THE JOURNAL ON TECHNOLOGY AND PERSONS WITH DISABILITIES

A Tangible Manipulative for Inclusive Quadrilateral Learning

Scott George Lambert - Saint Louis University - scott.lambert@slu.edu,
Brett L. Fiedler - University of Colorado Boulder - brett.fiedler@colorado.edu,
Chloe S. Hershenow - University of Tulsa - csh4376@utulsa.edu,
Dor Abrahamson - University of California Berkeley - dor@berkeley.edu,
Jenna L. Gorlewicz - Saint Louis University - jenna.gorlewicz@slu.edu

Abstract

Over the last decade, extensive growth in digital educational content has opened up new opportunities for teaching and learning. Despite such advancements, digital learning experiences often omit one of our richest and earliest learning modalities - touch. This lack of haptic (touch) interaction creates a growing gap in supporting inclusive, embodied learning experiences digitally. Our research centers on the development of inclusive learning tools that can flexibly adapt for use in different learning contexts to support learners with a wide range of needs, co-designed with students with disabilities. In this paper, we focus on the development of a tangible device for geometry learning - the Tangible Manipulative for Quadrilaterals (TMQ). We detail the design evolution of the TMQ and present two user studies investigating the affordances of the TMQ and the user strategies employed when explored in isolation and in tandem with a two-dimensional touchscreen-based rendering of a quadrilateral. Findings illustrate the affordances of the TMQ over traditional, static media and its ability to serve as an inclusive geometry learning tool.

Keywords

Blind/low vision, educational tools, mathematics learning

© 2022 California State University, Northridge

Introduction

Enabled by advancements in technology access and bolstered by online delivery of courses, digital formats are becoming the praxis in education. This shift to the digital space, however, introduces new challenges for students, particularly for those with blindness or a visual impairment (BVI). Acknowledging the gap between tangible and virtual learning opportunities, several research initiatives have investigated how to bring touch back in meaningful ways (Ding & Gallacher, 2018; Martinez et al., 2016). Several studies have focused on improving the efficacy of digital graphics through vibrations and audio feedback (Tennison et al., 2016) or by creating more sensory-rich interactions on touchscreens via variable friction displays (Xu et al., 2019). Others have built educational devices, leveraging kinesthetic learning in novel ways. Devices such as the Haptic Paddle and Hapkit demonstrated the efficacy of 3D printed devices to elucidate complex STEM topics that include dynamics (Martinez et al., 2019). Other systems have highlighted the importance of haptics in kinesthetic learning systems across STEM disciplines (Grow et al., 2007).

Taking these ideas further, several learning tools have been developed with a focus on inclusivity. Buzzi et al. (2015) developed a geometry learning application that leverages vibro-tactile feedback on a touchscreen to display more immersive graphics for students with BVI. As learning materials transition to digital space, interest in accessible formats for digital graphics also grows. Sallnäs et al. showed the effectiveness of adding haptics via commercially available haptic devices (e.g. Phantom Omni) to interactive mathematics simulations. In an angle-teaching simulation, students preferred the haptic tool to their existing systems (Sallnäs et. al., 2007). Similarly, a paired learning tool called 'Clicks' was developed as a modular geometry device

© 2022 California State University, Northridge

that connected with a tablet application to demonstrate various shapes in two and three dimensions (Adusei, Lee, 2017).

Our research centers on the development of inclusive learning tools that can flexibly adapt for use in different learning contexts to support learners with a wide range of needs, co-designed with students with disabilities. Here, we focus on the development of a tangible device for geometry learning. The learning tool will ultimately span coupled physical and digital interactive components, each consisting of an interactive quadrilateral that is extensible, allowing learners to extend and contract the sides of the object to continuously explore different quadrilaterals (e.g., parallelogram, rectangle, trapezoid, etc.), their invariant properties (e.g., angle congruence), and the relationships between them (e.g., square-rectangle definitions).

We chose to focus initially on a quadrilateral-shaped tool as it allows exploration of many shapes mentioned above and serves as a simple case to demonstrate basic geometric concepts such as parallel lines and right angles. The research team looks forward to leveraging the design acumen developed through this project toward taking on more advanced concepts in geometry curriculum. In particular, characterizing students' emergent multimodal sense-making strategies with two-dimensional shapes will inform how the team expands its technological offerings toward three-dimensional geometries of solids, which draws on perceptuomotor cognitive foundations in two-dimensional content.

The design rationale of this experimental learning tool applies theoretical perspectives from the embodiment turn in cognitive science (Newen et al., 2018) to re-motivate and implement longstanding pedagogical argumentation for multimodal interaction as the cognitive grounding of conceptual knowledge in the disciplines (Abrahamson 2014; Abrahamson et al., 2021). In particular, this perspective seeks to empower sensorially diverse STEM students by

© 2022 California State University, Northridge

centering their modal strengths (Tancredi et al., in press). By coupling the physical and digital components, teachers and learners will have flexible access to visual, auditory, and haptic displays and a wide range of traditional and alternative inputs, accessed with their available technology resources and adaptable to meet diverse learning needs.

Discussion

Hardware Design

This paper presents the Tangible Manipulative for Quadrilaterals (TMQ) shown in Figure 1. The TMQ is a 3D-printed reconfigurable device that allows a learner to explore the relations between different quadrilaterals through movement of the extensible sides and flexible vertices. The device can be used as a stand-alone tool for investigating relationships between quadrilaterals or as a paired device providing a tangible component to a physical/virtual coupled learning system. The envisioned experience created by the TMQ-touchscreen system is that students with BVI will be able to move back and forth between the touchscreen rendering of a quadrilateral and the TMQ itself. This leverages tangible interaction through kinesthetic learning in three-dimensional space and two-dimensional representation through digital formats that are ubiquitous and readily available.

Several design iterations of the TMQ were developed in collaboration with students with BVI. Initial prototypes tested the reception of size changes, cross-section shape, and presence of locking mechanisms. Rapid prototyping through 3D-printing allowed positive design changes to be quickly implemented into each subsequent prototype which can be printed and assembled in one day. The current device (Figure 1) is hand-held and adjustable in side length and angle, allowing a learner to create and interact with a range of four-sided shapes. Opening one angle to

© 2022 California State University, Northridge

180 degrees results in a limited set of triangles. The side lengths can be locked in place and tactile measurement markers are indented into the sides as reference lines (Figure 1).

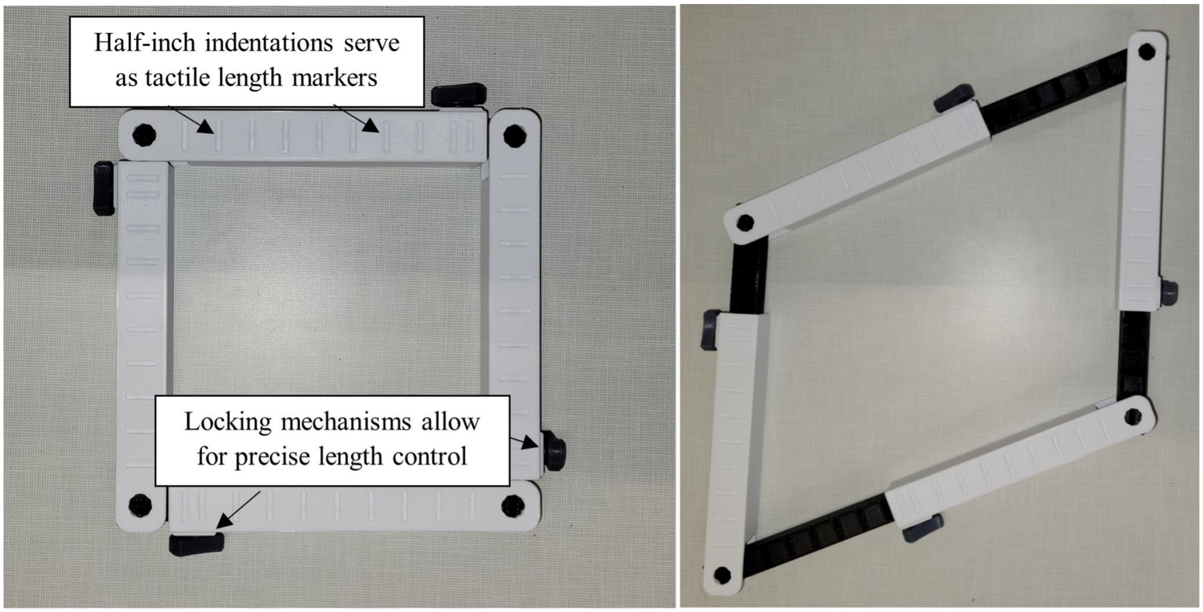

Fig. 1. The Standard TMQ Can Create Numerous Shapes and Features Length-Locking Mechanisms and Tactile Measurement Indents.

In order to connect to multiple modalities, a "smart" version of the TMQ (sTMQ) was created to enable future communications with software applications on a touchscreen tablet. This allows a learner to quickly transition between shapes as two-dimensional renderings displayed on the touchscreen and three-dimensional representations from the sTMQ. The sTMQ is embedded with length and angle sensors: four Force-Sensitive Linear Potentiometers (2730, Pololu Robotics and Electronics) and one rotary potentiometer (EVU-F2AF30B14, Sparkfun) giving a user immediate access to exact length and angle information. An Arduino Uno microcontroller was used for acquisition and calibration of sensors and connection to digital environments. Validation of the accuracy of the sensors was assessed by comparing the physical value of length and angle to the sensor-acquired value. Each length was expanded and contracted 10 times and

© 2022 California State University, Northridge

measured every 0.5 inches. For the angle sensors the same process was repeated at 45° increments, and the sensors demonstrated consistent accuracy within 2% of the physical values.

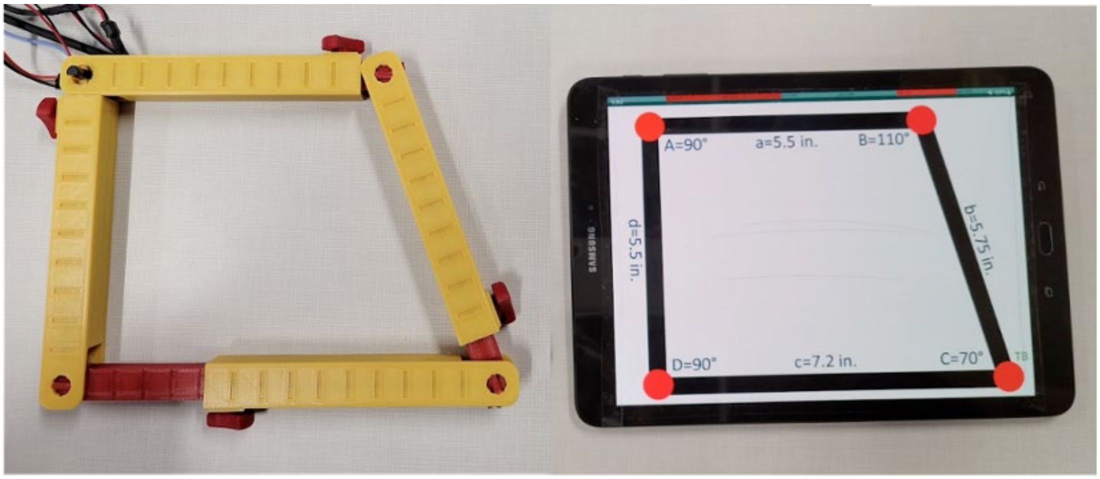

Fig. 2. The sTMQ Gives Learners Exact Length and Angle Values will Serve as a Three-Dimensional Shape Exploration Tool Alongside Two-Dimensional Graphics.

Research Study Design

The TMQ device was used to investigate two foundational research questions supporting the development of inclusive learning tools: (1) In the context of quadrilaterals, what affordances does a tangible manipulative offer over a static representation?; and (2) What core design attributes of the TMQ support user exploration and interaction? We conducted two studies: a comparative study between the TMQ and static graphics displayed on a touchscreen tablet or embossed graphics with 18 college-aged participants (16 blindfolded and 2 BVI), and an exploratory study with 7 participants with BVI aged 18-22. Informal exploration and discussion were done with 5 high school students with BVI.

Study 1

In the first study, we sought to investigate what affordances a tangible manipulative might offer over or in conjunction with multimodal, touchscreen-based renderings of quadrilaterals. To do this, 18 participants (16 sighted-hearing, blindfolded and 2 individuals who

© 2022 California State University, Northridge

are BVI) completed two tasks using the sTMQ and a touchscreen tablet and additionally, embossed graphics for individuals who are BVI. In consultation with an experienced teacher of students with visual impairments, embossed graphics were created using Nemeth Math Braille labeling following the guidelines from the American Printing House.

The first task was shape identification: on each medium, how accurately could participants identify the following shapes: various trapezoids, right-angle quadrilaterals, and parallelograms. Participants were given either the sTMQ set into a specific shape configuration, the touchscreen tablet displaying a shape (as shown in Figure 2, right), or embossed printouts (only participants who are BVI) of a shape and asked to identify the shape. Shapes were randomized from a library of quadrilaterals, trapezoids, and parallelograms; several configurations of each shape were created to avoid process-of-elimination answers as this task was completed three times on each medium.

The second task was shape recreation: participants were presented with a shape on the touchscreen (or an embossed graphic for participants who are BVI) and asked to replicate the geometry on the sTMQ as accurately as possible (as shown in Figure 2). Length and angle values were available as audio readouts on the touchscreen and recorded on the sTMQ via the Arduino Uno upon completion.

In the identification task, learners were about 1.4 times more successful in identifying the correct shape when using the sTMQ as opposed to the touchscreen or the embossed graphic. Parallelograms proved most difficult to identify as participants mistook slightly obtuse or acute angles as right angles, yet most successfully identified quadrilaterals. Stark differences were observed in the strategies employed by participants across mediums. While more serial exploration procedures were used to identify value readouts on the touchscreen-based graphic,

© 2022 California State University, Northridge

participants employed several two-handed referencing strategies when manipulating the sTMQ. Participants who are BVI differed from those blindfolded in exploration strategies on the touchscreen: participants who are BVI traced out the shapes on the touchscreen (a dominant strategy for embossed graphic exploration) while blindfolded participants hunted for value readouts. For participants who are BVI, exploration of the sTMQ was similar to that of tactile graphic explorations in that participants put both hands-on top of the device to establish a global understanding of the object through “hand scanning”. The most common strategy for blindfolded participants estimating side-lengths on the sTMQ was to use one’s thumb and another finger to estimate distance. We observed more variance in estimating angle measurements, with strategies including making “L-shapes” with the hands and comparing it to the orientation of the sTMQ.

In the shape recreation task, all participants were able to manipulate the sTMQ into a desired shape with an overall angle and length accuracy rate of 94%. Error rates by shape are further broken down in Table 1, which shows that parallelograms proved most challenging to accurately recreate, likely due to the inaccuracy of angle estimations through extended usage of “L-shapes.” New exploration strategies emerged here, such as the use of the width of a thumb, finger, or knuckle as reference measurements for adjustment estimations. Participants also began to lean into the tangible device more fully, using the space created between the adjustable lengths to determine how much they had adjusted the device. With a focus on fine-tuning individual angles and sides, we observed participants moving back and forth frequently and quickly between the touchscreen and the sTMQ to gather the correct measurements. Interestingly, when participants could not get a side or angle to match the desired touchscreen representation, they switched to the opposite side or angle, noticing that side lengths could affect angles and angles could affect line orientation.

© 2022 California State University, Northridge

This study validated our design approach: users were more successful in correctly identifying quadrilaterals on the sTMQ than either of the two-dimensional mediums (touchscreen and embossed graphics). Further, across 54 observations (3 shapes per participant, 18 participants), users were able to replicate quadrilaterals on the sTMQ with 94% accuracy from two-dimensional representations. Taken together, we observe several affordances the sTMQ offers: two-handed exploration and manipulation which enables users to establish relationships between changes in angles and changes in lengths, the ability to provide kinesthetic references (e.g. usage of fingers for taking measurements), and the ability to quickly create and transform between shapes which highlights their geometric relationships.

Table 1: Recreation Error Rates (%).

Shape (total observations)	**Overall**	**Angles**	**Side Lengths**
All Shapes (54)	6	6	7
Parallelogram (54)	10	11	9
Trapezoid (54)	6	7	6
Right-Angle Quadrilateral (54)	4	3	4

Study 2

In the second study, we sought to investigate the exploration strategies a learner with BVI might employ when using the TMQ. The study was conducted with 7 students (ages: 18-22) who were enrolled in a program for students with vision loss. Participants were first given the tangible and encouraged to explore its functionality before being asked to create and identify shapes. We observed one- versus two-handed exploration, usage of the device on or off the tabletop, rotation of the device, and tilting strategies for angle modification: single hand stabilization or mirrored action (Figure 3). Any shapes that learners naturally made were also noted.

© 2022 California State University, Northridge

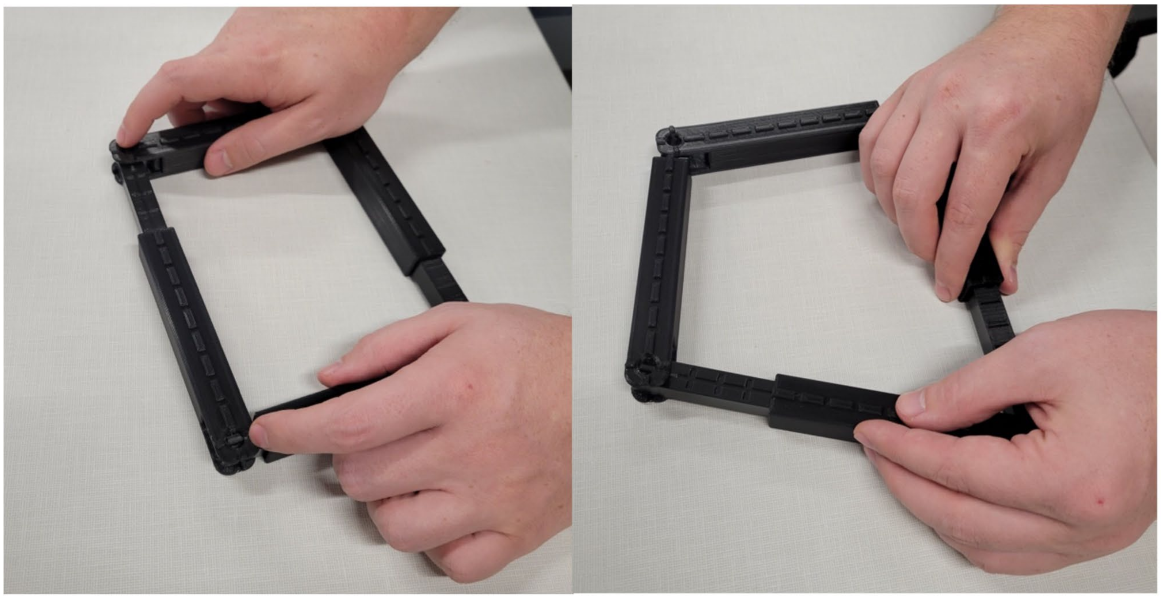

Fig. 3. Popular Exploration Strategies: Most participants utilized mirrored hand motions (left image), but in some cases used one hand to stabilize the TMQ while the other hand manipulated the shape (right image).

We learned some common strategies used by these learners and what shapes the participants made easily (such as parallelograms) and which shapes they struggled to create (such as trapezoids). Nearly all participants immediately made rectangles or parallelograms, and a few participants identified them as such without prompting. Once prompted to create and identify shapes, all participants were able to manipulate the TMQ into three or more distinct shapes. In exploration, most learners utilized mirrored motions. That is, what they did with one hand was done by the other hand, such as elongating the side lengths to create a wide rectangle. A few participants used one hand to stabilize the device while the other hand modified angles or lengths. Two-handed exploration was dominant for garnering a global understanding of the device, as well as for manipulation of the TMQ. About half of the participants explored the TMQ off the table for some part of their interview, with most preferring on-table exploration, likely

© 2022 California State University, Northridge

due to the lack of a locking mechanism resulting in unintentional length changes when the device was suspended above the table.

In the last part of the study, we asked participants to transform the TMQ between shapes in three different scenarios, with an emphasis on using smooth motion when possible. The first scenario was to transform from any rectangle to any parallelogram. The second scenario was mirroring the previous parallelogram without changing the side lengths. The third scenario was transforming from a small square to one twice as large. The first task was the simplest, with all participants creating a parallelogram within three attempts. Pedagogically, we found that, in the second task, parallelogram to mirrored parallelogram transformations were particularly challenging for many learners, with about half requiring further clarification on how to achieve a mirrored shape (e.g. "reverse the shape, make it point the other way."). Most completed the mirroring task in separate steps rather than smooth motions, changing side lengths and achieving imperfect mirrored shapes. Similar actions were shown in the square scaling task: only a couple participants scaled the shape smoothly, with most expanding the left and right sides together, then the top and bottom.

Taken together, these studies illustrate the core design attributes of user exploration and interaction with the TMQ. First, the extendable side lengths were critical for participants to transform between shapes and investigate scaling. We note, however, that because the side lengths of the TMQ did not have a locking mechanism, participants preferred on-table exploration, and often had to use their hands to secure the sides at specific lengths, confounding exploration. This observation prompted the team to add locking mechanisms to the side lengths as shown in Figure 1. Future investigations will explore how individuals choose to work in the horizontal or vertical plane when able to lock the lengths. Second, we observed that most users

© 2022 California State University, Northridge

pushed or pulled on the side lengths of the TMQ opposed to operating at the joints. We hypothesize that this could be resultant of the tasks themselves or how participants thought out transformations between shapes. Future investigations will probe this phenomenon of side versus angle interaction to understand the methodology at which participants are approaching such transformation and scaling tasks.

These challenges in exploration suggest areas for improvement for the TMQ including leveraging other dynamic non-visual feedback (e.g. vibrotactile feedback when a right angle is achieved or audio callouts of length / angle values when desired) to assist in shape accuracy. When prompted for thoughts on the device and its potential uses, many recounted their geometry learning experiences, stating that such a device “would have been really helpful in class” and “better than shapes on a page.” Overall, users enjoyed exploring shapes using the TMQ and its ability to transform between shapes rapidly.

Conclusion

In this paper, we present an inclusive learning tool, the TMQ, which highlights the geometric relationships among quadrilaterals. Two initial investigations with sighted and BVI learners illustrate the potential affordances of such a tool, both as a stand-alone device that can be tangibly transformed and queried and as a device paired with software applications which otherwise often have limited touch interaction. This work is situated within a larger project focused on developing a set of physical/virtual coupled inclusive learning tools that support action-based embodied learning opportunities (Abrahamson, 2014). Our design efforts recognize that all students, regardless of their sensory profile, could avail of opportunities to interact with haptic–tactile models of geometric forms and, more broadly, mathematical objects. As such, we

© 2022 California State University, Northridge

are informed by, and eager to contribute to, a general intellectual shift in the humanities to "reclaim cognition" (Núñez & Freeman, 1999).

Future work will focus on continued iteration of the TMQ design and novel devices to represent other polygons. Expanding on this existing TMQ/virtual app coupling, we will continue collaboration with PhET Interactive Simulations (phet.colorado.edu), focusing on enabling real-time, bi-directional communication between the TMQ device and a quadrilateral virtual simulation. PhET simulations are open source and widely-used for K-12 STEM learning, with existing infrastructure for description / speech reader access and other inclusive features to support non-visual access. Additional work will focus on expansions of this initial concept into a larger suite of inclusive educational tools that promise to bring a new dimension of movement and touch to digital learning experiences to help make learning more accessible to all.

© 2022 California State University, Northridge

Works Cited

Abrahamson, D. (2014). Building educational activities for understanding: An elaboration on the embodied-design framework and its epistemic grounds. *International Journal of Child–Computer Interaction, 2*(1), 1–16. https://doi.org/10.1016/j.ijcci.2014.07.002

Abrahamson, D., Dutton, E., & Bakker, A. (2021). Toward an enactivist mathematics pedagogy. *The Body, Embodiment, and Education*, 156–182. https://doi.org/10.4324/9781003142010-9

Adusei, M., & Lee, D. (2017). "Clicks" appcessory for visually impaired children. *Conference on Human Factors in Computing Systems - Proceedings*, *Part F127655*, 19–25. https://doi.org/10.1145/3027063.3049275

Buzzi, M. C., Buzzi, M., Leporini, B., & Senette, C. (2015, September). Playing with geometry: a multimodal android app for blind children. In *Proceedings of the 11th Biannual Conference on Italian SIGCHI Chapter* (pp. 134-137).

Ding, S., & Gallacher, C. (2018). The haply development platform: A modular and open-sourced entry level haptic toolset. In Extended Abstracts of the 2018 CHI Conference on Human Factors in Computing Systems (Paper D309, pp. 1–4).

Grow, D. I., Verner, L. N., & Okamura, A. M. (2007). Educational haptics. *AAAI Spring Symposium - Technical Report*, *SS-07-09*, 53–58.

Martinez, M.O., Morimoto, T.K., Taylor, A.T., Barron, A.C., Pultorak, J.D. A., Wang, J., Calasanz-Kaiser, A., Davis, R.L., Blikstein, P., & Okamura. A.M. (2016). 3-D printed haptic devices for educational applications. In 2016 IEEE Haptics Symposium (HAPTICS). 126–133. https://doi.org/10.1109/HAPTICS.2016.7463166

© 2022 California State University, Northridge

Orta Martinez, M., Nunez, C. M., Liao, T., Morimoto, T. K., & Okamura, A. (2019). Evolution and Analysis of Hapkit: An Open-Source Haptic Device for Educational Applications. IEEE Transactions on Haptics, X(X), 1–14. https://doi.org/10.1109/TOH.2019.2948609

Newen, A., Bruin, L.D., & Gallagher, S. (Eds.). (2018). *The Oxford handbook of 4E cognition*. Oxford University Press.

Núñez, R. E., & Freeman, W. J. (Eds.). (1999). *Reclaiming cognition: The primacy of action, intention, and emotion (Journal of Consciousness Studies 6, 11-12)*. Imprint Academic.

Sallnäs, E. L., Moll, J., & Severinson-Eklundh, K. (2007). Group work about geometrical concepts among blind and sighted pupils using haptic interfaces. *Proceedings - Second Joint EuroHaptics Conference and Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems, World Haptics 2007*, 330–335. https://doi.org/10.1109/WHC.2007.61

Tancredi, S., Chen, R. S. Y., Krause, C. M., & Siu, Y. T. (in press). The need for SpEED: Reimagining accessibility through Special Education Embodied Design. In S. L. Macrine & J. M. Fugate (Eds.), *Movement matters: How embodied cognition informs teaching and learning*. MIT Press.

Tennison, J. L., & Gorlewicz, J. L. (2016). Toward non-visual graphics representations on vibratory touchscreens: Shape exploration and identification. *Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*. https://doi.org/10.1007/978-3-319-42324-1_38

Xu, H., Peshkin, M. A., Member, S., & Colgate, J. E. (2019). UltraShiver: Lateral Force Feedback on a Bare Fingertip via Ultrasonic Oscillation and Electroadhesion. *IEEE Transactions on Haptics*, 1–29.

© 2022 California State University, Northridge

Xu, H., Peshkin, M. A., Member, S., & Colgate, J. E. (2019). UltraShiver: Lateral Force Feedback on a Bare Fingertip via Ultrasonic Oscillation and Electroadhesion. *IEEE Transactions on Haptics*, 1–29.

© 2022 California State University, Northridge