New Tools for Modeling Quantitative Variation in Complex Systems: Design and Preliminary Classroom Study

Michelle Wilkerson-Jerde and Uri Wilensky
Northwestern University

We are interested in supporting students as they explore what mathematical patterns reveal about the events and individuals that comprise a system. Toward this end, we have developed a set of enhanced modeling and analysis tools within the NetLogo (Wilensky, 1999) agent-based modeling environment. These tools draw from established findings regarding how students learn about complex systems and quantitative change over time. They include two major features: (1) a *construction kit* that allows students to build agent-based models by combining behaviors that each contribute to patterns of change in different ways, and (2) an *analysis interface* that dynamically links graphs and visuospatial representations of models and allows users to directly compare the results of multiple model runs or multiple models.

In this paper/poster, we will describe the motivation and design of these tools in more detail, and describe the first full-classroom implementation of this environment. For the purposes of this short proposal, we include a brief sample analysis and description of emerging results. More detailed analyses including more groups and events will be included in the final poster/paper.

Background and Objectives

Understanding how changing quantities can be used to explore and predict changes in the world is becoming an important life skill (Roschelle, Kaput & Stroup, 2000). But many of the core ideas of scientific literacy – for example, growth and decay, equilibrium, stability and change, cycles, and feedback – are characteristic of interconnected systems that exhibit different behaviors at different levels (Sabelli, 2006; AAAS, 1991). Important figures in everyday life – unemployment rates, global temperatures, or economic patterns – are similarly complex. As the world and the way we study it, becomes more dynamic and interconnected, educators are advocating for inclusion of systems thinking and complex systems principles at all levels of education (Jacobson & Wilensky, 2006; NRC 2010).

We are studying how students make sense of such trends from the perspective of the "mathematics of change and variation in systems" (MCVS). The acronym MCVS is adapted from Roschelle, Kaput and Stroup's term "mathematics of change and variation (MCV)" (p.47, 2000). We add the term "systems" to emphasize that often, mathematical change reflects complex interactions between many entities and events within a system (Sterman, 1994). As such, it requires an awareness of issues such as randomness, nonlinearity, chaotic behavior, and other features that are characteristic of complex systems (Guckenheimer & Ottino, 2008) but that might not be encountered in a traditional exploration of the mathematics of change. In Table 1, we expand on core ideas of mathematics of change and variation (as summarized in Stroup, 2002), and add those of particular importance in complex systems (in *italics*).

how much	A quantity that changes over time.
how fast	A quantity's rate of change over time.
reversibility	The accumulation of a rate of change over time, or a rate of change derived from a quantity's trend over time.
randomness	A pattern of change may feature "noise" as the result of local / stochastic conditions.
chaos	A trend may exhibit one of multiple possible outcomes, and its progress is sensitive to small changes in systemic conditions.
multiple inputs	A single changing quantity can reflect the result of multiple, sometimes conflicting, behaviors and entities.
nonlinearity	A trend can reflect feedback loops, limiting and/or enabling conditions, etc.
multi-level implications	Agent-level implications of a pattern of change may be different than system-level implications.

Table 1. Key Aspects of the Mathematics of Change and Variation in Systems

Our objectives for this specific poster/paper are to **describe new designed artifacts** that draw from existing literature on both complex systems education and mathematics education, **illustrate how those artifacts can support those core ideas** introduced in Table 1, and **share ongoing qualitative analysis of students' engagement with those artifacts and ideas** in a classroom context as they consider patterns of change in population ecology.

Theoretical Framework

Two major threads of existing literature inform our work. *Thinking in levels* posits that a key component of understanding complex systems involves understanding how individual and system level behavior (which is often seemingly contradictory) interact in a system, and suggests that agent-based modeling helps students to understand both of these levels (Wilensky & Resnick, 1999; Wilensky, 2003). *Qualitative calculus* posits that students are able to understand quantitative change over time by interacting with meaningfully linked representations of a quantity and its rate of change, often through enacted or embodied means (Nemirovsky, Tierney & Ogonowski, 1993; Kaput, 1994; Stroup, 2002).

Combining these two literatures suggests that it is important to consider the nature of systemic change at *four key levels* (see Table 2): *behaviors*, *multiple individuals*, a *quantity*, its *trend over time*. Our hypothesis, supported by literature (Kaput, 2003; Repenning, Ioannidou, & Phillips, 1999; Goldstone & Wilensky, 2008) is that agent-based modeling is one way to provide students with opportunities to develop such a model for thinking and learning about quantitative change in systems. Specifically, it can provide enactments and representations of a system at each of these levels by simulating the behaviors of individual agents in a system, and providing tools for measuring quantities in that system as they change over the course of the simulation.

Thinking	**Behaviors**	Different behaviors can affect the same quantity in different ways.
	Multiple Individuals	Multiple individuals interact and affect the same quantity.
	Single Quantity	A population level measure tracks a quantity that is increased or decreased each time unit.
	Trend Over Time	A measure changes over many time units, enabling prediction and analysis of quantitative trends.

Table 2. Four Levels of Mathematical Change and Variation in Systems.

We argue that considering different connections between these four levels can emphasize certain system-specific aspects of MCVS outlined in the Background, such as randomness or nonlinearity, and can also provide a window into how different modeling environments support those aspects. Note that although they are ordered in the list, connections need not be made in this order. Hence, agent-based modeling may emphasize how behaviors as enacted by multiple individuals reflect stochasticity in a population, and this may prompt a user to consider how randomness is characteristic of systemic change. On the other hand, system dynamics modeling can emphasize how multiple behaviors contribute via *rates* to a trend over time, which may background stochasticity but emphasize multiple inputs or feedback.

Design

To support learners' thinking about change at and across multiple levels, we have designed two additional tabs for use within the NetLogo interface. These tabs are designed to align with the levels described above, and leverage design principles from their corresponding literatures. We describe each of these components very briefly here; a poster/paper will allow for a more detailed description.

First, a block-based programming tab called *DeltaTick* is designed to emphasize the connection between agent-level behaviors and resulting quantitative trends as they emerge per "tick", or time unit. To use it, learners open a library that includes a number of pre-specified model building blocks. They can then create one or more populations of *actors*, a collection of homogeneous entities that all behave similarly. Learners can then add one or more pre-specified *behaviors* for each actor type that will execute during each unit of the model's simulated time or "tick". Behaviors can also be placed inside of *conditions*, which must be true for the behavior to occur. Finally, learners can add one or more *graphs*, each with one or more quantities of interest that they wish the graph to feature. This environment is inspired by similar behavior libraries (Kahn, 2007; Repenning, 1997); however, are different in that the behaviors in a given library are designed for specific disciplinary explorations and to relate to specific potential mathematical patterns.

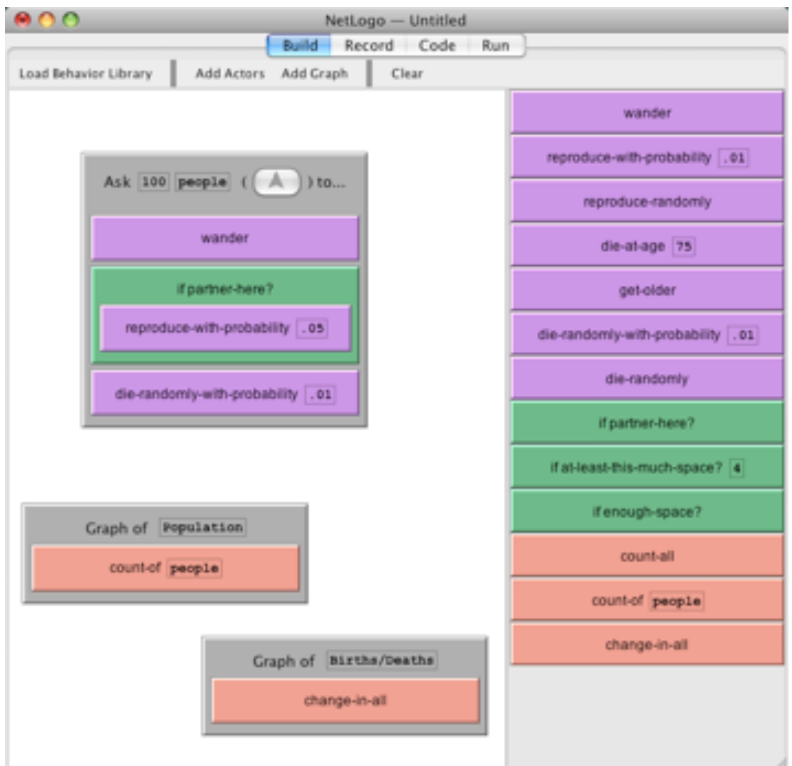


Figure 1. *DeltaTick.* This model begins with 100 "people" agents. During each time unit each person will wander, if they encounter another agent nearby (if partner-here?) they will have a 5% chance of reproducing. They also each have a 1% chance of dying. The model will graph the total population of agents (Population), and the total change in the population (Births/Deaths).

Second, learners have the opportunity to analyze their models using the *HotLink Replay* tool, which includes a visualization of the model and resulting graphs. These representations are dynamically linked, so that learners can click on feature of a graph and see its corresponding time in the simulation, or replay the simulation over time as a cursor indicates the corresponding area on the plot. Learners can also use this environment to directly compare multiple runs of the same or different models. The user can switch back and forth between visualizations of each different run of the model; and the graphs for the specific run that they have selected is dark while the rest are faded (or not shown).

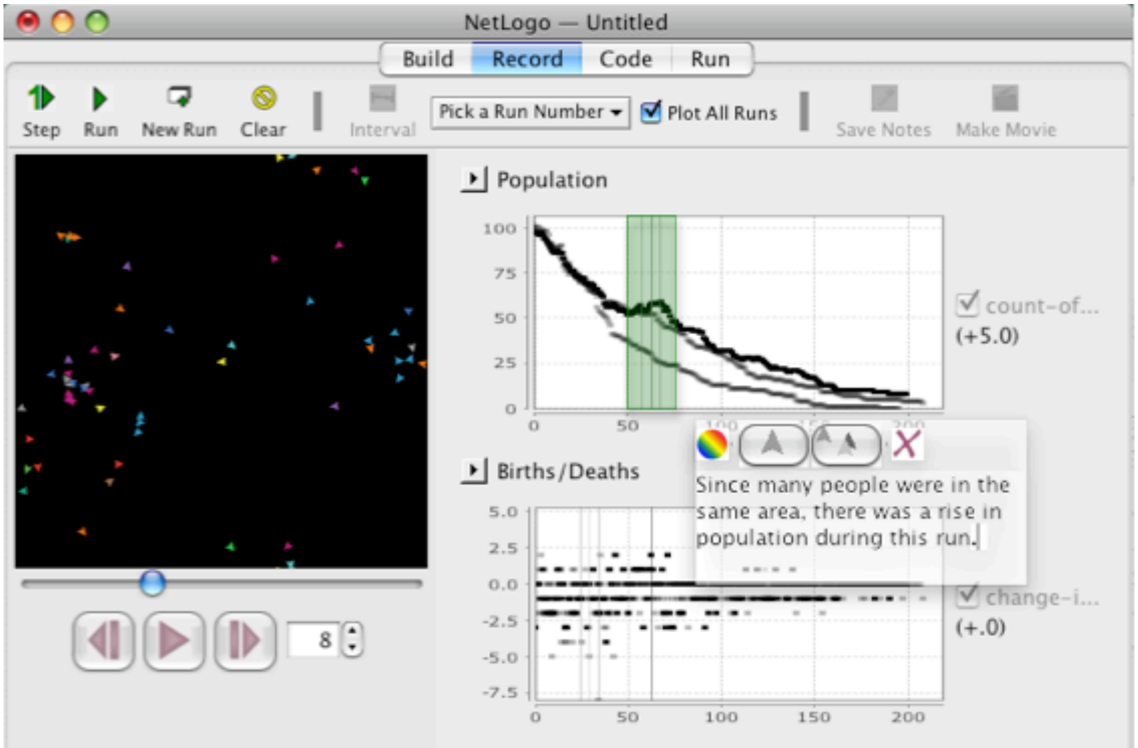


Figure 2. ***HotLink Replay.*** This features multiple runs of the model constructed in Figure 1. Since people only reproduce if they find a partner nearby, population increases only when there are clusters of agents. A point during which the population rose is highlighted in green, and a short annotation is attached. The visualization displays a cluster of agents at the corresponding time.

Methods

To explore how our first iteration of these tools and accompanying activities were used by students, we conducted a 200-minute (two 100-minute sessions) implementation during the final weeks of the 2010 school year with two classes of students (n=45) enrolled in AP Biology at a high performing (99% of students meet or exceed state standards), socioeconomically diverse (34.8% identified as low income) high school. During the first 100 minutes, students were introduced to the software, and completed a series of guided activities including (a) building and comparing an agent-based model of exponential population growth to its equation-based mathematical counterpart, and (b) finding out what combination(s) of agent behaviors would produce specific provided graph shapes. During the second 100 minutes, students completed a model-building session in which they chose some phenomenon of interest to them in the domains of population growth or predator-prey dynamics, researched that phenomenon, and create models to represent it.

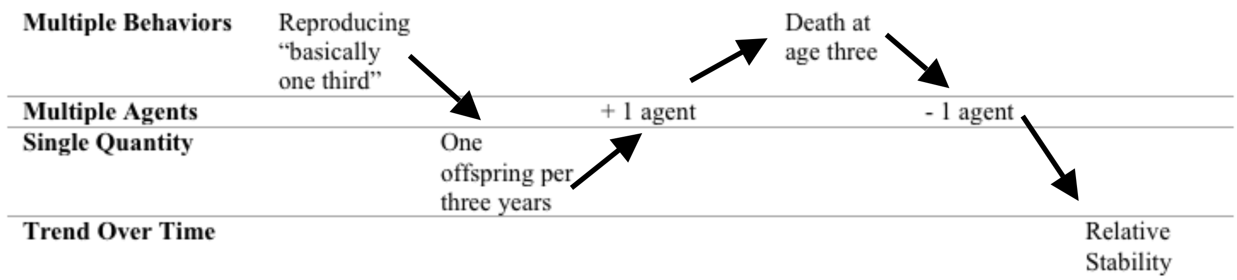
Learners completed these activities on laptop computers in groups of 2 or 3 in their normal classroom. Their actions on the computer screen, as well as their "live" discussions as they interacted with the software, were captured synchronously (via screen capture and webcam; TechSmith, 2010).

Analysis

Analysis is ongoing, and implements the four levels introduced in our Theoretical Framework. We attempt to model dynamic connections between students' own models and their simulated ones (in the spirit of Lesh, 2006). Specifically, we will focus on students' alignment of their own descriptions of those levels of the systems they explore with the representations provided in the models they create. For brevity of this proposal, we illustrate one such analysis below, and describe another (further analysis across multiple groups solving similar problems will be available in the final poster/paper).

In general, we are finding that students productively navigate between these levels when they find points of connection between their own descriptions and the representations provided by the environment, or connections between different representations in the environment that enable them to eventually connect back to their own descriptions.

Case 1. Agent-Generated Stability
In Case 1, a group of three students are attempting to construct a model in which the population of agents remains relatively steady. Their first attempt results in rapid growth, and their second in extinction. During model refinement, they consult their behavior blocks to determine individual behavior in the model, the visualization to determine whether births and deaths are occurring, and graphs to determine general population trends such as stability. In the third model, they reach stability, and one student in the group reasons: "I think basically, like, the fact that they're reproducing at .325, that's basically like one-third. And they're dying every three years, means they're stable, right?"



Case 2. Connecting Description with Representation
In Case 2, a group of 2 students are observing a simple model where each agent in the simulated world is reproducing with 1% change per "tick" of time. They are asked to discuss why a graph of "change in all", the total measure of change in the population from tick to tick, falls at times rather than only rising. These students are aware that the behavior blocks they have used to construct the model define the each agent's behavior, and state so explicitly. However, these blocks do not align with what these students think of when they think of factors that affect a population – factors such as birth control or natural disasters. We argue that this is where a disconnect occurs, and the environment is no longer valuable to the students for solving this problem. In the poster/paper, we will contrast this case with another group who, while working on the same problem, does not experience this disconnect and is able to carry on to make sense of the randomness generated in the plot.

Discussion
We hope that our continued development of the *DeltaTick* and *HotLink Replay* tools introduced in this proposal can contribute to complex systems and mathematics education in two respects. First, we see these tools as a context within which we can explore students' descriptions of mathematical change at these different levels, and develop *theory* about students' thinking and learning about complex quantitative change. Second, we see them as a *practical* contribution to the continuing effort to make complex systems thinking and modeling more accessible and feasible in education.

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