics students must rely on accessible materials such as tactile diagrams to learn mathematics. However, these compensatory materials are frequently found to offer students inferior opportunities for engaging in mathematical practice and do not allow sensorily heterogenous students to collaborate. Such prevailing problems of access and interaction are central concerns of Universal Design for Learning (UDL), an engineering paradigm for inclusive participation in cultural praxis like mathematics. Rather than directly adapt existing artifacts for broader usage, UDL process begins by interrogating the praxis these artifacts serve and then radically re-imagining tools and ecologies to optimize usability for all learners. We argue for the utility of two additional frameworks to enhance UDL efforts: (a) enactivism, a cognitive-sciences view of learning, knowing, and reasoning as modal activity; and (b) ethnomethodological conversation analysis (EMCA), which investigates participants' multimodal methods for coordinating action and meaning. Combined, these approaches help frame the design and evaluation of opportunities for heterogeneous students to learn mathematics collaboratively in inclusive classrooms by coordinating perceptuo-motor solutions to joint manipulation problems. We contextualize the thesis with a proposal for a pluralist design for proportions, in which a pair of students jointly operate an interactive technological device.

Keywords Blind · Embodiment · Enactivism · Ethnomethodological conversation analysis · Technology · Visually impaired

1 Background and objectives: seeking equitable design for blind mathematics students

Providing sensorily diverse learners equitable access to quality mathematics instruction is an enduring educational problem, evidenced by persistent achievement gaps that cannot be explained by differences in mathematical aptitude (Healy et al. 2016). de Freitas and Sinclair (2014) argue two key issues are at stake:

> (1) the loss of one sense may change the way other senses are used, which may lead to certain opportunities that often go untapped; and (2) mathematics itself changes under different sensory organisations. (p. 148).

We join this call to rethink mathematical praxis in rethinking access. In so doing, we have found strong resonance with the objectives, principles, and efforts of Universal Design for Learning. The *Universal Design* movement was first developed by architects and product developers

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(Goldsmith 1963, 1997; Mace et al. 1991) and later applied to education as *Universal Design for Learning* (UDL, Rose and Meyer 2002). In rethinking mathematical praxis, here we argue that UDL efforts could be enhanced by additional theoretical and analytical approaches. Following de Freitas and Sinclair, we submit that stepping forward with instructional design for sensorily heterogeneous students requires first stepping back to question enduring assumptions about what it means to learn and what roles *sensory* perception and *social* interaction play in the learning process. We propose *enactivism* and *ethnomethodological conversation analysis* (EMCA) as essential tools for contemplating mathematics learning environments that are perceptually pluralistic and capable of supporting collaborative meaning-making for sensorily diverse learners. This conceptual paper illustrates the combined utility of enactivism and EMCA for enhancing both (1) design—re-imagining mathematics learning environments that can support sensorily heterogeneous learners and (2) theory—understanding how learning occurs in such environments.

First, we will argue that enactivism allows us to more fully appreciate mathematics learning as a process of expanding one's capacity for situated sensorimotor activity, wherein the sensory modalities involved need not always involve the optical, but may be kinesthetic, proprioceptive, haptic, and so on. Then, we demonstrate EMCA's potential for refining our understanding of how participants achieve mutual understandings that lead to new multimodally constituted mathematical ideas in perceptually pluralistic learning ecologies: We come to know what we consider to be shared reality by means of the mundane interactional work we do to engage in successfully coordinating activity together, including displaying, repairing, and revising our ongoing interpretations of the world.

In Sect. 2, we will present the general research problem, giving blind and visually-impaired students equitable access to mathematics learning experiences. Then, in Sect. 3 we will outline how UDL responds to this problem. Section 4 develops our perspective on how enactivism and EMCA could infuse UDL with productive argumentation and solutions. To conclude our argument, we propose an instructional activity for ratio and proportion, in which interactive tools would enable sensorily heterogeneous students to collaborate in achieving the enactment, mutual sensation, and mathematical signification of coordinated movements that thus come to instantiate target concepts and practices. Section 5 summarizes our paper.

2 Research problem: lost in translation—blind students' inequitable access to mathematical concepts and educational activities

Blind and visually impaired[1] students significantly underperform academically compared to sighted peers. This achievement gap is greater in mathematics (Brothers 1973; Clamp 1997; Rapp and Rapp 1992), with spatial problems seeming to present a particular challenge (Morash and McKerracher 2014). However, there is no evidence that visual disability causes or correlates with impaired numeracy or quantitative reasoning, suggesting that impaired mathematical aptitude is not to blame. Instead, we join de Freitas (2016) to maintain that this observed gap is likely the result of sub-optimal opportunities—including technological resources, learning activities, and instructional methodology—for teaching and communicating about mathematics and spatial concepts using common non-visual accommodations (see also Healy et al. 2010 on integrating emerging inclusive technologies into classrooms).

Blind students must rely on alternative tools and materials for learning mathematics, including braille, tactile diagrams, description, and text-to-speech. However, these "accessible" materials present inferior opportunities for learning when compared with visually based instructional materials. Important information, from the subtle to the significant, can be lost in recasting visual materials. For example, print and braille representations of mathematical expressions have significant notational and structural differences, visual graphics are often simplified when they are converted into tactile formats, and visuo-spatial representations are replaced with cognitively demanding verbal descriptions. Moreover, braille is highly space-consuming, and page-space limitations often make it impossible to take advantage of spatial notation. Even refreshable braille displays are only able to present a single line of braille at a time. Space limitations motivate compact, rather than spatial, modes of notation and eliminate the potential for vertical spatial structuring of expressions. Students are forced to rely on notational rather than spatial conceptual chunking of expressions. Expressions and equations can also be verbalized, either by a human reader or with the use of text-to-speech (TTS) technology, but limitations on working memory and cognitive capacity makes this approach largely impractical. Tactile graphics can be used to graph equations, illustrate geometry problems, tabulate data, and more, but again, limitations on tactile information density constrain the complexity of the graphics. Tactile

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[1] Only for purposes of text flow herein, we hence abbreviate this phrase to "blind." This should not be taken to imply disregard to important nuances of gradients in individuals' visual capacity.

lines, textures, symbols, and braille annotations cannot be too densely packed without seriously compromising the graphic's tactile readability, and the density tipping point for tactiles is far lower than it would be for corresponding visual graphics (Herzberg and Rosenblum 2014).

Beyond inferior tools and materials, the *learning experiences* that visually impaired mathematics students have access to also frequently suffer. Numerous studies have demonstrated that mathematics learning is enriched through collaborative, active participation in learning activities with peers and instructors, where meanings are negotiated and refined by way of argumentation, exploration, and sense-making (e.g., Asterhan and Schwarz 2009; Cobb et al. 2000; for a review see; Lerman 2000). These experiences are made possible by constellations of social and material resources (e.g., task goals, participation frameworks, tools, etc.) that form dynamic, emergent *ecologies* (Erickson 1996; Lemke 1998). However, currently, productive participation in most collaborative mathematics learning experiences requires joint *visual* attention to tools like whiteboards and other strictly visually accessible shared representational spaces and media. Mathematics learning ecologies that assume and require vision as a prerequisite for participation will always underserve blind learners and therefore need to be re-imagined as social and material ecologies that can support active engagement of sensorily heterogeneous students through multiple perceptual modalities in negotiating new mathematical ideas together (Quek and Oliveira 2013; Sedaghatjou 2018).

3 Embracing a perceptually pluralistic ontology and epistemology of mathematics

The impressive careers of blind scientists and mathematicians like Abraham Nemeth, Bernard Morin, Newell Perry, Lev Pontryagin, and Nicholas Saunderson demonstrate that vision is by no means a prerequisite for extraordinary achievement in the disciplines (q.v. Amalric et al. 2018). Understanding their success requires a shift in discourse on the modal and material constitution of mathematics knowing and learning and has implications for our current conceptualizations of dis/ability (Scherer et al. 2016). As we approach the design of learning experiences for blind mathematics students, we must query prevalent vision-based assumptions about mathematical learning, knowing, and discourse.

For example, when blind students learn mathematics through engaging in activities centered on artifacts, a striking manifestation of this learning occurs when students gesture to communicate their haptic, tactile, and kinesthetic experiences. Healy and Fernandes (2011) comment on the provenance of gestures in modal experiences that do not include vision:

> [G]estures [of blind mathematics students] are illustrative of imagined reenactions of previously experienced activities.... [T]hey emerge in instructional situations as embodied abstractions, serving a central role in the sense-making practices associated with the appropriation of mathematical meanings (p. 157).

Non-visual modalities, such as the kinesthetic, proprioceptive, tactile, and haptic, thus readily enable the development of mathematical knowledge. Moreover, these non-visual perceptual modalities are not *substitute* ways of knowing mathematics, because for the students in question they do not stand in the stead of anything—they are bonafide, legitimate, subjectively meaningful, and conceptually adequate constitutions of the notions that come forth for the students as they engage in activities designed for them to appropriate mathematical meanings. As de Freitas (2016) submits, in theorizing blind students' modal experiences with mathematical artifacts, "The human body becomes differently abled when we consider how contemporary assemblages of human and non-human engender new kinds of experiences" (p. 198).

In this section we: (3.1) problematize ocularcentric epistemologies and ontologies of mathematics; and (3.2) introduce UDL philosophy and solutions.

3.1 What you see is not what you get: unpacking sensory and representational modalities

In mathematics education, visual modes of perceiving and knowing are frequently privileged over other modalities, consistent with a long standing "hegemony of vision" or "scopic regime" pervasive in Western epistemology (Jay 1993; O'Loughlin 2006). The embodied and sociocultural turns in mathematics education have seriously challenged this scopic regime by interrogating the nature of mathematics and what it means to do and know it. These turns have led to a shift away from a view of mathematics as a purely intellectual realm divorced from human experience and recast it as collective disciplinary praxis distributed over bodies, tools, and individuals, not unlike other domains of human expertise like masonry or animal husbandry (Barnes et al. 1996; Saxe 2012; Urton 1997; Wittgenstein 1953).

Educational research informed by this turn in the philosophy of mathematics examines how cultures develop, foster, and regulate participation in activities that attend to logico–quantitative relations among objects, where these objects may be either actual or imagined assemblages (e.g., Saxe 2012). One consequence of reclaiming mathematics as a human practice has been a comprehensive search for its corporeal, multimodal, and social provenance and constitution (Hall and Nemirovsky 2012; Núñez et al 1999; Radford 2009). In response, some educational researchers have been seeking to redefine the role of vision in mathematical learning and knowing (e.g., de Freitas 2016). In particular, Abrahamson et al. (2014) have argued that if we are to continue using the term *visualization* in reference to mathematical sense-making, we must take it metonymically to encompass also the panoply of nonvisual sensory phenomenology. Indeed, spatial relationships constituting mathematical structures can be directly apprehended through auditory, kinetic, tactile, and haptics means (e.g., Fortin et al. 2008).

In practice, however, non-visual sensory modalities continue to be rarely capitalized upon. Instead, "visual" and "spatial" representations are often conflated, so that most technological resources for mathematics learning communicate spatial relationships strictly via optically accessible features. For example, geometry concepts and practices are often introduced with videos or drawings of 3-dimensional figures, and these same forms of learning media are also utilized to convey algebraic concepts with spatial representations. However, these modes of representation are primarily visual and thus inaccessible to someone who cannot see. Alternatively, the illustration of mathematics concepts can be more accessibly accomplished with 3-dimensional models that leverage multiple sensory modalities (e.g., including touch) and can just as adequately, if not superiorly, capture the spatial concept (e.g., see Horvath and Cameron 2017, on algebra).

While vision has advantages for engaging with the environment, it is not *sine qua non* for the mathematical work of imagining spatially vested objects, structures, and transformations. To overturn this scopic regime in mathematics education, cognitive scientists, education researchers, and practitioners must work harder to differentiate between *seeing* and *knowing* when designing and evaluating learning environments. By recognizing ocular access as merely one among many ways of apprehending and constructing spatial and quantitative relationships, possibilities for designing sensorily heterogeneous learning ecologies accessible to all emerge. As we now explain, there are promising alternative approaches to educational design that challenge prevailing scopic ontology of abstract mathematical objects.

3.2 Universal Design for Learning (UDL)

Inspired by Universal Design (UD) paradigms from architecture and product design (Goldsmith 1963, 1997), the UDL framework was first developed by David H. Rose and subsequently adopted by the Center for Applied Special Technology (CAST; Rose and Meyer 2002). UDL calls for instructional design where: (1) information is made perceptually accessible in a multitude of ways; (2) students have multiple means of engaging with information that are customized to their needs, interests, and motivations; and (3) students have opportunities to express their understanding of this information in a multitude of ways.

Rather than add specialized and potentially ostracizing accommodations for "atypical learners," UDL holds that educational experiences should be designed in the first place to be usable and accessible to the widest variety of learners possible, removing accessibility barriers ab initio from informational displays and artifacts (see also the "born accessible" movement, Capiel 2014; Summers et al. 2012). Although many digital learning materials have proliferated in the educational market such as infographics, video, and animations, that claim to embody Universal Design (UD), they may not be truly accessible (Siu 2016). Successful UDL solutions emerge from close collaboration with practitioners whose pedagogical expertise in working with students with accessibility needs provide crucial insight to design (Siu 2016).

UDL philosophy resonates with systemic perspectives on human behavior like ecological psychology (Gibson 1977) that view individuals as actively forming functional relations within larger activity structures, which include the material environment as well as other people. UDL's objective is to remove or mitigate environmental constraints on individuals' access to information (Burgstahler 2001). This systemic view of learning shifts the burden of access from individuals with heterogeneous physical, intellectual, or sensory ability to the social and material ecologies that can enable or constrain efforts to form functional relations (Iwarsson and Stahl 2003).

When educational settings, products, and practice are geared to serve accessible learning experiences to a plurality of diverse students, classrooms become more inclusive and less differentiated. Such accessibility solutions de-pathologize physiological heterogeneity and de-dichotomize diagnostic labeling by benefiting all students. For example, the use of closed captions makes video content more accessible not only to hearing-impaired students but also second language learners, students sitting far away from audio speakers, and so on (Braun 2008; Ely et al. 2006; Encelle et al. 2011; Krejtz et al. 2012; López 2010; Packer et al. 2015).

We propose that creating and analyzing successful UDL for mathematics could be enriched through its substantiation in enactivist theories of learning and the use of ethnomethodological conversation analysis to investigate sensorily diverse learners' collaborative meaning making practices in such environments. In the next section we draw on these perspectives to warrant a perceptuo-pluralistic design rationale that: (1) instantiates perception-for-action as the cognitive grounding of mathematical concepts (inspired by enactivism); as it (2) foregrounds multimodal social interaction as a nexus where perception-for-action is consolidated towards normative disciplinary praxis (inspired by ethnomethodological conversation analysis).

4 Towards new solutions: how enactivism and ethnomethodological conversation analysis can inform the development and evaluation of perceptually pluralist mathematics UDL

As we strive to re-imagine instructional materials for sensorily heterogeneous students, we have been inspired by enactivist and ethnomethodological approaches to learning. These perspectives foreground the critical roles pre-symbolic multimodal activity and social sense-making practices play in constituting mathematical reasoning. As such, these approaches help us reconsider what it means to learn new mathematical concepts and practices, so that we can develop more equitable, inclusive, post-deficit, and perceptually pluralistic design, facilitation, analysis, and scholarly discourse around instructional processes. Here we outline these perspectives (Sects. 4.1 and 4.2), and then we propose their integration in practice (Sect. 4.3), by demonstrating how these two approaches, combined, may inform the design of equitable mathematics learning activities.

4.1 Enactivism: cognition emerges from recurring patterns in perceptually guided action

Enactivism, an embodiment perspective on cognition, holds that knowing the world comes forth in, through, and for engaged activity. Sensory perception is *irreducibly intertwined* with the action it is serving and is served by, and stable cognitive structures emerge from recurrent patterns in this perception–action entwining (Varela 1999). Like ecological psychology (Gibson 1977) and phenomenological philosophy (Merleau-Ponty 2005), enactivism takes a post-dualist, systemic view of cognition as constituted dynamically in functional relations across malleable complexes of people and materials. Objects in the world reveal themselves to us not directly or as representations, but as enactive horizons. These new possibilities for engaging the environment suggest themselves through active and purposeful processes of assembling available interrogations of shape, texture, spatial position, color, smell, sound and taste (e.g. Gibson 1977). Different individuals privilege different modalities for perceptually guiding their actions based on their sensory access to the situation (e.g., a sighted individual might recognize an object in a darkened room as a table by feeling it with their hands; a visually impaired person might recognize the object as a table by *listening* for it via echolocation; see also Sinclair and de Freitas 2014). Thus, to understand how objects (including mathematical objects) emerge for individuals with heterogeneous sensory capacities, enactivism holds that we must attend to individuals' *active processes* of configuring affordances for action.

Over the last decade, enactivism has been successfully adopted by mathematics education (see Reid et al. 2015). Hutto et al. (2015) put forth an enactivist approach to conceptualizing mathematics learning, knowing, and reasoning as emerging in the form of ecologically situated, goal-oriented *multimodal* sensorimotor activity. This approach casts mathematical semiosis (including the use of technical vocabulary, inscriptions, and material instruments) as a means for coordinating and consolidating diversely constituted multimodal notions both within and across individuals. Following this enactivist reconceptualization of mathematics, we suggest that the goal of mathematics education should not be for students to have identical experiences of particular concepts, but instead to expand their distinct and even idiosyncratically meaningful ways of participating in distributed mathematical practice.

Abrahamson (2009, 2014) has developed a heuristic framework that is consistent with tenants of enactivism for creating instructional artifacts and activities that can be implemented within educational institutions. Central to Abrahamson's *embodied design* framework is a methodology for building *fields of promoted action* (Reed and Bril 1996), which are socially facilitated physical settings for participants to engage in movement-based activities by which they develop capacity to enact some culturally valued practice, namely moving in a new way. Abrahamson's embodied design takes up the call for "new developments in input and output technologies to restructure fundamentally the immediacy of possible interaction with various mathematical representations" (Jackiw and Sinclair 2009, p. 419). While these technologically enabled instructional environments were originally designed to shift classroom mathematical discourse away from the symbolic register toward non-symbolic, animated, interactive, and explorative multimodal experiences for grounding target concepts, we now recognize that they could constitute *perceptually pluralistic environments* with potential to support sensorily heterogeneous students in exploring and developing mathematical ideas.

As an example, The Mathematics Imagery Trainer (Abrahamson and Trninic 2015; see Fig. 1 for several versions) is an instrumented field of promoted action, wherein students develop sensorimotor coordinations for enacting bimanual movement shown to constitute forms of reasoning about target mathematical concepts (Abrahamson et al. 2012, 2014, 2016). Concepts implemented to date in the Trainer activity architecture include proportions (Abrahamson et al. 2014; Fig. 1a), the Cartesian field (Duijzer et al. 2017), speed (Flood 2018), geometrical area (Shvarts 2017), parabolas (Shvarts and Abrahamson 2018; Fig. 1c), and trigonometry

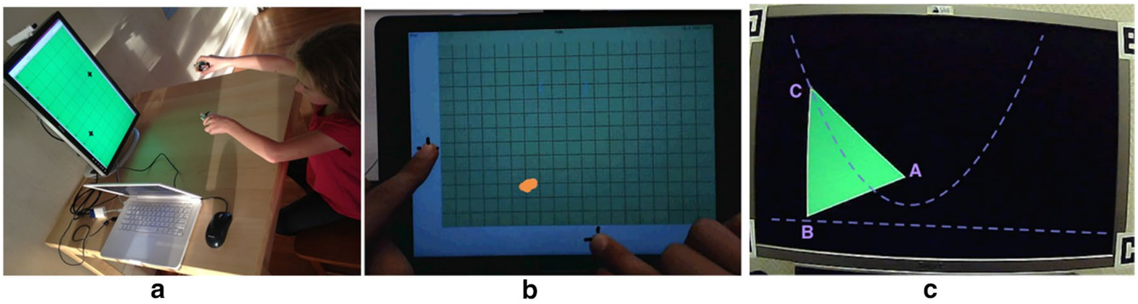

Fig. 1 The Mathematics Imagery Trainer. An assortment of design implementations (still visually biased) for students to coordinate new movement forms that keep a screen green: **a** a trainer for proportion, in which a child is learning to raise her hands at different speeds; **b** a trainer for two-dimensional Cartesian space, with an eye-tracking fixation point marking the child's gaze toward an imagined line connecting the two index fingers as they move simultaneously along orthogonal axes; and **c** a trainer for parabolas, in which the student manipulates Point C, while Point B is vertically below it on the directrix, and Point A remains fixed at the focus (the student is not shown the dotted lines). The task is to move Point C, keeping the triangle green, and it requires keeping the triangle isosceles

(Alberto 2018). Using the Mathematics Imagery Trainer, sighted students have learned to move in new ways, interacting haptically, kinesthetically, tactilely, as well as visually and auditorily with the system as well as a tutor and other students in order to discover and describe mathematical patterns.

When students engage in activities centered on the Mathematics Imagery Trainer, they confront problems of enacting complex movements, such as an unfamiliar bimanual movement (Fig. 1a). They solve these movement problems by determining in the perceptual field new phenomenal objects, *attentional anchors*, by which to coordinate the position of their hands (Abrahamson and Sánchez-García 2016; Abrahamson et al. 2016). For example, the child in Fig. 1b is imagining a line connecting his index fingertips; by intending to move this line rightward at a constant angle to the *x*-axis, he is able to move his individual fingers at different speeds along orthogonal vertices. We conjecture that, if provided appropriate sensory feedback and frames of reference, blind students would also construct phenomenal objects that draw on kinesthetic sensation of their hands' relative positions as the hands are moving (see Sect. 4.3).

The design rationale of the Trainer is that students are tasked to transform a problematic situation, which bears no symbols, into some prescribed goal state (e.g., in Fig. 1, making a screen green). Students are offered utensils to perform this task, and yet the technology places conceptually strategic constraints on their attempts to do so, so that students' existing operatory sensorimotor schemes prove inadequate. To accomplish the task, these schemes must be reorganized. As viewed by the field of coordination dynamics (synergetics), which resonates strongly with constructivist and enactivist philosophy, a systemic *shift* in students' activity within the field is insufficient. That is, the students cannot recognize the environment as affording existing forms of engagement. Instead, a *bifurcation* is required (Kelso 1984, 1995), what Piaget calls reflective abstracting (Abrahamson et al. 2016). That is, the environment needs to come forth for the students as affording qualitatively different forms of engagement (Heft 1989). Students develop these new movement forms as evidenced in their spontaneous configuration of attentional anchors. In developing and calibrating these forms, they appropriate the pedagogically targeted cultural affordances (Abrahamson and Bakker 2016; and see; Vérillon and Rabardel 1995 on instrumental genesis).

The multimodality of Trainer activities could potentially enable sensorily heterogeneous students' alternative ways of exploring and knowing target mathematical concepts. These fields of promoted action do not supplement ocularcentric design with proxy or substitute affordances for blind students to learn mathematical concepts as near as possible to how sighted students would. Instead, these designs are consistent with UDL principles, by creating spaces where varifold, intellectually compatible, alternative ways of knowing and expressing mathematical concepts can come together and mutually elaborate one another. While the Trainers have not been created for blind students, they could be readily re-designed so that visual access is not necessary for operation (see Abrahamson 2012 for a prototype tablet application that sonifies interaction feedback by triggering auditory instead of visual stimuli). In such re-designs, vision is no longer privileged epistemologically (i.e., seeing is no longer the only way to believing) but becomes just one among several possible modal constitutions of the target concept. By requiring coordinated dyadic enactment of task-oriented joint actions, these fields of promoted action could create an interpersonal participatory nexus, where multimodal affordances are negotiated and refined into new, perceptually pluralistic, disciplinary normative mathematics ideas.

We recognize that there is much work ahead for us to engage seriously in UDL reconfiguration of the Mathematics Imagery Trainer (see Sect. 4.3, farther below).

However, our enactivist approach to conceptual design, embodied in the Trainer, puts forth a view of conceptual meanings as grounded in new ways of moving, thus reconceptualizing mathematics as ways of knowing that are accessible to all learners. This theoretical reconceptualization of mathematics creates opportunities for researchers and designers to imagine enactivist learning experiences that can cater to *multi*modal, and not just visual, sensorimotor activity.

4.2 Ethnomethodological conversation analysis: mathematics learning as a sensory-pluralistic interactional achievement

Ethnomethodological conversation analysis (EMCA) provides an important analytic complement to enactivism for both investigating and informing the design of sensory pluralistic mathematics learning experiences that are able to support students with heterogenous perceptual faculties. EMCA starts from the premise that during social encounters, perceived order and intelligibility in the world is ongoingly produced by the situated interactional work of participants in the moment. Shared realities—our mutually intelligible experiences of objects and processes—emerge from ceaseless reciprocal efforts to shape each other's perceptions of surroundings and circumstances through a relentless process of displaying, repairing, and ratifying our unfolding interpretations of the world we engage together (Garfinkel and Sacks 1970; Heritage 1984; Mondada 2012). EMCA's project is bringing the fine details of these processes into relief.

EMCA studies have established that participants make use of a vast variety of local, perceptually available resources (e.g., talk, gesture, objects, body posture, etc.) for organizing their conduct together (Garfinkel and Livingston 2003; Goodwin 2000; Mondada 2011). These resources are always finely customized to the situation at hand and participants' ongoing analysis of what their fellow interlocuters are able to apprehend (Goffman 1964). On the telephone, for example, people do not have visual access to one another and rely on subtle verbal resources to signal and coordinate closing conversations (Schegloff and Sacks 1973). However, when participants are co-present, coordinating a mutual farewell makes use of bodily and material methods and resources like picking up one's coat (a visually observable process) or moving towards the front door (a spatially apprehendable process) (c.f., Broth and Mondada 2013; Heath 1986). In addition, participants frequently draw on modalities beyond the seen (e.g., visual gesture) and heard (e.g., speech) to coordinate mutually intelligible activity together (Mondada 2016): Numerous EMCA-inspired investigations have demonstrated the central importance of haptic, tactile, and kinesthetic resources in communication (Goodwin 2017; Nishizaka 2007; Streeck 2013; Becvar Weddle and Hollan 2010) from family interactions (Goodwin and Cekaite 2018) to doctor consultations (Heath 1989) that have been overlooked by other logocentric and occularcentric approaches to communication.

As a result, EMCA has been able to make key contributions towards revealing the complex and nuanced communicational practices of sensorily and neurologically heterogeneous persons (e.g., Avital and Streeck 2011; Iwasaki et al. 2018; Goodwin 2018). EMCA investigations have also revealed how participants with heterogenous access to communication resources (e.g., visually-impaired and non-visually impaired participants) use a wide array of creative methods and resources to coordinate all kinds of activities together from giving directions to exploring art (Due and Lange 2017; Friedman 2012; Garfinkel 2002; Goode 1994; Saerberg 2010; vom Lehn 2010). For example, in the case of art museums, vom Lehn (2010) demonstrated how visually impaired and non-visually impaired persons develop interesting and unique insights about art pieces together by negotiating and reconciling their tactile and visual observations of exhibits. Inspired by these investigations, we seek to better understand (1) the resources and methods that sensorily heterogenous students can use to negotiate mathematical meanings together and (2) what materials and tasks would best support this process.

In mathematics education, recent studies have moved away from deficit models of disability to more closely examine sensorily diverse students' experiences in mathematics and have produced important initial insights into such processes. These authors have shown how blind students (Healy et al. 2016) and deaf students (Krause 2015) co-construct new mathematical ideas in ways that are different from those of sighted and hearing students (de Freitas 2016). Different languages and representational modalities have different affordances for mathematics reasoning and problem-solving, making it crucial for educational designers and researchers to explore their potentials for meaning-making. For example, sign language may offer advantages for collaboratively discovering new mathematical relationships in its use of physical space and motion to capture and organize ideas as compared with the linear constraints of spoken American English. Flood (2018) has also demonstrated the beneficial effects that exchanging, recycling, and transforming multimodally expressed ideas through revoicing can have on advancing new mathematical understandings. In addition, the use of haptic and tactile ways of communicating (e.g., conveying an idea by tracing a pattern on another person's body—c.f. Azevedo and Mann 2018) may also lead to unique new discoveries and appreciations of mathematical phenomena.

While EMCA-inspired investigations have produced numerous insights into traditional mathematics classrooms with sighted and hearing sensorily homogenous learners (e.g., Forrester and Pike 1998; Ingram et al. 2015) currently, we know very little about how sensorily heterogenous learners could collaborate and negotiate mutually intelligible interpretations of phenomena in UDL-inspired sensory-pluralistic learning environments. Much more EMCA-inspired research is needed to document, interpret, and support the achievements of diverse communicational praxes in UDL-inspired mathematics classrooms with learners with heterogenous perceptual faculties. In particular, EMCA's concern with the fine details of endogenous social "sense-assembly" procedures make it a powerful framework for understanding and informing the design of collaborative, sensory-pluralistic fields of promoted action for sensorily heterogenous learners. History is littered with examples of design disasters that failed to take into consideration unexpected, "seen-but-unnoticed" yet absolutely essential methods people use to coordinate their activities together in a diverse variety of settings (e.g., Dourish 2001; Heath and Luff 2000), and EMCA has a rich history of making critical contributions to user-experience research and design (Dourish and Button 1998; Koschmann et al. 2007; Suchman 1987). Rather than assume or attempt to predict the methods and resources participants will use in a setting, EMCA holds that such information can only be discovered in actual circumstances through rigorous observation and analysis of people's actual practices.

Appreciating how mathematical understandings emerge in UDL-inspired fields of promoted action between sensorily heterogenous mathematics students will require careful attention to participants' practical, situated, embodied, multimodal methods and resources for building and maintaining intersubjectivity (Flood et al. 2016; Flood 2018; Koschmann and Mori 2016; Wittmann et al. 2013). As part of this process, participants reciprocally work to render their experiences publicly available to others, using novel resources and methods that are perceptually available to their collaborators. In the case of perceptually pluralistic fields of promoted action like the Mathematical Imagery Trainer, these multimodal social interactions become a nexus where subjective perception-for-movement and attentional anchors are negotiated and refined into disciplinarily recognizable ways of knowing and representing the world mathematically (Flood et al. 2016). As students explore and develop new repertoires for interacting with Mathematical Imagery Trainers, a key challenge for them becomes sharing and communicating their perceptual experiences and modes of action with others. This requires creative, multimodal strategies of communicating on their part, for example, by guiding another's hands to recreate a particular physical sensation (Abrahamson et al. 2012; Becvar Weddle and Hollan 2010). Understanding how these unique interactions lead to new collaboratively negotiated multimodal mathematical ideas requires EMCA to both reveal the affordances of sensorily pluralistic UDL and enacvist-inspired fields of promoted action for mathematics learning and inform their iterative re-design. In the next section, we outline our sensory-pluralistic framework for educational design and its investigation.

4.3 A proposed sensory-pluralistic design for proportion

We conclude the article by proposing an empirical context that would enable us to evaluate our thesis concerning the educational potential of perceptually-pluralist design for collaborative mathematics learning. Here we briefly outline a hypothetical classroom UDL experience for the inclusion and mutual growth of perceptually diverse students. In particular, this proposed technology-enabled and task-oriented experience is designed to expand on predominantly visual modes of access to quantitative relation structures by affording multimodal engagement with objects via manipulation contemporary with multi-sensory feedback. Students of sensory diversity participating in this activity collaborate on solving movement problems demanding coordination of action and the negotiation of emergent meanings. In so doing, they are compelled to render their subjective perceptions publicly available so as to co-construct communicational methods in semiotic registers that are mutually intelligible and pragmatically actionable. In a sense, students ultimately can come to reflect on their activity not by thinking *about* the sensory experience of their diverse peers but by inhabiting their peers' perceptual orientation (Goodwin 2018). In such situations, "Attention is not just attention to- or even shared attention to- a common object but *attention with-* by which the perception of the object can be jointly constructed" (Katharine G. Young, personal communication, Dec. 2, 2017; our italics).

Figure 2 portrays two students co-operating a Mathematics Imagery Trainer for Proportion. The pair could either both be blind, both sighted, or could be a blind and sighted student working together. The students are standing opposite each other. Immediately between them and within hand's reach is a tall device mounted on a desk. The device is vertically oriented—it is an elongated rectangular plane rising from the desk upwards. On this plane there are two parallel vertical tracks that share a baseline (the desk). Each of the two tracks has a joystick-like control handle (a knob), nested in its track groove, that protrudes on both sides. Each handle can slide up or down its vertical groove, and each student can operate each of these two handles. When one raises or lowers the handle, it remains at whatever height it was placed. The handles are a material interface of a computer-based technology. Each of the handles constitutes a manual input implement, so that the interface has two inputs, and computer algorithms evaluate relations between these inputs, and vis-à-vis changeable values set by the facilitator on a

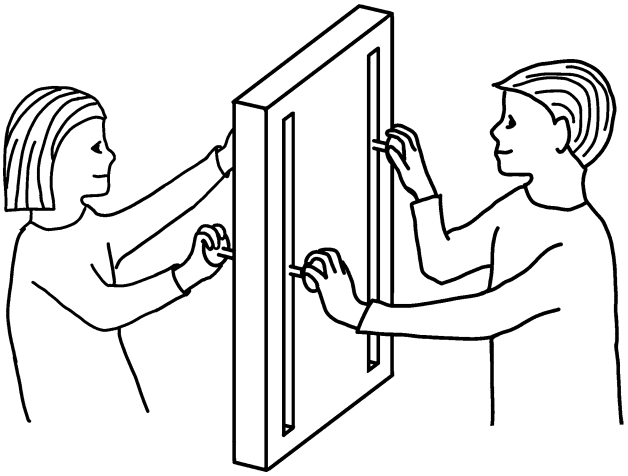

Fig. 2 A Mathematics Imagery Trainer for joint problem solving in classrooms of sighted and blind students

console dash-board, to determine appropriate feedback to the users.

The task is interactive and has a well-defined, simple goal for a joint operation: The students need to work together to place the handles at particular heights above the baseline so as to get "success" feedback on this placement. The feedback is primarily haptic. Both handles can rattle (felt as a buzz in the hand) depending on their vertical displacements relative to one another. In some handle orientations the rattle is more vigorous, and in others, the rattle subsides as the students find a pair of locations that are a "success." Unbeknownst to the students, the rattle gradually subsides and stops only when the handles approach a particular preset height ratio with one another. Once they have found several "rattle-free" locations, they are asked to move both handles continuously so as to keep them within the no-rattle success zone.

Like the original Mathematics Imagery Trainer for Proportion (Fig. 1a), the task is bimanual and the two handles must be controlled simultaneously. For example, a student on each side of the device could control one of the handles while lightly holding the other, so that each student is agent on one of the handles and patient on the other (imagine in Fig. 2 that the girl controls her right-hand handle and the boy controls his right-hand handle, and that they each let their left-hand handle be guided by the other). Holding both handles allows each student to have proprioceptive and kinesthetic sensations of the (changing) location of both handles and the distance between them, as well as letting them mutually monitor each other's actions. Each student might come to anticipate the other student's intentions and constantly evaluate this prediction. Students would need to negotiate their respective agency, possibly agreeing to trade off different roles (e.g., one person controls both handles while the other hangs on) or co-enact movements using both implements simultaneously. In this proposed design, an instrumented field of promoted *joint* action would emerge, in which two students couple into a single two-person complex dynamical system to explore for effective sensorimotor patterns (Ishigaki et al. 2017; Sebanz and Knoblich 2009; Shvarts and Abrahamson 2018; Solfo and van Leeuwen 2018 in press).

As they attempt to solve the problem together, students would engage in spontaneous, multimodal conversation. They will bring to each other's attention what they have noticed about the mechanism, what particular functions, dimensions, and properties they were attending to, and what sense they made of those phenomena with respect to their actions. They will plan their next joint actions and negotiate the significance of the events they experience together. In real time, they comment on features of the event and correct each other. Over time, selected properties of the dynamic interactive events emerge and are highlighted and coded in their interaction with one another as solutions to the practical problem of coordinating joint actions in an unfamiliar context and terrain. With time, the teacher interpolates into the working environment mathematical instruments, such as measurement tools (e.g., evenly spaced tactile "unit" notches along the vertical tracks). The students appropriate these tools as frames of reference bearing apparent potential for enhancing aspects of their joint performance. In so doing, they shift into quantitative propositions to describe and coordinate their activity together. Thus a set of meanings that had been generated to solve together intersubjective action coordination problems become expressed in, and transformed through normative disciplinary form, preparing the grounds to adopt shared semiotic practices for encoding and expanding on these meanings.

Transpiring in a mathematics classroom, this collaborative motor-coordination activity is pre-framed for the participants within the socio-epistemic norms of conceptual learning (see Brousseau 1997 on the didactical contract). As members of this community of practice, students are aware that their personal and interpersonal negotiations with the material mechanism constitute mathematical sense-making and will be generalized as such through guided, collective semiotic reification into symbolic registers. What they are learning to do is marked and endorsed by the community as legitimate ways of knowing the curricular mathematical concepts (just as in cases of sighted students working on a different implementation of this design, see Abrahamson and Trninic 2015; Abrahamson and Sánchez-García 2016). The design would include activities for collective adoption of normative mathematical signs that encode these multimodal methods.

With the proposed design above, we hope to have demonstrated how our enactivist rethinking of mathematics learning as developing new patterns of sensorimotor engagement, coupled with an EMCA rethinking of mathematics learning as constituted in making action mutually intelligible, can frame and enrich UDL efforts in mathematics education. Our design represents just one possibility among many and we hope it will inspire other designs of perceptually pluralistic learning ecologies that are able to engage sensorily heterogenous learners with different aspects of mathematics. We look forward to future research projects that will allow us to evaluate the exciting new prospects of rendering instruction accessible to all learners, and we hope to encourage other mathematics education researchers to follow suit.

5 Summary

Universal Design for Learning is a powerful framework organizing the design of material and computational solutions for learning experiences that are truly accessible to all. Here, we have argued that elaborating and enhancing UDL with enactivism and ethnomethodological conversation analysis will guide our design and understanding of better solutions able to incorporate current rethinking of what it means for individuals to learn disciplinary concepts and practices. Because educational practice is itself a complex activity system, we imagine that enacting our proposal will require rethinking additional facets, such as assessment and teacher training, as well as continued negotiation of instructional methodology for coordinating multimodal enactment with extant sociocultural practices involving historically evolved mathematical signs (Healy et al. 2010). Thus, stemming from this proposal are many new avenues for research and design on this path toward perceptual pluralism in mathematics education.

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References

Abrahamson, D. (2009). Embodied design: Constructing means for constructing meaning. *Educational Studies in Mathematics, 70*(1), 27–47.

Abrahamson, D. (2012). Mathematical Imagery Trainer—Proportion (MIT-P) IPhone/iPad application (Terasoft): iTunes. Retrieved from https://itunes.apple.com/au/app/mathematical-imagery-trainer/id563185943.

Abrahamson, D. (2014). Building educational activities for understanding: An elaboration on the embodied-design framework and its epistemic grounds. *International Journal of Child-Computer Interaction, 2*(1), 1–16.

Abrahamson, D., & Bakker, A. (2016). Making sense of movement in embodied design for mathematics learning. In N. Newcombe & S. Weisberg (Eds.), Embodied cognition and STEM learning [Special issue]. Cognitive Research: Principles and Implications (Vol. 1, No. (1), pp. 1–13).

Abrahamson, D., Gutiérrez, J. F., Charoenying, T., Negrete, A. G., & Bumbacher, E. (2012). Fostering hooks and shifts: Tutorial tactics for guided mathematical discovery. *Technology, Knowledge, and Learning, 17*(1–2), 61–86.

Abrahamson, D., Lee, R. G., Negrete, A. G., & Gutiérrez, J. F. (2014). Coordinating visualizations of polysemous action: Values added for grounding proportion. *ZDM - The International Journal on Mathematics Education, 46*(1), 79–93.

Abrahamson, D., & Sánchez-García, R. (2016). Learning is moving in new ways: The ecological dynamics of mathematics education. *Journal of the Learning Sciences, 25*(2), 203–239.

Abrahamson, D., Shayan, S., Bakker, A., & Van der Schaaf, M. F. (2016). Eye-tracking Piaget: Capturing the emergence of attentional anchors in the coordination of proportional motor action. *Human Development, 58*(4–5), 218–244.

Abrahamson, D., & Trninic, D. (2015). Bringing forth mathematical concepts: Signifying sensorimotor enactment in fields of promoted action. *ZDM Mathematics Education, 47*(2), 295–306.

Alberto, R. (2018). *Design research of pedagogical constraints on coordination dynamics: Actions, perceptions, and reasoning in learning mathe*matics. In D. Abrahamson (Conference chair), Coordination dynamics of mathematics education. University of California, Berkeley, October 25–26, 201.

Amalric, M., Denghien, I., & Dehaene, S. (2018). On the role of visual experience in mathematical development: Evidence from blind mathematicians. *Developmental Cognitive Neuroscience, 30*, 314–323.

Asterhan, C. S. C., & Schwarz, B. B. (2009). The role of argumentation and explanation in conceptual change: Indications from protocol analyses of peer-to-peer dialogue. *Cognitive Science, 33*, 373–399.

Avital, S., & Streeck, J. (2011). Terra incognita: Social interaction among blind children. In J. Streeck, C. Goodwin & C. LeBaron (Eds.), *Embodied interaction: Language and body in the material world* (pp. 169–181). Cambridge: Cambridge University Press.

Azevedo, F. S., & Mann, M. J. (2018). Seeing in the dark: Embodied cognition in amateur astronomy practice. *Journal of the Learning Sciences, 27*(1), 89–136.

Barnes, B., Henry, J., & Bloor, D. (1996). *Scientific knowledge: A sociological analysis*. Chicago: University of Chicago Press.

Becvar Weddle, A., & Hollan, J. D. (2010). Professional perception and expert action: Scaffolding embodied practices in professional education. *Mind, Culture, and Activity, 17*(2), 119–148.

Braun, S. (2008). Audiodescription research: state of the art and beyond. *Translation Studies in the New Millennium, 6*, 14–30.

Broth, M., & Mondada, L. (2013). Walking away: The embodied achievement of activity closings in mobile interaction. *Journal of Pragmatics, 47*(1), 41–58.

Brothers, R. J. (1973). Arithmetic computation: Achievement of visually handicapped students in public schools. *Exceptional Children, 39*, 575–576.

Brousseau, G. (1997). Theory of didactical situations in mathematics (N. Balacheff, M. Cooper, R. Sutherland & V. Warfield, Trans.). Boston: Kluwer Academic Publishers.

Burgstahler, S. (2001). *Universal design of instruction*. Arlington: National Science Foundation.

Capiel, G. (2014). Born accessible. *Journal of Electronic Publishing*. https://doi.org/10.3998/3336451.0017.121

Clamp, S. (1997). Mathematics. In H. Mason, S. McCall, C. Arter, M. McLinden & J. Stone (Eds.), *Visual impairment: Access to education for children and young people* (pp. 218–235). New York: David Fulton Publishers.

Cobb, P., Yackel, E., & McClain, K. (Eds.). (2000). *Symbolizing and communicating in mathematics classrooms—Perspectives on* *discourse, tools, and instructional design*. Mahwah: Lawrence Erlbaum.

de Freitas, E. (2016). Material encounters and media events: What kind of mathematics can a body do? *Educational Studies in Mathematics, 91*(2), 185–202.

de Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. New York: Cambridge University Press.

Dourish, P. (2001). *Where the action is: The foundations of embodied interaction*. Cambridge: MIT Press.

Dourish, P., & Button, G. (1998). On "technomethodology": Foundational relationships between ethnomethodology and system design. *Human-Computer Interaction, 13*(4), 395–432.

Due, B., & Lange, S. B. (2017). The Moses effect: The spatial hierarchy and joint accomplishment of a blind person navigating. *Space and Culture, 21*(2), 129–144.

Duijzer, A. C. G., Shayan, S., Bakker, A., Van der Schaaf, M. F., & Abrahamson, D. (2017). Touchscreen tablets: Coordinating action and perception for mathematical cognition. *Frontiers in Psychology. 8*, 144.

Ely, R., Emerson, R. W., Maggiore, T., Rothberg, M., Connell, O., T., & Hudson, L. (2006). Increased content knowledge of students with visual impairments as a result of extended descriptions. *Journal of Special Education Technology, 21*(3), 31.

Encelle, B., Ollagnier-Beldame, M., Pouchot, S., & Prié, Y. (2011). Annotation-based video enrichment for blind people: A pilot study on the use of earcons and speech synthesis. In *The Proceedings of the 13th International ACM SIGACCESS Conference on Computers and Accessibility* (pp. 123–130). ACM.

Erickson, F. (1996). Going for the zone: The social and cognitive ecology of teacher-student interaction in classroom conversations. In D. Hicks (Ed.), *Discourse learning and schooling* (pp. 29–62). Cambridge: Cambridge University Press.

Flood, V. J. (2018). Multimodal revoicing as an interactional mechanism for connecting scientific and everyday concepts. *Human Development, 6*, 145–173

Flood, V. J., Harrer, B. W., & Abrahamson, D. (2016). The interactional work of configuring a mathematical object in a technology-enabled embodied learning environment. In C.-K. Looi, J. Polman, U. Cress, & P. Reimann (Eds.), Proceedings of the International Conference of the Learning Sciences (ICLS) (Vol., 1, pp. 122–129). Singapore: ISLS.

Fortin, M., Voss, P., Lord, C., Lassonde, M., Pruessner, J., Saint-Amour, D., Rainville, C., & Lepore, F. (2008). Wayfinding in the blind: Larger hippocampal volume and supranormal spatial navigation. *Brain, 131*(11), 2995–3005.

Friedman, A. M. (2012). Believing not seeing: A blind phenomenology of sexed bodies. *Symbolic Interaction 35*(3), 284–300.

Garfinkel, H. (2002). *Ethnomethodology's program: Working out Durkheim's aphorism*. New York: Rowman & Littlefield.

Garfinkel, H., & Livingston, E. (2003). Phenomenal field properties of order in formatted queues and their neglected standing in the current situation of inquiry. *Visual Studies, 18*(1), 21–28.

Garfinkel, H., & Sacks, H. (1970). On formal structures of practical actions. In J. C. McKinney & E. Tiryakian (Eds.), *Theoretical sociology: Perspectives and developments* (pp. 337–366). New York: Appleton-Century-Crofts.

Gibson, J. J. (1977). The theory of affordances. In R. Shaw & J. Bransford (Eds.), *Perceiving, acting and knowing: Toward an ecological psychology* (pp. 67–82). Hillsdale: Lawrence Erlbaum Associates.

Goffman, E. (1964). The neglected situation. *American Anthropologist, 66*(6), 133–136.

Goldsmith, S. (1963). *Designing for the disabled*. London: RIBA Publications.

Goldsmith, S. (1997). *Designing for the disabled: The new paradigm*. New York: Routledge.

Goode, D. (1994). *A world without words*. Philadelphia: Temple University Press.

Goodwin, C. (2000). Action and embodiment within situated human interaction. *Journal of Pragmatics, 32*(10), 1489–1522.

Goodwin, C. (2018). *Co-operative action*. New York: Cambridge University Press.

Goodwin, M. H. (2017). Haptic sociality. In C. Meyer, J. Streeck & J. S. Jordan (Eds.), *Intercorporeality: Emerging socialities in interaction* (pp. 73–102). New York: Oxford University Press.

Goodwin, M. H., & Cekaite, A. (2018). *Embodied family choreography: Practices of control, care, and mundane creativity*. New Jersey: Routledge.

Hall, R., & Nemirovsky, R. (2012). Introduction to the special issue: Modalities of body engagement in mathematical activity and learning. *Journal of the Learning Sciences, 21*(2), 207–215

Healy, L., & Fernandes, S. H. A. A. (2011). The role of gestures in the mathematical practices of those who do not see with their eyes. *Educational Studies in Mathematics, 77*(2), 157–174.

Healy, L., Jahn, A. P., & Frant, J. B. (2010). Digital technologies and the challenge of constructing an inclusive school mathematics. *ZDM - The International Journal on Mathematics Education, 42*(3–4), 393–404.

Healy, L., Ramos, E. B., Fernandes, S. H. A. A., & Peixoto, J. L. B. (2016). Mathematics in the hands of deaf learners and blind learners: Visual–gestural–somatic means of doing and expressing mathematics. In R. Barwell, P. Clarkson, A. Halai, M. Kazima, J. Moschkovich, N. Planas, M. Setati-Phakeng, P. Valero, M. Villavicencio & Ubillús (Eds.), *Mathematics education and language diversity: The 21st ICMI Study* (pp. 141–162). Cham: Springer.

Heath, C. (1986). *Body movement and speech in medical interaction*. Cambridge: Cambridge University Press.

Heath, C. (1989). Pain talk: The expression of suffering in the medical consultation. *Social Psychology Quarterly, 52*(2), 113–125.

Heath, C., & Luff, P. (2000). Technology and social action. In *Technology in action* (pp. 1–30). Cambridge: CUP.

Heft, H. (1989). Affordances and the body: An intentional analysis of Gibson's ecological approach to visual perception. *Journal for the Theory of Social Behaviour, 19*(1), 1–30.

Heritage, J. (1984). *Garfinkel and ethnomethodology*. New York: Polity Press.

Herzberg, T. S., & Rosenblum, L. P. (2014). Print to braille: Preparation and accuracy of K-12 mathematics materials. *Journal of Visual Impairment & Blindness, 108*(5), 355–367.

Horvath, J., & Cameron, R. (2017). *3D printed science projects (Vol. 2): Physics, math, engineering and geology models*. Berkeley: Apress.

Hutto, D. D., Kirchhoff, M. D., & Abrahamson, D. (2015). The enactive roots of STEM: Rethinking educational design in mathematics. In P. Chandler & A. Tricot (Eds.), *Human movement, physical and mental health, and learning [Special issue]. Educational Psychology Review*, (Vol. 27, No. 3, pp. 371–389).

Ingram, J., Pitt, A., & Baldry, F. (2015). Handling errors as they arise in whole-class interactions. *Research in Mathematics Education, 17*(3), 183–197.

Ishigaki, T., Imai, R., & Morioka, S. (2017). Association between unintentional interpersonal postural coordination produced by interpersonal light touch and the intensity of social relationship. *Frontiers in Psychology. 8*, 1993.

Iwarsson, S., & Stahl, A. (2003). Accessibility, usability and universal design—positioning and definition of concepts describing person–environment relationships. *Disability & Rehabilitation, 25*(2), 57–66.

Iwasaki, S., Bartlett, M., Manns, H., & Willoughby, L. (2018). The challenges of multimodality and multi-sensoriality: Methodological issues in analyzing tactile signed interaction. *Journal of Pragmatics*. https://doi.org/10.1016/j.pragma.2018.05.003

Jackiw, N., & Sinclair, N. (2009). Sounds and pictures: Dynamism and dualism in dynamic geometry. *ZDM Mathematics Education, 41*, 413–426.

Jay, M. (1993). *Downcast eyes: The denigration of vision in twentieth-century French thought*. Berkeley: University of California Press.

Kelso, J. A. S. (1984). Phase transitions and critical behavior in human bimanual coordination. *American Journal of Physiology: Regulatory, Integrative and Comparative, 246*(6), R1000–R1004.

Kelso, J. A. S. (1995). *Dynamic patterns: The self-organization of brain and behavior*. Cambridge: MIT Press.

Koschmann, T., & Mori, J. (2016). "Its understandable enough, right?" The natural accountability of a mathematics lesson. *Mind, Culture, and Activity, 23*(1), 65–91.

Koschmann, T., Stahl, G., & Zemel, A. (2007). The video analyst's manifesto (or the implication of Garfinkel's policies for studying instructional practice in design-based research). In R. Goldman, R. Pea, B. J. Barron & S. Derry (Eds.), *Video research in the learning sciences* (pp. 133–144). Mahwah: Lawrence Erlbaum.

Krause, C. M. (2015). *The mathematics in our hands: How gestures contribute to constructing mathematical knowledge*. Wiesbaden: Springer Spektrum.

Krejtz, I., Szarkowska, A., Krejtz, K., Walczak, A., & Duchowski, A. (2012). Audio description as an aural guide of children's visual attention: evidence from an eye-tracking study. In *Proceedings of the Symposium on Eye Tracking Research and Applications* (pp. 99–106). ACM.

Lemke, J. L. (1998). Multiplying meaning: Visual and verbal semiotics in scientific text. In J. R. Martin & R. Veel (Eds.), *Reading science: Critical and functional perspectives on discourses of science* (pp. 87–113). London: Routledge.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Westport: Ablex.

López, A. P. (2010). The benefits of audio description for blind children. *Approaches to Translation Studies, 33*, 213–225.

Mace, R. L., Hardie, G. J., & Place, J. P. (1991). Accessible environments: Toward universal design. In W. E. Preiser, J. C. Vischer & E. T. White (Eds.), *Design intervention: Toward a more humane architecture* (pp. 155–175). New York: Van Nostrand Reinhold.

Merleau-Ponty, M. (2005). Phenomenology of perception (C. Smith, Trans.). New York: Routledge. (Original work published 1945).

Mondada, L. (2011). Understanding as an embodied, situated and sequential achievement in interaction. *Journal of Pragmatics, 43*(2), 542–552.

Mondada, L. (2012). The conversation analytic approach to data collection. In J. Sidnell & T. Stivers (Eds.), *The handbook of conversation analysis* (pp. 32–56). Boston: Blackwell Publishing Ltd.

Mondada, L. (2016). Challenges of multimodality: Language and the body in social interaction. *Journal of Sociolinguistics, 20*(3), 336–366.

Morash, V., & McKerracher, A. (2014). The relationship between tactile graphics and mathematics for students with visual impairments. *Terra Haptica, 4*, 13–22.

Nishizaka, A. (2007). Hand touching hand: Referential practice at a Japanese midwife house. *Human Studies, 30*(3), 199–217.

Núñez, R. E., Edwards, L. D., & Matos, J. F. (1999). Embodied cognition as grounding for situatedness and context in mathematics education. *Educational Studies in Mathematics, 39*(1), 45–65.

O'Loughlin, M. (2006). *Embodiment and education: Exploring creatural existence*. Dordrecht: Springer.

Packer, J., Vizenor, K., & Miele, J. A. (2015). An Overview of Video Description: History, Benefits, and Guidelines. *Journal of Visual Impairment & Blindness, 109*(2), 83–93.

Quek, F., & Oliveira, F. (2013). Enabling the blind to see gestures. In P. Marshall, A. N. Antle, E. v.d. Hoven, & Y. Rogers (Eds.), *The theory and practice of embodied interaction in HCI and interaction design [Special issue]. ACM Transactions on Human–Computer Interaction* (Vol. 20, no. (1), pp. 1–32).

Radford, L. (2009). Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings. *Educational Studies in Mathematics, 70*, 111–126.

Rapp, D. W., & Rapp, A. J. (1992). A survey of the current status of visually impaired students in secondary mathematics. *Journal of Visual Impairment & Blindness, 86*, 115–117.

Reed, E. S., & Bril, B. (1996). The primacy of action in development. In M. L. Latash & M. T. Turvey (Eds.), *Dexterity and its development* (pp. 431–451). Mahwah: LEA.

Reid, D., Brown, L., Coles, A., & Lozano, M.-D. (Eds.). (2015). Enactivist methodology in mathematics education research [Special issue]. *ZDM Mathematics Education, 47*(2).

Rose, D. H., & Meyer, A. (2002). *Teaching every student in the digital age: Universal design for learning*. Alexandria: Association for Supervision and Curriculum Development.

Saerberg, S. (2010). Just go straight ahead. *The Senses and Society, 5*(3), 364–381.

Saxe, G. B. (2012). *Cultural development of mathematical ideas: Papua New Guinea studies*. Cambridge: Cambridge University Press.

Schegloff, E. A., & Sacks, H. (1973). Opening up closings. *Semiotica, 8*, 289–327.

Scherer, P., Beswick, K., DeBlois, L., Healy, L., & Opitz, E. M. J. Z. (2016). Assistance of students with mathematical learning difficulties: How can research support practice? *ZDM Mathematics Education, 48*(5), 633–649.

Sebanz, N., & Knoblich, G. (2009). Prediction in joint action: What, when, and where. *Topics in Cognitive Science, 1*(2), 353–367.

Sedaghatjou, M. (2018). Advanced mathematics communication beyond modality of sight. *International Journal of Mathematical Education in Science and Technology, 49*(1), 46–65.

Shvarts, A. (2017). Eye movements in emerging conceptual understanding of rectangle area. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 268). Singapore: PME.

Shvarts, A., & Abrahamson, D. (2018). *Towards a complex systems model of enculturation: A dual eye-tracking study*. Paper presented at the annual conference of the American Educational Research Association, New York.

Sinclair, N., & de Freitas, E. (2014). Rethinking gesture with new multitouch digital technology. *Gesture, 14*(3), 351–374.

Siu, Y. (2016). Designing for all learners with technology. *Educational Designer, 3*(9). Retrieved December 17, 2017 from http://www.educationaldesigner.org/ed/volume3/issue9/article34/index.htm.

Solfo, A., & van Leeuwen, C. (2018). From adult finger tapping to fetal heart beating: Retracing the role of coordination in constituting agency. *Topics in Cognitive Science, 10*(1), 18–35.

Streeck, J. (2013). Interaction and the living body. *Journal of Pragmatics, 46*(1), 69–90.

Suchman, L. A. (1987). *Plans and situated actions*. New York: Cambridge University Press.

Summers, E., Langston, J., Allison, R., & Cowley, J. (2012). Using SAS/GRAPH to create visualizations that also support tactile and auditory interaction. In *SAS Global Forum*.

Urton, G. (1997). *The social life of numbers: A Quechua ontology of numbers and philosophy of arithmetic*. Austin: University of Texas Press.

Varela, F. J. (1999). *Ethical know-how: Action, wisdom, and cognition*. Stanford: Stanford University Press.

Vérillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of thought in relation to instrumented activity. *European Journal of Psychology of Education, 10*(1), 77–101.

vom Lehn, D. (2010). Discovering "experience-ables": Socially including visually impaired people in art museums. *Journal of Marketing Management, 26*(7), 749–769.

Wittgenstein, L. (1953). *Philosophical investigations (G. E. M. Anscombe, Trans.)*. Upper Saddle River: Prentice Hall.

Wittmann, M., Flood, V., & Black, K. (2013). Algebraic manipulation as motion within a landscape. *Educational Studies in Mathematics, 82*(2), 169–181.