

RUNNING HEAD: DIALECTICS AND GENERATIVE DESIGN

A Dialectic Analysis of Generativity:
Issues of Network Supported Design in Mathematics and Science

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Title

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Abstract

New theoretical, methodological, and design frameworks for engaging classroom learning are supported by the highly interactive and group-centered capabilities of a new generation of classroom-based networks. In our analyses, networked teaching and learning are organized relative to a dialectic of (1) seeing mathematical and scientific structures as fully situated in socio-cultural contexts and (2) seeing mathematics as a way of structuring our understanding of and design for group-situated teaching and learning. This dialectic provides a unique way of constructing understandings of the relationship between content and social activity. We believe that our approach to generative design, centering on the notion that classrooms have multiple agents interacting at various levels of participation and engagement, makes the best possible use of the plurality of emergent ideas found in classrooms. We review networked classroom examples that helped provoke our theoretical analyses. Features are presented for what we call generative design in terms of the respective “sides” of the dialectic. Additional implications for activity design are then addressed. We close with an examination of how this dialectic framework also can support constructive critique of both sides of the dialectic in terms of content and pedagogy.

Introduction

The highly interactive and group-centered capabilities of a new generation of classroom-based networks support the development of new theoretical, methodological, and design frameworks for engaging classroom learning. We are interested in how mathematical and scientific content can frame the design of classroom activities and supportive technologies. Content in this sense is an organized body of knowledge developing over time and enacted through activity. In exploring the mutually constitutive relationship between content and classroom activity, we structure this analysis of network-supported teaching and learning in terms of a dialectic of (1) seeing mathematical and scientific structures as fully situated in socio-cultural contexts and (2) seeing mathematics and science as ways of structuring our understanding of, and design for, group-situated teaching and learning. This work is intended to have practical and theoretical value for researchers and classroom educators.

Practically, we hope to avoid the possibility that the promise of next generation network capabilities will be less than fully realized. Simply aggregating student responses to multiple choice questions, while relatively easy to do in a networked environment and certainly useful on some occasions, misses the many ways in which day-to-day classroom practice can be improved by a more thorough engagement with generative design. Accordingly, in this paper we attend both to some of the generative design principals that can help guide classroom-focused activity design and to some of the potential pitfalls that can compromise the realization of the mediating potential of these next generation systems. Theoretically, we believe the approach to generativity discussed in this paper – centering on the idea that classrooms have multiple agents participating at richly interactive levels of engagement and agency – makes better use of the diversity of identities, the plurality of conceptual structures, the shifting of collective

understandings, and the evolution of disciplinary concepts found in classroom activity. Attending to this plurality of expressive activity serves to deepen our insights into the emergence and development of ideas and, simultaneously, into the designs that can best support the advancement of mathematical and scientific thinking for all our students. Critical to our engagement with this multiplicity of expressive forms is the framing of our analyses in terms of a dialectic of seeing content as both socially structured and socially structuring.

We begin our analysis of network-supported generativity by introducing our intended meaning for dialectic, as well the particular dialectic we use as the central organizer of our efforts. Specific features of networked classroom technologies then are described and, briefly, we answer the question of what makes these next generation networks unique in terms of the mediating role they may assume. Next we illustrate the use of mathematical and scientific ideas as foundations for the design and analysis of classroom activity across a number of network-based projects. Then to generalize from and also to reflectively critique this use of the content domains, we introduce four features of generative design: (a) space-creating play and (b) dynamic structure (see 2 in Figure 1); and (c) agency and (d) participation (see 1 in Figure 1). This focus on design is intended to highlight the ways in which the dialectic between social and mathematical /scientific structures illuminates new possibilities for generative classroom practice. We close by drawing out specific implications, including critiques of current practice relative to group-oriented pedagogy and design, as well as social analyses of school-based learning.

Dialectic as Creative Tension

Many researchers now embrace the idea that content and learning exist in, and emerge from, social activity. These analyses have made significant contributions to our understandings

of teaching and learning, and have heralded a new era of not just greater subtlety in the ways we look at the classroom, but also in what it means to teach and learn. Teaching and learning have come to be understood as forms of participation in activities and processes much larger than individual comprehension or engagement (cf., Gutierrez & Rogoff, 2003; Lave & Wenger, 1991; Moll, 1990). We propose a next step in this analysis is to see how mathematical and scientific ideas are not just organized by social activity (1 in Figure 1), but also how they can play a structuring role in the social space of classrooms (2 in Figure 1) and thus become involved in organizing social activity.¹

{Insert Figure 1 about here}

At the center of this dialectic analysis is the genesis and interaction of ideas as they evolve in relation to generative, network-supported, classroom activity (see Figure 1).

Our use of the term dialectic follows its use in ancient Greek thought. Unlike the more recent Hegelian use that anticipates a synthesis of opposites, we look to revitalize an earlier sense of dialectic that predates Plato and that views dialogue, discourse and disputation themselves as deepening our understandings of the world². Dialectic is a kind of juxtaposition of ideas, often

¹ Prawatt, 1996, advances a similar sense of ideas participating or having what Dewey [1925/1981] called “sentiency”.

² Parmenides’ (510 BC) foundational poem is seen as a starting point for the ongoing development of the idea of dialectic: “There is need for you to learn all things ... both the unshaken heart of persuasive Truth and the opinions of mortals, in which there is no true reliance ... that the things that appear must genuinely be, being always, indeed, all things” (Diels & Kranz, 1951, p. 246).

literally a debate, rather than a resolution or synthesis. Understandings emerge via the activity of holding in creative tension even ideas that seem paradoxical. We believe that careful examination of this dialectic (Figure 1) can have implications for evolving notions of generative design supported by next generation network technology. As will be discussed more fully, our use of the term generative refers to orchestrating classroom activity in ways that occasion productive and expressive engagement by participants, characterized by increased personal and collective agency.

We see in this dialectic a potentially useful tension and interdependency between the structuring role of mathematics and science and the structuring role of social activity. Designing with this dialectic in mind moves the focus away from having to decide between the two, to productively leveraging the dialect's generative potential. Attending to this dialectic is intended to present us with a novel approach to addressing the question raised by Shulman's (1987) notion of pedagogical content knowledge: What is the relationship between content and the activities of teaching? Content, in the proposed framework, is seen to dynamically structure the network-supported learning activity itself. In a significant sense, then, the content becomes the pedagogy. Reciprocally pedagogy, understood as emergent ways of coming to participate in communities of mathematical and scientific practice, develops content. Accordingly, we can begin to explore how learning activity becomes dynamic, enacted, or lived content. We can also use the dialectic to address the question of the relationships between the classroom community's developing insights and our situated participation in larger communities of mathematical and scientific practice (Lave, Smith, & Butler, 1988; Newman, Secada, & Wehlage, 1995; Resnick & Rusk, 1996). The dialectic is offered as a way of engaging these issues as well as pragmatic design challenges related to working with next-generation classroom networks. The overall sense is that,

too often, social-constructivists ignore the role of mathematical and scientific structure, and, too often, content specialists ignore the structuring role of social activity. Rather than continuing to talk past each other, our hope is that this dialectic analysis can help engage us all with the ways in which content and social activity – together – mutually constitute meaningful teaching and learning activity.

Next-Generation Networks as New Mediating Tools

We believe next-generation network functionality marks a significant break with traditional uses of technology in classrooms in terms of pedagogical vision and technical capability, provoking a new round of theorizing and design. As such, these next-generation capabilities have the potential both to support and to advance our understandings of generative practices in classrooms. But given that traditional networks, with their highly individualistic focus, have been used in classrooms for decades, the question for this section becomes: What might be said to be truly “new” or distinctive about the mediating role this next-generation, group-oriented, network functionality can have in classrooms?

To begin with, these systems are typically designed specifically for the classroom. Rather than simply import traditional network capabilities associated with business or other less learning-centered environments, these systems are specifically optimized for classrooms: places where learners and teachers come together as groups, in a physically contiguous space, with the goal of advancing meaningful learning. Instead of constraining the learning experience to be narrowly individualistic, this technology supports socially situated interaction and investigation. Moreover, the group itself owns the learning trajectories and the processes of knowledge construction, rather than outside experts or programmers.

The implementations may vary, but the top-level design features of these systems are remarkably similar and typically include: individual devices or “nodes”; support for a range of topologies for real-time or near-real time interaction (e.g., peer to peer, peer to group including whole-class, or group to group); wireless flexibility and portability; a core set of meaningful functionality in each device (e.g., at least that of a graphing calculator); and a mixture of public and private display spaces (e.g., the public space can be a computer projection system as with participatory simulations [Wilensky & Stroup, 1999] or a calculator Viewscreen™ with some of the SimCalc materials [Kaput, Roschelle, Tatar, & Hegedus, 2002] and the private space can be the students’ own individual displays on a calculator or laptop computer). The network experience is “author-able” in that it allows teachers, students or others to create new activities or change the flow of a given activity. Participants can exchange both group and individual artifacts/data-types including text, strings, numeric values, ordered pairs, lists, matrices, individual and whole class graphs, images and, in some cases, sounds or video. New forms of gesture are enabled. Just as traditionally students can raise their hands to signal their desire to make a comment, in a networked activity, students can, for example, use the arrow keys on the calculator to move a simulated agent on the projected screen.

With a few important exceptions (cf., *Computer-Supported Intentional Learning Environments* [CSILE], Scardemalia, 1993; Scardamalia & Bereiter, 1991; Scardamalia, Bereiter, & Lamon, 1994; Scardamalia, Bereiter, McLean, Swallow, & Woodruff, 1989; collaboratory notebook, Edelson, Pea, & Gomez, 1996; *ClassTalk*, Abrahamson, 1998; Mestre, Gerace, Dufresne, & Leonard, 1997; Hake, 1998), classroom networking has served primarily in two ways: (a) As a portal to sources of information or interactivity centered outside the classroom (e.g. visiting the *Cable News Network* [CNN] web site or filling out a web-based

questionnaire), or (b) To implement Computer Assisted Instruction (CAI) or tutoring environments. Two features characterize these uses of networking in classrooms. First, the experience is fundamentally individual – most of the activity could be carried out at home or in a local library as little or no use is made of the social space of the classroom. Second, the knowledge and/or the trajectory of learning is owned by a distant expert for web content or the computer programmers for CAI/tutoring environments. Especially for low socio-economic status students who have had their experience of school-based technology skewed heavily toward the use of drill and practice CAI environments (e.g., see Wenglinsky, 1998), a fundamental tenet of mathematics and science reform – that the classroom should be a community of inquiry characterized by joint ownership and construction of understanding – is undermined.

Due to the group-focused interactivity and data collection capabilities of next generation networking, we now have a new mediating tool to explore the dialectic relationship between content and activity in designing for the dynamics of classroom teaching and learning. This paper is not about technology *per se*, but rather about a specific instance of the interaction and co-evolution of design, technological affordance, and theory (Dewey, 1938; Hickman, 1990). Because a significant number of these networks are about to become widely available and are poised to become a major presence in classroom learning, it is important to understand the role these next-generation designs have in advancing the evolution of what it means to teach and learn. Our hope is that this article will serve as an invitation to researchers, designers, and practitioners to participate in shaping this evolution.

As will be illustrated in the examples that follow, new network capabilities can support generative approaches to classroom activity and these approaches move well beyond simply using networks to access outside sources of information or to implement computer assisted

instruction. In embodying classroom-focused, physically proximate capabilities and design, the technology enables important new ways for teachers and students to act, interact, represent, display and communicate in the classroom. At many levels of organization in the classroom, the artifacts and activities of the domains themselves become the medium for important forms of social interaction. Of course, the network alone does not determine the nature of the learning experience (Papert, 1990). However, a technological resonance can exist between technological affordances and the activities of teaching and learning. The sense is that some technologies more readily support generative group explorations (i.e., the “right tool” for the job).

Like other theoretical lenses used to examine classroom activity, ours draws the observer’s eye to particular features of the activity. For example, the theoretical lens of symbolic interactionism advanced by Cobb and Yackel (1996) draws our attention to specific features of teachers’ and students’ roles at individual and group levels of activity. An understanding of this theoretical stance allows educators and researchers to explore, characterize and act on the interrelations of these roles and levels of analysis. In a similar way, our dialectic framework invites the reader to attend to the ways in which mathematics and science are used to structure social activity and in so doing provides a new framework for understanding the dynamic relationship between content and pedagogy. We provide the following examples to illustrate the side of the dialectic relationship where mathematical and scientific concepts are used to structure the social activity of classrooms (2 in Figure 1).

Mathematics and Science Structuring the Social Space

Reflecting on a number of early projects involved in working with next-generation network capabilities, an interesting pattern emerges. Many of the researchers and developers responded to the challenge of designing activities using next-generation capabilities by using

mathematical/scientific ideas to organize the classroom-based activities and analyses. As illustrated in the following examples, the interactions in these group activities are designed to embody or enact domain-related ideas at various levels of organization and agency. This use of domain-related “big ideas” (cf., Papert, 1980) to organize and analyze group learning is what has been discussed as “mathematics structuring the social sphere” (*MS3*; Stroup, et al., 2002, pp. 195-213). Such an engagement with mathematics and/or science goes well beyond thinking of these domains as content to be learned by individual students. Consistent with the work of other researchers (e.g., Lesh & Yoon, in press), mathematical activity and scientific activity are seen to be as much about interpretation, integration, differentiation and revision as they are about computation and deduction. The brief examples that follow are intended to help make visible the sense in which mathematics and science are, by design, complex, lived processes that can be well-supported by next-generation network capabilities.

Parametric space

The SimCalc Project has been pushed to think anew about the design of classroom-based activities as they move from developing software for individuals to learn calculus ideas to now working in a group-oriented network space (cf., Hegedus, & Kaput, 2002). They have begun to use the idea of a parametric space to organize network-supported activity. For example, to help understand the roles of the “m” and “b” in linear functions of $y = mx + b$ (slope-intercept) form, each group of students is assigned its own value of “b” (the y-intercept), which controls the starting point of the group’s “mascot” in a “race.” In this race, groups are asked to finish in a tie with all the other groups’ mascots at a given time (six seconds) and position (twelve meters). They then must determine the velocity (slope) that accomplishes this task for their given initial conditions. In submitting their solutions to a public display space, the resulting aggregation of

graphs has properties that none of the individual graphs have: A fan of lines all meeting at the same place is created. When simulated characters associated with each of these lines are animated, the characters all come together at (6, 12) and, if continued for additional time, they then spread apart again in a kind of mathematical dance. In this way, the SimCalc Project has begun to use the mathematical idea of parametric space as both the content and the cognitive organizer for the network-supported learning activity to foster the creation and analysis of mathematical objects. Mathematics is thus both emerging from students' activity in the social space of the classroom and is at the same time structuring that space because properties of linear function are organizing students' activity. (For further specific examples, see Kaput & Hegedus, 2002).

Emergence and Systems Theory

In a similar way, content becomes pedagogy and design for the Participatory Simulations Project (Wilensky & Stroup, 1999; 2000)³. Participatory simulations are networked activities where learners act out the roles of individual system elements and observe how the behavior of the system as a whole emerges from their individual behaviors. These emergent results then become the focus of discussions and analyses. Using network technology with a public display space, students can, for example, become agents in a population where a disease is introduced

³ As is noted in these publications, possibly the first major instance of where a participatory simulation was used in the context of systems dynamics and systems learning was *The Beer Game* as developed by Jay Forrester and his systems dynamics group at MIT in the early 1960's. These participatory simulations were called "flight simulators," in a way that alluded to the use of simulator environments in WWII. There is a significant literature related to *The Beer Game* and interest in this participatory simulation has been recently revitalized as a result of its appearance in Senge's widely read *The Fifth Discipline* (1994). Diehl (1990) appears to have been the first to use the phrase "participatory simulations" to describe these activities. Over the years different technologies have been used to implement participatory simulations. These implementation range from the use of simple paper and pencil (e.g., Senge, 1994; Stor & Briggs, 1998), to the use of electronic badges (so-called "Thinking Tags," see Colella et al, 1998; Borovoy et al, 1996; 1998), handheld technologies (Stroup, 1997b, 1998 [a TI-83 graphing calculator]; Soloway et. al., 2001 [using a Palm OS device]), and a new, network-based HubNet architecture (Wilensky & Stroup, 1999b, 2003). Our focus herein is on participatory simulations as implemented in next generation classroom networks.

and be part of the system as the disease spreads. In another simulation they each can control a stoplight in a simulated city's traffic grid and together work toward improving the traffic flow. Not only is dynamic-systems modeling the content being introduced into the curriculum, the learning itself is organized in terms of the classroom *becoming* the dynamic system. By assuming iconic roles in a system, mathematical ideas like emergence, feedback, and complexity are literally embodied by the network-supported learning activity.

When introduced to the gridlock activity, students are presented with the following scenario: The mayor of the City of Gridlock is unhappy with the traffic congestion in town and she has commissioned the class to improve the situation (Wilensky & Stroup, in review). The goal of the activity is for the students to find ways of optimizing traffic flow for the simulated city. Students are asked what they know about traffic flow. In response to the teacher asking, "What are some of the things that you guys listed that would be indications that traffic would be good or bad to you?" the learners articulate a wide range of factors that can impact complex phenomena like traffic. Some of these responses involve behaviors of individual drivers (slow drivers) and others are related to the structure of the roadways or the context for the behavior (e.g., barrier walls too close, lights too long). Collectively, these lists suggest that students do have an initial appreciation of how agent behavior in an environment can have consequences for the emergent features of a complex system. During the simulation, students control individual traffic lights and also call out directions and strategies to each other aimed at optimizing flow.

Afterward, whole-class analysis of traffic systems involves students exploring the dynamic relationship between individual or sub-group behavior and the resulting emergent, complex dynamic system. For traffic these insights come from using both their first-person experience in the world in a way that can lead to more robust, incisive and powerful understandings of

complex phenomena, as well as their participation in creating and analyzing an emergent system. As with the SimCalc example above, mathematical and scientific understandings are both emerging from activity *and* structuring of the activity itself.

Proof

Mack (2002) has begun significant network-based research related to how teachers' notions of mathematical proof organize their real-time decision making about where to go with classroom learning, especially as related to moments of mathematical uncertainty (e.g., when students ask questions that teachers are uncertain about). Teachers' notions of aesthetics for mathematical proof – their ideas of what characterizes good mathematical reasoning – serve as an instructional resource (Lave, 1988) for making sense of and validating (or disproving) student-generated mathematics claims. What is important here is the way in which the mathematical idea of proof is used to organize Mack's analyses of classroom activity. The content – proof in this case – is enacted as a core organizing feature of the teacher's pedagogy.

In Mack's work with participatory simulations (Wilensky & Stroup, 1999) significant instances of uncertainty emerge when, for example, each student uses the arrow keys on a networked calculator to move an individual point around on his/her screen. At the same time, this point and all the others from the class move on a computer screen projected at the front of the class. The teacher asks the students to move according to a rule like "move until your y-value is two times your x value." Once a line forms in the public display space, the points are then aggregated and sent back to the students with the challenge to use the calculator to find equivalent functions that go through these points. The student functions can then be collected and displayed. Mack (2002) has found that, in the course of this activity, students will submit functions where the teacher is unsure of their equivalence. The teacher then has to make

decisions about how to proceed based in part on his/her notions of mathematical proof. The group's learning experience is structured by the teacher's ideas or aesthetic of how mathematics reasoning is carried out. Again, the content of mathematics is emerging from and structured by students' networked activity in the social space of the classroom, and again that social space is also structured by the mathematics embodied in teachers' enacted notions of proof.

Complex Adaptive Systems

Peter Senge's (1994) influential book, *The Fifth Discipline*, discusses systems dynamics in organizational behavior. To illustrate important ideas related to organizational learning he presents *The Beer Game* (pp. 27-54), a classic role-playing, inventory management game. Players assume the roles of retailer, wholesaler, distributor and factory and exchange orders to manage beer inventory and to minimize costs. The structure of the activity, including the interactions among participants, creates an emergent system. This dynamic system exhibits properties that can then be analyzed to help learners understand important implications for managing organizational behavior.

More recent work related to networked classrooms uses *The Beer Game* to engage students in learning about dynamic systems. Rather than analyze the interactions through the lens of the systems dynamics literature following from Jay Forrester (1968), Hurford extends Holland's (1995) analyses of complex adaptive systems (CAS) to classrooms as students participate in the network-supported version of *The Beer Game* activity. Holland analyzes the mechanisms by which CAS evolve and adapt including (a) tags which "enable [CAS] to observe and act on properties" in their environment (p. 13), (b) building blocks, the "component parts" (p. 34) of (c) internal models, or the "mechanism for anticipation" (p. 31) in CAS. Within this framework the building blocks – elements of the emergent internal model that the class develops – are captured

and used to infer the tagging mechanisms – features of the classroom activity – that learners are using to make sense of complexity in distribution systems. Tags, building blocks and internal models structure the reasoning of both the individuals and the group. Thus, complex adaptive systems are what is learned and also a vehicle for learning.

Statistics

Statistical reasoning has long been a way of talking about learning activity. Only relatively recently, however, have statistical ideas been used as a way to structure learning activity in a networked classroom. One networked approach to statistics comes from a tradition that is closely associated with the creation of a dynamic geometry environment called *Sketchpad*TM where learners act directly on mathematical objects, e.g., select and move the vertex of a given triangle (Jackiw, 1995). In a similar way, *Fathom*TM, a dynamic statistics environment, has students act directly on the objects of statistics (Finzer, 2001). Using a computer mouse, learners can drag data, graph axes, sliders, and histogram bars to change them and the effected objects are updated *during the drag*. The student can, for example, select the edge of a bin in a histogram and stretch it to explore the resulting changes in the distribution. Extending this idea of acting directly on the mathematical objects in the network space, groups of students can submit new data or actions to manipulate – in real time – a jointly constructed, publicly projected histogram (Finzer & Erikson, 1998; Finzer & Stroup, 2000). The conjecture is that this kind of socially situated activity – where mathematical actions and meanings are made visible and are to invite negotiation among learners and the teacher – lowers obstacles to student understanding of certain key ideas in data analyses: representativeness of a sample, variability from one sample to another, the connection between a case and a collection, and the meaning and importance of data types in modeling real

world problems.⁴ Statistical reasoning is both the content and the activity through which content emerges.

We move now from examples of content structuring activity to exploring features of generative design. The focus shifts from the mutually constitutive relationship between content and activity to exploring implications for our understanding of, and design for, generative teaching and learning.

Generative Design Features – Play and Dynamic Structure

A number of approaches to generative teaching and learning have been developed (e.g., Freire, 1998; Senge, 1994; Learning Technology Center, 1992; Wittrock, 1991),⁵ and these do focus on forms of activity that support the continuous improvement of individual and group functioning. With this paper we are working to reframe some of these ideas in terms of the proposed dialectic. We also look to make generativity more explicitly about designing technologies and activities for highly interactive *group* spaces in a way that moves beyond simply scaling individual models of learning to the group. We now further explore part (2) of the dialectic shown in Figure 1: the use of the scientifically and mathematically structured ideas of *space-creating play* and *dynamic structure*. This extends previous understandings of generative teaching and learning in ways that are well supported by next generation network capabilities.

Some aspects of this approach to generative design share features with aspects of thought

⁴ More recent work at the Center for Connected Learning parallels some aspects of this use of a network (Abrahamson & Wilensky, 2003) and draws on previous agent-based approaches to statistical reasoning (cf. Wilensky, 1995, 1993).

⁵ Generative teaching, as discussed by Wittrock (1991), involves students' ability to create artifacts that embody their constructed understandings. In a closely related way researchers from the Learning Technology Center at Vanderbilt emphasize aspects of creating "shared environments that permit sustained exploration by students and teachers" in a manner that mirrors the kinds of problems, opportunities, and tools engaged by experts (1992, p. 78). Senge claims, "for a learning organization, 'adaptive learning' must be joined by 'generative learning', learning that enhances our capacity to create" (Senge, 1994, p. 14). Perhaps the most significant in its connections to the sociocultural and critical analyses taken up more directly later in this paper is Paulo Freire's use of "generative

revealing activity as discussed by Lesh and others within a “models and modeling” framework (Kelly, 2003; Lesh, Carmona, & Post, 2002; Lesh, Hoover, Hole, Kelly, & Post, 2000). Content-related expressivity is highlighted in all these approaches.

Space-Creating Play

If students are asked to create and display functions that are the “same as 4x,” they are generating a *space* or collection of related mathematical objects. In the networked version of this activity, the space of related objects is the result of students’ play-full explorations of possibilities, using their individual computing devices, and then using the network to share their examples and to learn from the examples of others.

In considering how play actually works for children it is important to emphasize that play is not simply an “anything goes” state of affairs. Instead, play is an organized form of activity:

The premise that Durkheim, Vygotsky, and Piaget share ...is that thinking and cognitive development involves participating in forms of social activity constituted by systems of shared rules that have to be grasped and voluntarily accepted. ... The system of rules serves, in fact, to constitute the play situation itself. In turn, these rules derive their force from the child’s enjoyment of, and commitment to, the shared activity of the play-world. (Nicolopoulou, 1993, p. 14)

Thinking and cognitive development related to play involve participation: the activity is structured by a system of shared rules that need to be understood and accepted. The power of this form of activity comes precisely from the children’s dedicated engagement related to being part of the “play-world.” Generative teaching and learning as discussed in the prior literature have had some aspects of space-creating play associated with them, but they underutilized the space of

words” in ways that explore the “creative play of combinations” to create new words as part of developing literacy with adult learners in Brazil (1998, p. 87). (See also Callahan, 1999; Stroup, 1997a).

behaviors and artifacts for classroom-based (group) learning and teaching. The network projects discussed earlier in this paper are more group focused. They are also more overtly engaged with the sense in which learners are seen to be “playing” in mathematically and scientifically structured spaces. This play then creates a space of objects or emergent behaviors that embody students’ understandings of the mathematical or scientific content, and that can serve as the objects of attention and analysis for the class.

For the activity to be play-full in the ways intended, the space is created and owned by the students and expresses their emergent, collective understandings of the domain. These student-generated instances are distinct from teacher or textbook generated examples in which students have little or no space-creating agency. Generative activity is also distinct from over-scripted activities that constrain students’ opportunity to play and/or the space they can populate. For example, a generative activity would ask students to find functions that are the same as “ $4x$ ” while an over-scripted activity would ask students to create pairs of single-digit, positive integer terms involving “ x ” that sum to “ $4x$.” The sense is that the structure of the activity should emerge from what students create in a way that they find expressive, that they can make use of, and that is not overly constrained from outside. In the former case students may feel like the conversation can continue or that there is more they can do and explore, whereas in the latter case the sense is of having found the limited set of correct examples. The generative use of mathematical and scientific ideas to structure classroom activity doesn’t collapse the space of possibility but instead opens up ways of instantiating and constructing understanding.

Dynamic Structure

When students in a networked space create and then display functions that are “the same as $4x$,” the emergent structure of the activity is itself *lived* or brought into being by what the

learners do and come to understand. Unfortunately in mathematics research, the idea of structure has come to be understood in a relatively static way. Notably, the process-object analyses of mathematics learning have tended to collapse structure to a relatively static kind of object (Dubinsky, 1991; Sfard, 1991). For example, Sfard highlights this static idea of structure as “object”: “[T]he structural conception is static, instantaneous, and integrative, the operational is dynamic, sequential, and detailed”(Sfard 1991, cited in Kieran, 1993, p. 193). In contrast, social constructivists have tended to respond critically to this idea of structure.

When Soviet psychologists speak of the ‘structure of an activity,’ they have in mind something very different from what has come to be known as ‘structuralism’ in Western psychology [and mathematics education]. The units are defined in terms of the function they fulfill rather than of any intrinsic properties they possess. (Wertsch, 1979, p. 19)

Dynamic structure is intended to point to a functional or operational sense of structure, not fixed or intrinsic attributes. A space of functions is created by the “4x” activity, and in a significant sense this space is brought into being by learners’ own understandings of the mathematical ideas of equivalence. Their mathematical ideas are what *dynamically structure* the learning activity. This socially animated understanding of structure fits well with the ideas Soviet psychologists – and especially Vygotsky (1934/1986) – bring to learning.

What may come as a surprise to some, however, is the sense in which this lived and larger-than-the-individual meaning of dynamic structure may be exactly what Piaget (1970) was pointing to in suggesting that formal mathematical constructs (e.g., the algebraic group) could serve as the prototype of what he meant by emergent cognitive structure⁶. Structures emerge in

⁶ Piaget takes the idea of the group, as it is found in abstract algebra, as the “prototype of structures in general”(1970, p. 19), and explicitly links it to his characterization of constructivist learning: “It is because the group concept combines transformation and conservation that it has become the basic constructivist tool” (1970, p. 21).

relation to the activity of individuals as well in the development of mathematics and science as domains (Piaget, 1970; Piaget & Garcia, 1983). Individuals and larger communities of mathematical and scientific inquiry are different levels at which we can attend to creative agency. Structure, understood in this dynamic way, is a *patterning* or *coordination* in the kinds of operations on elements of the system. It is this larger dynamic sense of structure that allows Piaget to talk about group-situated learning as “co-operation” (Montanegro & Maurice-Naville, 1997, p. 140). Students are exchanging, adding on to, and transforming understandings in ways that mirror formal operations in mathematics. This dynamic, emergent sense of structure found in both Vygotsky’s and Piaget’s work is consistent with the sense of generativity that follows from the notion of mathematics and science structuring social activity. Viewed this way, there are striking parallels and forms of complementarity between Vygotsky’s analyses of language and Piaget’s analyses of operational thought that can be brought to the task of thinking about, and designing for, activity in classrooms supported by next generation networked functionality. In summary, emphasizing this side of the dialectic, generative learning and teaching come to be understood as organized by space-creating play and dynamic structure.

Generative Design Features – Agency and Participation

In this section we focus on agency and participation as two socially significant features of classroom activity closely related to the ideas of space-creating play and dynamic structure. In so doing we explore that side of the dialectic emphasizing the structuring function of social activities in characterizing generativity (1 in Figure 1). Acceptance of the social construction of mathematics and science that develops from the interactions of teachers and students with content and with each other allows for new insight into the nature of what is learned and taught

in network-supported classrooms. The following analyses are intended to more fully situate these interactions and the content domains themselves within sociocultural activity.

The network-based projects reported on earlier represent important examples of the unique affordances of such designs because they highlight one aspect of the dialectic, namely the structuring of the classroom social space by mathematics or science proper (2 in Figure 1). However, notions of learning and the roles of students in relation to content (Lave & Wenger, 1991; Moll, 1990; Rogoff, 1995), and the function of tool-mediated activity (Vygotsky, 1987; Wertsch, 1991) are under-specified in those projects. As we continue to explore generative design features, we move next to the structuring role of activity in the social space of classrooms. The notion of learning here extends what was referred to earlier as “participating in forms of social activity constituted by systems of shared rules that have to be grasped and voluntarily accepted” (Nicolopolou, 1993, p. 14) to make explicit that participation involves not only understanding rule systems, but also shaping and creating shared rules through social interactions in classrooms. Learning in this sense is also transforming individual and collective participation in important practices of communities (e.g., mathematics and science) (see Lave & Wenger, 1991; Rogoff, 1995). Thus, agency and participation (see 1 in Figure 1) are explored next in examining how mathematics or science get constructed, and by whom; and the mutually constitutive relation between content, activity, and tools in next-generation networks.

Together, participation and agency in networked activity are important elements of generativity, expanding the social space of the classroom by inviting active, creative engagement in the construction of mathematics and science. This notion of expanded social space highlights two dimensions of classrooms: 1) content and representation, and 2) ways of participating, including the use of social and cultural practices.

Levels of Agency: Expanding Content and Representations

Students' opportunities to assume agency at both the individual and collective levels in these new social spaces are markedly different from traditional classrooms. To be a visible and necessary participant in real-time, public construction of knowledge is a significant form of agency that is unique to next-generation network design. The nature of the activity and of the generative designs themselves are both important features that expand students' and groups' agency in structuring the mathematics and science that emerge from their collective engagement with content. To date, two design features seem to be particularly important: (a) anonymity and (b) opportunity to expand the content and representations that are the focus of activity.

Many networks allow students to submit contributions to the emergent system to be considered by the class without their identities being publicly associated with that information. Davis (2002, p. 208) shows that, freed from *who* sent in a response, students are able to explore *what* the mathematical activity represents, whether it is sending in functions that are the same as $4x$ or controlling a traffic light in a simulation. Thus, anonymity facilitates the group's ability to explore mathematical and scientific concepts in a non-threatening way. From the teacher's perspective: "It just promotes a lot of discussion and everybody's free to discuss it because kids can be criticizing an equation that they themselves wrote and nobody would know" (Davis, 2002, p. 208). At the individual level, students identify with their responses, icons, and data that show up in the group display, focusing on themselves as part of the collective construction of mathematical and scientific objects. Also, students indicated that the representation of self in relation to the group space gives them a sense of how they were doing relative to the class as a whole. At the group level, efforts center on the co-construction and analysis of content. Thus,

opportunities to reflect anonymously on their respective insights contribute to students' and groups' increased sense of agency in engaging the learning activities.

In addition to being invited to contribute to network activities because anonymity reduces the risk of doing so, students have the opportunity to expand both the content and representations involved. For example, in one participatory simulation used by a teacher to explore positive/negative integers, students also recognized and explored concepts of slope and rate, as well as their representation in graphs (Ares, 2003). Here, students used the mathematics of the network-mediated activity itself to expand the content and representations involved. This aspect of agency, or control of the means by which something is accomplished, is linked to Lave and Wenger's (1991) notion of legitimacy of participation wherein individuals' substantive contributions involve them in shaping practice, or systems of shared rules. Gaining legitimacy involves making contributions to critical aspects of practice, as in students' expansion of the activity to include concepts of rate, slope, and representation. They gained legitimacy in mathematical activity by exercising the agency afforded them in the networked activity to construct mathematical practice and content. Thus, the space of mathematical objects was enlarged through their play-full engagement in the generative activity.

Participation: Expanding The Use of Cultural and Social Practices

Careful consideration of what kind of participation in the construction of what kind of content is important in order that the classroom learning is generative. Students and groups of students not only learn in ways that foster powerful, dynamic understandings of mathematics and science but also learn in ways that support all students developing knowledge and skills that foster their successful participation in mathematical and scientific activity in the larger world. If one of the aims of network-related projects is to open mathematics and science to more students

(e.g., Kaput, 1994), then questions about the nature of the activity from which such learning emerges are essential.

A diversity of avenues of participation is available in network-mediated activity, including text, physical and electronic gestures, as well as verbal contributions to classroom dialogue (e.g., conjecture, prediction, observation, and explanation). Moreover, the collaborative character of participation in those modes of contribution, using a variety of representations of phenomena (texts, graphs, visual displays of emergent systems, language), and in inquiry-oriented discussion and analyses invites multiple ways of belonging.

Together, the varied modes of participation and joint construction of knowledge mean there is unique potential in networked classroom technologies to draw on students' cultural and social practices to support learning in mathematics and science. Lee & Fradd (1995) note that communication patterns vary across cultural groups, and that, "Students from diverse language backgrounds often have different interpretations of verbal communication and paralinguistic expression.... alternative communication patterns can provide ... students with powerful ways of demonstrating their knowledge and understanding" (p. 17). For instance, during the participatory simulation described above, two Latinas collaborated in Spanish throughout the simulation, sitting next to two European American boys who did their own work and then compared their results in English. Both pairs' interactions were important and appropriate, expanding the ways of participating seen in more conventional teaching. In addition to Spanish and English serving as cultural resources, choosing to collaborate versus working independently may also have had gendered and/or cultural roots. The dynamic structuring of their activity occurred not only through the mathematics involved, but also through the students' use of individual and collective

social, cultural, and academic resources. Thus, being able to draw on varied ways of participating made good use of important resources the students brought to the task.

Mediating Function of Networks: Expanding the Social Space

We can further analyze this example of how students used distinct cultural resources – including language – in network-supported activities to construct powerful insights. In the subsequent whole-class discussion both pairs of students’ hypotheses, predictions, and explanations about *jointly constructed mathematical objects* were important *contributions* that shaped the whole class’ content understanding. Thus, the dual dimensions of (a) content and (b) the ways of participating were mediated by the technology in important ways that provided an opportunity for students to draw on their social and cultural resources in engaging the full range of concepts and representations embodied in the participatory simulation. As Vygotsky (1978) noted:

The use of artificial means, the transition to mediated activity, fundamentally changes all psychological operations, just as the use of tools limitlessly broadens the range of activities within which the psychological functions may operate. (p. 55)

Figure 2 captures this sense that activity, mediated by generative networks, functions to expand the social space of classrooms (where the circles represent the example discussed above) in relation to less generative designs and activities that often narrow curriculum and participation (e.g., the white circles representing working independently or working with the whole class on textbook examples of rate and slope).

{Insert Figure 2 about here}

Students' participation in the construction of mathematical and scientific objects and content, as well as the modes by which they do so, are mediated in unique, inclusive and generally more expansive ways by this use of the networked technology (see solid circles in Figure 2).

Thus far, we have organized the paper in terms of the dialectic that we use to characterize the relationship between content and social activity. The central attribute of our approach to design is to encourage expressive diversity that promotes the evolution of ideas through the interactions of people and tools. This plurality of ideas can be generated, propagated, shared, contested, and advanced when there are multiple ways for those idea to emerge and develop. The dialectic allows us to heighten attention to, and make the best use, of this plurality. Having presented features of generative design as organized by the two sides of the dialectic, we now move on to draw implications that arise from seeing the dialectic process (Figure 1) as potentially integrative.

Design Implications for Network-Supported Classroom Activity

Four broad categories of design implications frame this section's examination of the network design implications of the dialectic. As part of the effort to more fully realize the potential of these next-generation systems, some current practices in design are also critiqued in terms of this generative framework.

Productive Play

In generative participatory settings, willing and doing are to be unified. Technology and teachers play a convening role, not a controlling role, where students are central to what expressive artifacts (e.g., a graph or a kind of motion for a simulated agent) and insights get produced. Students move to assume a convening role as well. Agency requires that "willing" is not just deciding to *play along*, but also to *play a part* in what the activity it is about, what

matters, and where the classroom activity should go next. The separation of willing and doing in most classroom activity contributes to student alienation in the classroom. Our belief is that the integration of willing and doing will meaningfully advance students' sense of agency and ownership, and that this ownership will scaffold engagement in authentic activity outside the classroom and outside school.

Relative to space-creation, what distinguishes playing along from playing a meaningful part includes, in some sense, the size of the space the students can explore. Playing along invokes a sense of constraint and limited possibility. Activities such as “solve this linear system by adding the same thing to both sides” and “simplify the expression $2x + 2x$ by combining like terms” are highly constrained. Playing a part, on the other hand, involves one's own explorations being juxtaposed to others', to the group's evolving notion of the domain, and to the more formalized insights of the dynamic communities of science and mathematics. The difference can be as simple as asking students to submit expressions, using a network, that are the “same as $4x$ ” or as complex as finding ways to improve traffic flow in a participatory simulation.

Traditional models of tutoring, including their embodiment in some forms of networked design, typically center on playing along. Even in more recent “cognitive” tutoring environments that are a clear improvement over traditional CAI environments, a relatively small space of correct expressions is evaluated as acceptable (Carnegie Learning, 2003; Heffernan, 2001, 2003). One of these cognitive tutoring environments begins by asking the student to find a linear expression describing the motion of a boat, rowed at a constant speed, crossing a river with a current having a uniformly constant rate of flow (Heffernan, 2001, 2003). Not only are students constrained to a small set of possible responses, they are not allowed to raise questions about the nature of the problem or its significance in rowing a real boat across a real river like the ones

they might find in their world outside of school. In essence the student is actively coached by these environments to converge to one of the possibilities in this small solution space.

Similarly, sending out to students a “family” of nearly identical tasks, ones varying only in terms of the highly constrained randomization of one or more parameters, is not generative even if, in principle, the space of possible tasks and solutions is large. For example, some recent activities mediated by new connected technologies allow one student to evaluate a peer’s attempt to describe the motion of a simulated elevator using either piece-wise constant velocity graphs or piece-wise linear position graphs (Roschelle & Vahey, 2003). Without going into the details, powerful ideas in calculus are intended to be, and can be, part of the matching activity. Each pair of students gets a unique matching task where each task has a single correct solution. While this work has focused on important social dimensions of peer-to-peer interactions (cf., Roschelle, 1990), variation is accomplished by having the technology select random pairs of graph segments for each duo of students to work on. According to the designers, a large space of possibilities can be explored because each student pair is working with a unique but related task. Peer-to-peer dialogue notwithstanding, from our perspective this is not generative because the technology itself is responsible for generating the space, not the students’ own thinking.⁷ Moreover, once the task is assigned there is only one right answer per group. And finally, the sense in which these pairs may be seen to be exploring a “family” of related functions is all but inaccessible to anyone but the programmer and, possibly, the teacher. With this design, larger insights about families of functions are unlikely to be visible to, or generated by, the students.

⁷ Recent iterations of this activity now give students the ability to choose the initial challenge and highlight the after-activity discussion wherein the students are asked to describe what clues were most useful in successfully completing the activity. This emergent space of clues is captured in a shared public space by having the teacher write down the list of useful clues as students call them out. In these ways the designers have increased the generativity of the activity (Roschelle, personal communication, November 6, 2003).

Agency and Authorability

Generative design, as we're using it, requires that what the activity is about emerges from participation and is brought into being by the participants. Agency is a necessary feature of this design. What the activity is about can't reside only "in" the teacher's head or "in" the technology. More than visibility and engagement, agency also includes a sense of being able to contribute to and/or change the flow of the activity. Next generation network design includes ways of acting at individual, small group, or whole class levels. At all these levels students can change the nature of their participation and influence the evolution of what the activity is about. For example, a group of students responsible for one vertical column of intersections in the traffic gridlock activity can decide to implement a column-specific strategy to cascade the lights. This has implications both for what they do individually and for their group's relations to the class' activity. Depending on what results, they can then shape the strategy and discourse of the larger group. Ideally, this sense of agency would extend to being able to author or alter the network-supported communication, forms of interactivity, and analyses. Students could, for example, notice that the gridlock simulation has cars leaving one side of the grid reappear on the opposite side (they "wrap"). Out of a desire to explore a more realistic simulation of traffic, they could reach into the relatively accessible computer code and try and make the cars' behavior more like what they know of the world around them (cf., Wilensky, 1999). Or they might decide to try and find examples of systems that come close to working in this wrap-around way (e.g., some aspects of an open, rectangular hallway system in their high school). What the activity is about and how the students can investigate it are changed in fundamental ways. The conversation and investigation can change directions and these trajectories are co-constructed by the participants.

Situating Communities

In generative designs, several planes of agency and participation are made active and visible both in the design of the technology and in the nature of the activity. In a sense we are alluding to the possible network design implications of Rogoff's (1995) "planes of analysis of sociocultural activity" (e.g., intrapersonal, interpersonal, and community; p. 142). The community that emerges locally in the classroom is critical to the mathematics and science that is constructed. More than this, we expect that participation in a generative classroom community will also have implications and consequences for the multiple communities that serve to situate the activities of students and teachers (e.g., communities of math and science, the workplace, and cultural groups). Communities are organized at different levels and through a variety of means, and the technology should play a mediating role in support of students' and teachers' meaningful interaction with these levels.

By engaging in multiple levels of participation emerging out of generative classroom activity, students are not cut off from the world – including the worlds of formal mathematics and science – but are better linked to other communities through their appropriation of authentic modes of participation. They are better linked because they understand, from having participated in it, that knowledge and insight emerge from active inquiry within communities. They understand this from the inside out, from having been a legitimate participant. They have negotiated consensus and difference within a given community. This scaffolds their meaningful engagement with the insights and formalism of, in this case, mathematics and science. The issue is not "real" science versus classroom science. Instead this approach replaces a static, canonical notion of what math and science are with a more active, evolving, and participatory

conceptualization. It is the static notions of math and science – as they are typically presented in traditional classrooms – that contrast most markedly with “real” science and mathematics.

The Dialectic Enables Constructive Critique

Up until this point we have used the dialectic as a way of drawing out implications for classroom activity as supported by next-generation network technologies. We now explore its utility as a framework for supporting a critique of the respective limitations of both sides of the dialectic in terms of content and pedagogy. What is at stake here is not the question of their respective structuring functions, or their significance in shaping notions of generative design, but the ways in which they are also dependent on each other. This dependency enables critique. And critique itself helps to further the generative potential of the designs that attends to the dialectic. Relative to network-supported design in particular, the sense of dialect introduced in this paper suggests that creative tension – including constructive critique – is what can serve to open up, and to help to more fully realize, the generative potential the dialectic between the structuring role of math and science and the structuring role of social activity.

A Social Critique of the Mathematical and the Scientific

As significant an improvement as all these network-supported forms of interaction are – where historical notions of teachers’ and students’ relation to content are altered – historical notions of content itself are still being maintained, tacitly, in some of network-mediated efforts reported herein. One might observe that an under-examined assumption of some of this work is that mathematics and science are homogeneous, monolithic, and unambiguous. Viewed in this way, even an updated notion of generativity can serve to lead only to a predetermined outcome. The very idea that mathematics and science are socially constructed – especially as situated in the classroom’s concrete, localized, and diverse construction processes – problematizes this

homogeneity and lack of ambiguity. From the dialectic perspective, generative activity can produce insights and growth possibilities for both actors *and* notions of content. Content is not static. Generative design takes seriously the sense in which mathematics and science are themselves evolving and structuring forms of activity and insight, not fixed entities. This evolving and structuring sense of content is a conspicuous feature of network-supported activity. Ambiguity and heterogeneity are inherent in the developing use of mathematical and scientific language and the evolution in what it means to know.

In a related way, as we move to more culturally situated notions of participation, a somewhat uncritical assumption in some designs for network activity seems to be that mathematics and science are universal languages. Vygotsky's idea of "scientific knowledge" (1987) can be seen to share this assumption even as it also attends to how this language comes to have meaning from social interaction and activity. In contrast, treating language as a dynamic mediating tool through which group-level and individual knowledge structures emerge poses interesting challenges to universality. The network-supported evolution of ideas and discourse in the classroom serves to problematize any notion of a universal, fixed language. The mediating tools of mathematical and scientific language change in both form and significance through activity. Thus, generative designs need to engage this evolution to tap into students' developing understandings. Clinging to notions of universality undermines this engagement.

Additionally, cultural ways of knowing, as embodied in the variety of students' languages and ways of participating, expands the possibilities for construction of content and participation, as well as the exercise of agency. Generativity, especially as it relates to space-creating play, makes good use of important contributions this linguistic and cultural diversity can bring to network-supported activity. As long as we hold on to notions of universal language as a gold

standard, there is little or no room for linguistic diversity to matter in and of itself, or for the generative potential of varied expressive forms of language and participation to be realized.

In moving from sociocultural to more critical theoretical perspectives, questions arise about whose community, culture, and history are the foci of the design activities. Students' lives are situated in social, cultural, historical, and political arenas. How are these connections to students' lives included in the network-supported activity of classrooms? For example, a tacit assumption of the traffic gridlock activity discussed earlier is that students' experiences in the world are important to the development of their traffic strategies. But there are many features of the simulation that are not like the world. Who decides if the simulation does or does not capture the significant relationships students might want to attend to? Additionally, is traffic flow itself truly a compelling issue for all or even most students? It is likely that for most students there are a host of more pressing and socially relevant challenges to address. The move to address the complexity in the world around them is a good one. But for mathematics and science to be seen as socially significant and powerful, the question of what complex phenomena are worth investigating must be negotiated with students. For personal and collective agency to be advanced by a particular activity, design must more overtly attend to the questions of does this really matter and for whom. How do the questions raised connect to students' history, culture, and community? In what ways do the activities facilitate students' development of meaningful ideas and insights in a way that can allow them to take action on their world? We believe these questions must be more explicitly engaged in our design efforts.

A Domain-Centered Critique of the Social Analyses

To be fair, while critiques of mathematics education as acultural, ahistorical, or apolitical are certainly reasonable, critiques of sociocultural theoretical frameworks are also important.

Socioculturalists have helped researchers and educators focus on the reality of the social construction of mathematics and science, however, they have done significantly less well in attending to the dialectic proposed herein because of inattention to the structuring roles of mathematical and scientific reasoning for learning (Lee, 2001). There are two sides to this inattention: 1) there is a blindness to content in the way that domain specialists are often blind to culture, language, and social activity, and 2) there is a blindness to the ways in which social critique and analyses often are themselves structured by mathematical and scientific ideas.

There has been little attention to date to the fundamental connections between activity and domain-specific content learning (but see Gauvain, 1998). Social analyses often make claims across disciplines in ways that ignore the unique features of domains (e.g., processes of learning in science are different from those in learning English). There is a sense of making universal claims, albeit about social dimensions of learning, that are not sufficiently situated in relation to what activity in mathematics and science classrooms claims to be about. Not only is this true at the level of the domain as a form of situated activity, but also at the level of the participants' account of the centrality of content in their own representations of what classroom activity is about. An English teacher would claim that his work is about teaching English, and a science teacher would make similar claims about what her work is about. The ways in which they orchestrate classroom activity involve both domain-specific and highly contextualized forms of social engagement. In assuming a near-universal voice, the claims of social critique may miss important aspects of the nature of the activity itself.

This paper also encourages educational researchers, theorists and practitioners to attend to the ways in which science and mathematical domains structure social analyses. For example, earlier we discussed using statistics as a way of structuring the social activity of the classroom.

But it is also the case that statistics has been central to the development of social analyses and critiques of school-based learning (cf., Bowles & Gintis, 1976). Sociology, for example, is heavily dependent on quantitative analysis. Formal aspects of statistical reasoning structure these analyses. Statistics shapes the questions posed, how these questions get answered, and, in part, what counts as knowledge. Critical approaches do not stand outside what we have referred to as content, but are themselves deeply situated relative to the dialectic attended to in this paper.

Discussion

Generativity centers on the diversity and interactivity of learners' ideas. Throughout this paper the dialect of seeing mathematics and science as both socially structured (1 in Figure 1) and socially structuring (2 in Figure1) has supported our exploration and extension of notions of generativity especially in relation to networked classroom technology. We do not view this use of dialect as a setting up a Hegelian synthesis of content and pedagogy but as a kind of holding in creative tension. Generativity emerges in relation to this tension through making visible, and fostering attention to, the mutually constitutive relationships between social activity and domain-related learning and insight. We've made a case for viewing such expressive activity and the ongoing development of ideas, framed in terms of the dialectic, as productive relative to theory and design for classrooms. In its fullest realization, next generation networking can help to support this dynamic vision.

Future efforts extending the productive use of generativity, situated in relation to the dialectic, need to do more to examine the role of language in classroom learning and in the development of domains. The mutually constitutive relation between the language of mathematics and science and the structure of networked classroom activity needs careful analysis. Language is seen to have a central role in both the standards supported by national

professional organizations and in classroom activity. As examples, the National Council of Teachers of Mathematics has advanced the idea of “mathematics as communication” (1989; 2000) as a major strand in the mathematics reform movement and a significant literature in classroom-based research has made increasingly clear the role discourse has in advancing domain-related learning (cf., Frykholm & Pittman, 2001; Rosebery, Warren, & Conant, 1992). Recent work has begun to explore the mutually constitutive relation between the language of the networked classroom activities and the structuring role of mathematics (see Ares, Stroup, & Schademan, 2004). Attention to the unique mediating role of new artifacts – e.g., real-time public displays of jointly constructed representations – in shaping classroom discourse offers important glimpses into the development of mathematical and scientific knowledge and reasoning.

Another potentially productive line of inquiry could examine the generative design possibilities for linguistically diverse classrooms. Within a generative framework the languages of mathematics and science and the diverse languages students speak create a vibrant space of possibility. We can inquire as to whether the network-supported activity has a potentially powerful mediating role in making good use of the academic and social resources available to teachers and students in culturally and linguistically diverse classrooms. Closely related is the question of whether the generative network activity design is effective in fostering diverse students’ sense of participation and agency relative to mathematics and science learning. Paying close attention to new avenues for socially and culturally specific communication and interaction can help to optimize the generative potential of the social spaces emerging in networked classroom environments.

Overall, we believe that the capabilities of next-generation classroom networks can assume a significant mediating role in advancing the evolution of what it means to teach and learn. Generative design, emerging in relation to the dialectic analyses presented herein, has both practical and theoretical value for researchers and teachers interested in the relationship between social interaction and the construction of content. Theories of learning, design, and the development of ideas are informed and restructured in important ways when we embrace the diversity and plurality of classroom activity.

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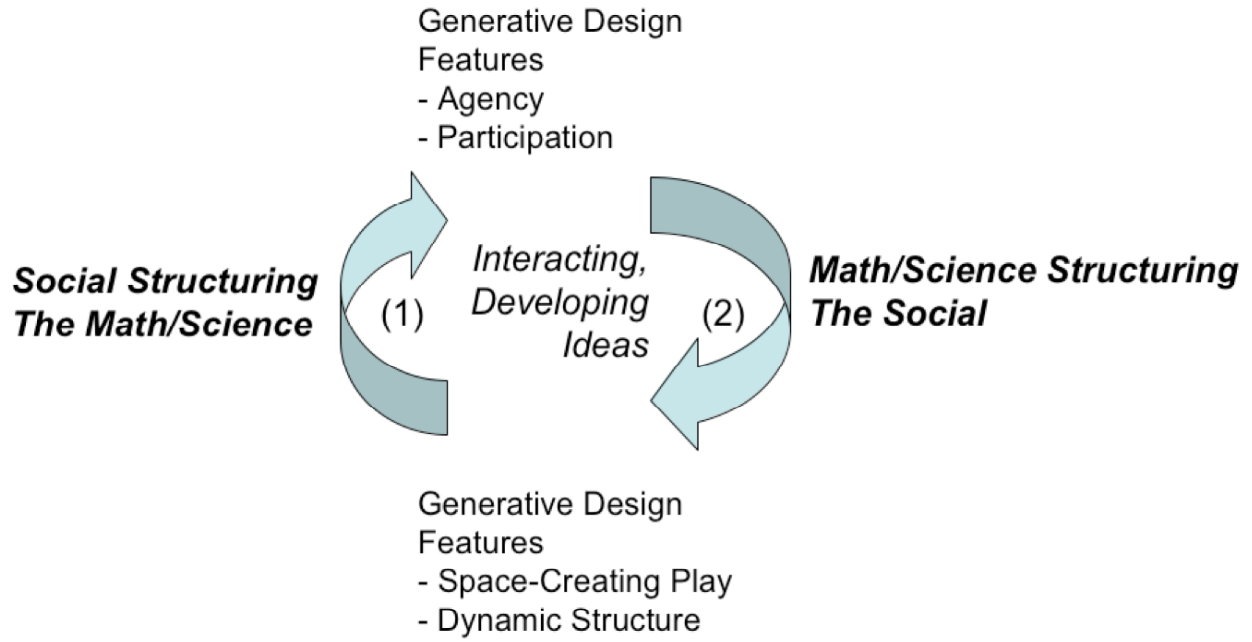


Figure 1. The development of ideas via the dialectic of mathematics and science as socially structured (1) and socially structuring (2).

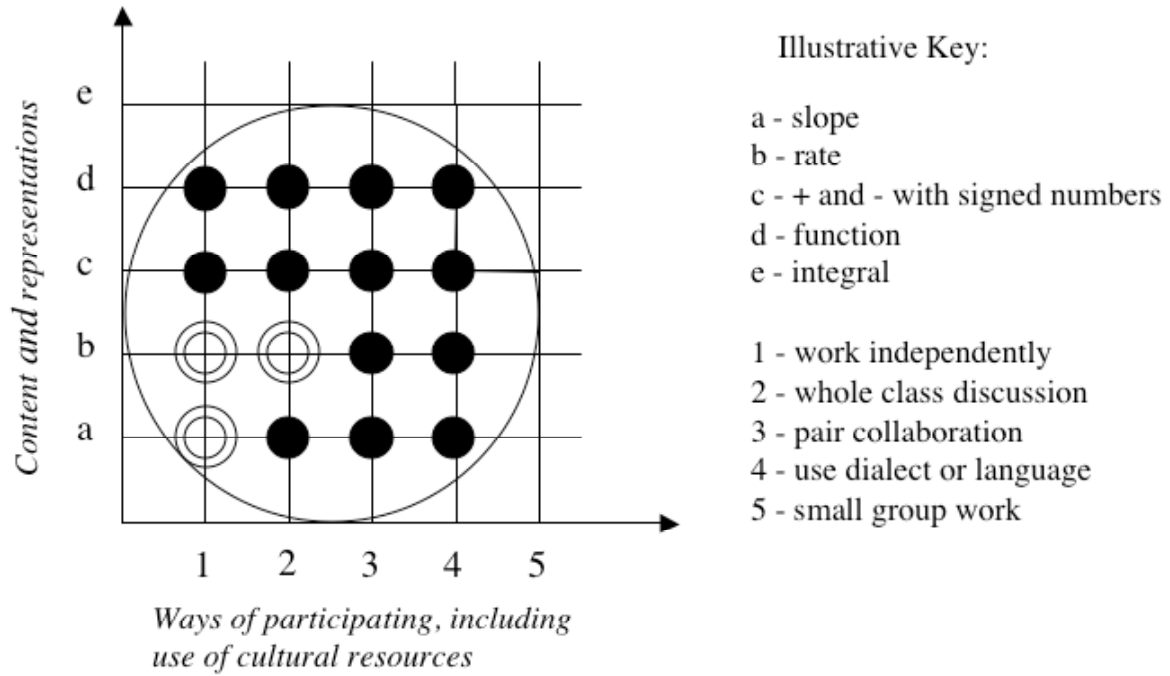


Figure 2. Expanded social space formed by the interaction of content and cultural dimensions.