Landscape Dynamics and Conspicuous Consumption [*]

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Abstract

We study evolutionary games with a continuous space of strategies A. The current state is the distribution F of players over A. The payoff function defines an adaptive landscape for each state. Each player continuously adjusts her strategy in A towards higher ground in the landscape. Consequently the distribution F changes continuously and so the landscape changes, and the players adjust again. Assuming gradient adjustment, this interplay between landscape and state is described by a partial differential equation or, equivalently, by a dynamical system on the infinite-dimensional space $\mathcal{F}$ of current states.

The paper illustrates these ideas using Veblen's notion of conspicuous consumption, i.e., one's payoff depends on where one's consumption x falls in the current distribution F. For simplicity we take $A = [0, 1]$, and obtain explicit solutions to the landscape dynamics for special cases. Two Propositions characterize the long run and short run dynamics for more general cases. Using the simulation package NetLogo we illustrate both Propositions.

1   Introduction

Evolutionary game models analyze strategic interaction over time. Equilibrium emerges, or fails to emerge, as players adjust their strategies in response to the payoffs they earn. Thus far the models have mainly considered situations in which players chose among only a few discrete strategies. The current paper presents evolutionary games in which players to choose within a continuous strategy space A.

The *current state* of such an evolutionary game is the distribution of all players' choices over A. In any particular application, the current state defines a payoff function on A, whose graph is called the *adaptive landscape*. Players respond to the landscape in continuous time by adjusting their strategies towards higher payoff. Hence the current state (the distribution of chosen strategies) changes, and this in turn alters the landscape. The interplay between the evolving state and the landscape gives rise to nontrivial dynamics. In particular, when players follow the gradient (i.e., steepest ascent in the adaptive landscape), the evolving state can be characterized as the solution to a nonlinear partial differential equation, or equivalently, a dynamical system on an infinite-dimensional space. In some simple cases the PDE can be solved explicitly, but in general one studies the dynamics by taking a discrete approximation. Dynamics then are given by time steps on a spatial lattice.

Such geometric (continuous or lattice) strategy spaces allow us to model a population of players with an n-dimensional continuum of strategies. Such situations are rather common in the social sciences. We illustrate with the dynamics of conspicuous consumption, first popularized by Thorstein Veblen (1899).

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There are already a few papers that treat evolutionary games with continuous strategy spaces. Friedman and Yellin (1997, 2000) lay the groundwork for our approach. Bomze (1990, 1991) and Oechssler and Riedl (2001, 2002) extend replicator dynamics to continuous strategy spaces, and Cressman and Hofbauer (2003) further develop the approach and connect it to recent work in theoretical biology. However, as explained in the next section, their approach has a different range of applicability than ours and cannot be interpreted in terms of adaptive landscapes.

The next section reviews basic evolutionary games and presents the elements of landscape dynamics. Section 3 applies the machinery to conspicuous consumption. It formalizes Veblen's insight into a particular utility function representing players' payoff. Assuming that players have identical income and preferences but differing initial consumption patterns, we show that gradient dynamics yield a partial differential equation generalizing Burgers' equation from fluid mechanics. Proposition 1 shows that the equation has a unique solution beginning at an arbitrary initial state, and that the solution converges asymptotically to the degenerate (or Dirac delta) distribution at a particular point. That is, in the long run everyone ends up with the same consumption pattern. Proposition 2 shows that the short run dynamics are interesting: the path typically involves a moving, interior compressive shock wave. The iterpretation is that a homogeneous middle class emerges and grows until ultimately it swallows up the entire population.

The next section discusses generalizations of the Veblen consumption dynamics and other economic applications. An online Appendix (Friedman and Abraham, 2004) collects all mathematical proofs and derivations.

2   Elements of Landscape Models

Our exposition begins in familiar territory, in order to establish notation and perspective. Recall that a basic evolutionary game concerns a single population of players, each with the same finite set of pure strategies. The instantaneous state of the system consists of the proportion of the population playing each of the pure strategies.

The state evolves in continuous time, following some dynamical process that respects three principles. The process is (a) monotone: higher payoff strategies become more prevalent over time; (b) continuous (or inertial): only small changes in the state are possible in a very short time period; and (c) players do not try to influence the future play of others, as in a game against nature (GAN).

In typical biological applications, the players are born, reproduce, and die. Each child has the same strategy as its parent. Meanwhile, players meet at random and play a fixed two-player game whose payoffs represent the players' fitness (number of offspring). An alternative narrative, more attuned to social science applications, is that the players learn and tend to switch to higher payoff strategies as they participate in some sort of ongoing interaction.

The mathematical description is as follows. Let k be an integer greater than one, and let $\Delta \subset R^k$ be the unit simplex, whose vertices $e^i = (0, ..., 0, 1, 0, ..., 0), i = 1, \ldots, k$ represent the pure strategies. The simplex is the state space of the dynamical system. That is, a state $s = (s_1, \ldots, s_k)$ is a tuple of population shares, where each $s_i \geq 0$, and $\sum s_i = 1$. A player choosing strategy i receives payoff $f(e^i, s)$ when the state is $s \in \Delta$. The dynamics are then given by a vectorfield on Δ that depends on the payoff function f. The standard biological example is *replicator dynamics*, given by $\dot{s}_i = (f(e^i, s) - f(s, s))s_i$. That is, the growth rate for each population share $\dot{s}_i/s_i$ is its fitness $f^i(s)$ relative to the population average $f(s, s) = \sum s_j f(e^j, s)$.

More general dynamics are helpful in social science applications. For example, sign-preserving dynamics simply require that the share growth rates have the same sign as the relative fitnesses $f(e^i, s) - f(s, s)$, and are consistent with the principles of monotonicity and inertia. In most of the literature it is assumed that the fitness function $f(x, s)$ is linear in both arguments. This is consistent with the biological interpretation that players meet at random to play a fixed two-player game, but is too restrictive for some biological applications and for many interesting social science applications. Here we use "playing the field" models with payoff functions f that are non-linear (but smooth) in the state variable s. It is conceptually straightforward to extend to multiple populations. For example, in biology one might have separate strategy sets for males

and females, and in economics one might have separate strategy sets for buyers and sellers. The state s now specifies the strategy shares in each population. Fitness is computed separately for each population but in general depends on the entire state s.

2.1 Adaptive landscapes

The narrative description of a basic adaptive landscape again concerns a single population of players, each with the same space of strategies, but the space now is at least one-dimensional and continuous. The instantaneous state of the system is the distribution of chosen strategies with the player population. A player's fitness (payoff) depends on her choice of strategy, as well as on the current state of the entire system. From the player's perspective, fitness looks like a landscape in which she seeks to go uphill.[1] As play evolves over time, all players continuously adjust their strategies to increase their fitness, so the population distribution changes, and consequently the landscape morphs. A player's fitness changes as the direct result of her own actions, and also indirectly as the state evolves. The dynamics arise from this interplay between state and landscape.

The mathematical description of the basic landscape model is as follows. Let the unit interval $A = [0,1]$ be the space of strategies, and let $\mathcal{F}$ be the space of all cumulative distribution functions on A, with the weak-star topology. That is, $F \in \mathcal{F}$ if F is a right-continuous nondecreasing function on A with range $[0,1]$, and countable discontinuities. Recall that $F(x)$ represents the fraction of players whose choices are no larger than x. A sequence of such functions $\{F^n\}$ converges in the weak-star topology if, for every continuous function g on A, the Stieltjes integrals $\int_A g(y)dF^n(y)$ converge in R. The extreme points of the state space $\mathcal{F}$ are the degenerate (or Dirac delta) distributions, which put all mass on a single point. Using the Heaviside notation $\Theta(y) = 0$ if $y < 0$ and $\Theta(y) = 1$ if $y \geq 0$, each extreme point can be written as $\Theta(x - x_o)$ for some $x_o \in A$. The space $\mathcal{F}$ is clearly the closure of its extreme points, and thus is an infinite-dimensional simplex. When F is differentiable on A, it is sometimes convenient to use the density $f = F'$ to represent the state. More details may be found in the online Appendix to this paper, (Friedman, 2004).

The *fitness* for any player choosing strategy $x \in A$ when the current state is $F \in \mathcal{F}$, is denoted by $\phi(x, F)$. The application dictates a particular fitness function $\phi : A \times \mathcal{F} \to R$. For any player holding strategy x, the function $A \to R;\ x \mapsto \phi(x, F)$ with F held constant, is the instantaneous *fitness landscape* for that player. It depends on F only, and we think of it as a landscape, or graph in $A \times R$.

The dependence of ϕ on F can take many forms. In some applications it is the expectation $\phi(x, F) = \int g(x, y)dF(y)$ of some two-player game payoff function $g(x, y)$. In some (possibly oversimplified) biological applications the dependence is only via the mean strategy $\mu_F = \int y dF(y)$. In fluid dynamics applications (and in the Veblen consumption application presented below) it depends only on the local value $F(x)$ at the chosen strategy x (e.g., Witham, 1974). In other economic applications the dependence on the current state F is quite nonlinear and arises from market-clearing prices.

Landscape dynamics are given as a vectorfield (defined almost everywhere) on the infinite-dimensional simplex. A general expression is

$$F_t(x, t) = \Psi(x, F, \phi) \tag{1}$$

where $F(x, t)$ is the cumulative distribution function over $x \in A$ for the state at time t, and F_t denotes the partial derivative of $F(x, t)$ with respect to t.

Consistent with the inertial principle of evolutionary games and with Darwin's dictum *Natura non facit saltum*, we assume that individual adjustment is continuous. That is, a discrete change in a player's strategy x takes a positive amount of time. This restriction might seem innocuous but it is violated by generalized replicator dynamics and other dynamics that don't respect the ordering of the action set $A = [0,1]$. The intuition for replicator dynamics is that individuals never adjust, and dynamics arise entirely from differing

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[1]The landscape metaphor goes back to Sewall Wright (e.g., 1949) and has been revived by Stuart Kauffman (e.g., 1993). Wright considered low dimensional continuous landscapes and Kauffman considers high dimensional sequence spaces of discrete-valued traits. Neither considers the dynamically changing (i.e., distribution dependent) landscapes examined here.

birth or death rates at different strategies. Apparent jumps occur because new births don't generally appear at the same location as recent deaths.

Consistent with the monotone principle of evolutionary games, we assume that the direction of adjustment is given by the sign of the gradient, i.e., uphill in the fitness landscape. If also the adjustment speed is proportional to the gradient $\phi_x = \partial\phi/\partial x$, we have a *gradient adjustment system*. In this case dynamics obey the *master equation*,

$$F_t(x,t) = -\phi_x(x,F)F_x(x,t) \tag{2}$$

where F_x and F_t respectively denote the partial derivatives of $F(x,t)$ with respect to x and t. This nonlinear partial differential equation simply states that probability mass is conserved: the rate of change $F_t(x,t)$ in population mass to the left of any point x is equal to the (negative of the rightward) flux past that point. The flux is the product of the density $f = F_x$ and the velocity given by the gradient ϕ_x.

3   Consumption Dynamics

Veblen consumption illustrates several of possibilities inherent in landscape dynamics (Friedman, 2001; Friedman and Yellin, 2000) and is interesting in its own right. Thorstein Veblen (1899) popularized the idea that some goods and services (think of Hummers or seldom-used second homes) are consumed largely to gain status, a theme pursued more recently by authors such as Duesenberry (1949), Frank (1985) and Ljundqvist and Uhlig (2000). Such consumption has the desired effect only to the extent that it exceeds the conspicuous consumption of other people, i.e., its utility is rank-dependent.

Consider a single population of consumers with identical incomes. Each consumer chooses a fraction $x \in [0,1]$ of income to allocate to ordinary consumption, and allocates the remaining fraction $1 - x$ to rank dependent consumption. The state is the cumulative distribution function $F(x)$ of ordinary consumption. Assume standard direct utility $c\ln x$ from ordinary consumption x, where the parameter $c \geq 0$ represents the relative importance of ordinary consumption. Suppose that rank dependent utility arises from envy, i.e., I compare my rank-dependent consumption $1 - x$ to everyone else's and am unhappy to the extent that it falls short. The shortfall is $\min\{0, y - x\}$ when your rank dependent consumption is $1 - y$. After integrating the expected shortfall by parts, one verifies that overall expected utility is

$$\phi(x,F) = c\ln x - \int_0^x F(y)dy, \tag{3}$$

with gradient

$$\phi_x = c/x - F(x). \tag{4}$$

Figure 1 shows two landscapes with $c = 0.1$ defined by this payoff function for two different initial distributions. In Figure 1A, the distribution F is uniform on $A = [0.0, 1.0]$. In Figure 1B, the distribution F is uniform on $A = [0.9, 1.0]$.

Dynamics are governed by the Master Equation (2). Insert the gradient (4) into (2) to obtain the partial differential equation

$$F_t = F_x[F - (c/x)]. \tag{5}$$

Our first result describes the asymptotic behavior of landscape dynamics for Veblen consumption. It turns out that (5) implies clumping, and all consumers end up with the same combination of ordinary and conspicuous consumption.

Proposition 1. Let the distribution $F(x,t)$ be a solution to (5) for a given initial condition $F_o(x) = F(x,0)$, and let $\widetilde{x} = \sup\{x \in [0,1] : xF_o(x) < c\}$. Then as $t \to \infty$, $F(x,t)$ converges pointwise to $\Theta(x - \widetilde{x})$.

Proof. See the Appendix, available on our website.

4

That is, (5) has a unique solution $F(x, t)$ starting from an arbitrary initial distribution, and as $t \to \infty$ the solution converges to the degenerate distribution at some point $\widetilde{x} \in [0, 1]$ whose value depends on the initial distribution and the parameter c.

For our purposes the transient dynamics are especially interesting. To illustrate, suppose $c = 0$ and the initial distribution is $F(x, 0) = 3x^2 - 2x^3$, i.e., the initial density is the symmetric unimodal (single-peaked) function $f(x, 0) = 6x(1 - x)$. The Appendix shows that $F(x, t)$ has a continuous unimodal density $f(x, t)$ for $t \in [0, 2/3)$. The mode (or peak) $x^*(t)$ decreases steadily from $1/2$ at $t = 0$ to $1/6$ at $t = 2/3$, and the height of the mode becomes unbounded as $t \to 2/3$. The intuition is that all consumers decrease ordinary consumption (recall that for $c = 0$ they care only about conspicuous consumption) but gradient dynamics dictate that the modal consumer adjusts more rapidly than consumers with initially lower x and he begins to overtake them at time $t^* = 2/3$ and consumption level $x = 1/6$. Given our assumption of identical underlying preferences and income, this consumer can't actually pass his rivals because his behavior is identical to theirs once he attains the same consumption level. Instead, he clumps together with them, and the clump grows as it overtakes consumers with x just below the mode and is overtaken by consumers with x just above the mode.

Thus, beginning at $t = 2/3$ we get a growing, moving mass of consumers with identical consumption patterns, a homogeneous middle class. To calculate its position $x^*(t)$ and mass $M(t)$ for $t > 2/3$, one uses techniques developed in fluid mechanics to deal with shock waves. The Rankine-Hugoniot conditions (see Smoller, 1994) impose conservation of mass and exploit the weak-star topology to obtain a unique distribution function $F(x, t)$ with a jump discontinuity. It turns out that for $t \in (2/3, 1]$ the position is $x^*(t) = (1 - t)/2$ and the jump size is $M(t) = \sqrt{\frac{3}{t^2} - \frac{2}{t^3}}$. Thus the middle class absorbs the entire population by the time it hits the boundary $x = 0$ at time $t = 1$. For $t > 1$, of course, everyone continues to neglect ordinary consumption.

Figure 2 shows the cumulative distribution function F for choices x, shown for time $t = 0$ and subsequent times. At $t^* = 2/3$ the distribution has a vertical tangent at $x^* = 1/6$. At later times the distribution has a jump discontinuity.

So much for the numerical example. What can one say for more general initial distributions and positive c? The next result guarantees very similar behavior when consumers put relatively little weight c on ordinary consumption.

Proposition 2. Let the initial distribution $F_o(x)$ be thrice continuously differentiable, with a regular strict maximum at $x = q \in (0, 1)$. Then for all sufficiently small $c > 0$, the solution of (5) has a moving interior shock. Up to first order in c, the shock emerges at time

$$t^*(c) = 1/f(q) + cT(q) + O(c^2)$$

and location

$$x^*(c) = q - F(q)/f(q) + cX(q) + O(c^2) \in (0, 1).$$

Proof. See the Appendix, which includes formulas for the functions $T(q)$, $X(q)$.

Numerical explorations reported below suggest that even for moderately large weights c shock waves will arise from any local maximum of the initial density far enough above the point $\widetilde{x}$ where the gradient is 0.

4 Numerical Simulations

We now translate the Conspicuous Consumption Model into the world of computational mathematics. We will treat, in order, the density f, the distribution F, the payoff ϕ, and the gradient, ϕ_x. Then, we describe Veblen, our NetLogo model that implements the integration of an arbitrary initial density, to simulate the Conspicuous Consumption Model. It will be convenient to collect here the basic equations from the preceding, adding numbering to which we may refer within our NetLogo code.
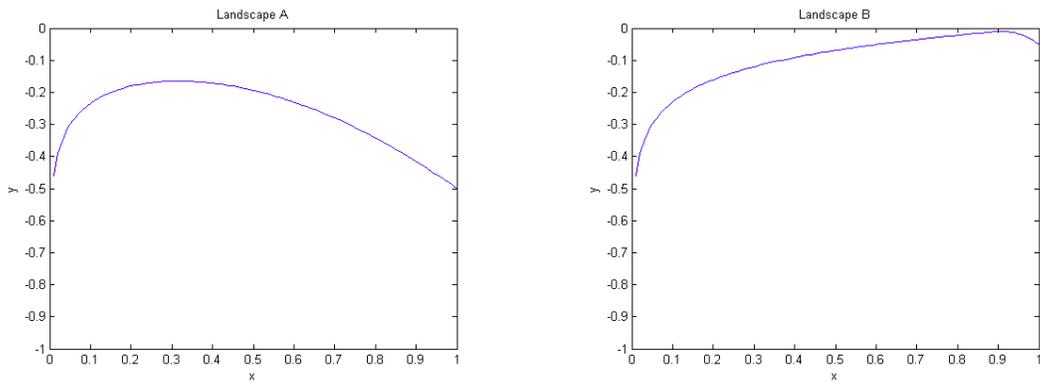
Figure 1: Two landscapes on the unit interval with $c = 0.1$, defined by our payoff function for two different initial distributions. On the left, the distribution F is uniform on $A = [0.0, 1.0]$. On the right, the distribution F is uniform on $A = [0.9, 1.0]$.
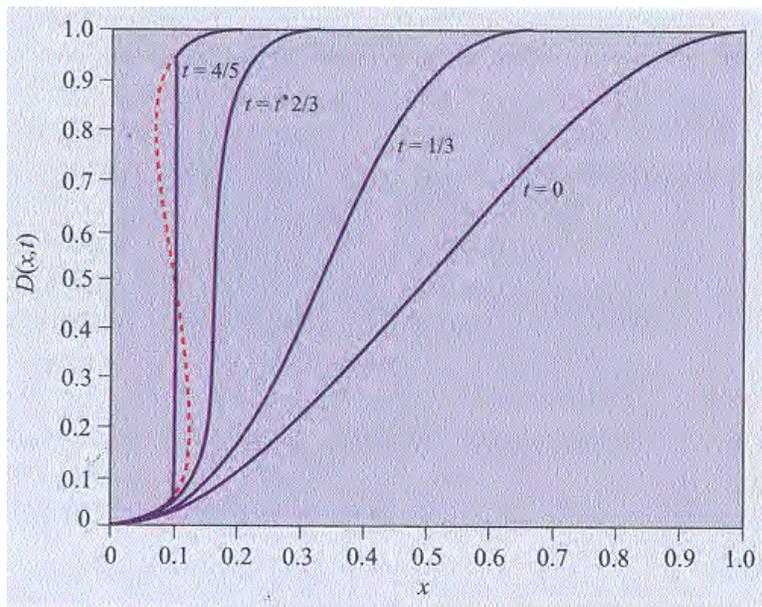

Figure 2: The cumulative distribution function F for choices x, shown for time $t = 0$ and subsequent times, showing a compressive shock wave. At $t^* = 2/3$ the distribution has a vertical tangent at $x^* = 1/6$. At later times the distribution has a jump discontinuity

6

The density, f, represents a probability measure,

$$\int_0^1 f(y)dy = 1. \tag{6}$$

This function is nonnegative. The cumulative distribution is its integral,

$$F(x) = \int_0^x f(y)dy, \tag{7}$$

This function is nondecreasing, with clearly, $F(0) = 0$ and $F(1) = 1$. The payoff, as a function of x and F, is,

$$\phi(x, F) = c \ln x - \int_0^x F(y)dy. \tag{8}$$

Here c is a nonnegative constant. This function is nonpositive on the interval $(0, 1]$. Note the second integral. The gradient of ϕ is,

$$\phi_x = c/x - F(x). \tag{9}$$

4.1 Discretization

Let us suppose that the number of consumers is M, and that the strategy space, $A = [0, 1]$, is divided into N equal intervals. The width of these intervals is thus $\Delta x = 1/N$. Let m_i denote the number of consumers in the $i - th$ interval. Note

$$\sum_{i=0}^{N-1} m_i = M.$$

Formally, these intervals are closed on the left end, and open on the right, except for the first one, which is open on both ends, and the last one, which is closed on both ends. Informally, we may ignore this subtlety, and just imagine that there be no consumer on an endpoint.

Let f_i denote the average density of consumers in the $i - th$ interval, $[i\Delta x, (i+1)\Delta x)$, where $i = 0, ..., N - 1$. Then we have,

$$f_i = m_i/(M\Delta x) = (m_i/M)N \tag{10}$$

As a check, we numerically integrate f over $[0, 1]$, obtaining,

$$\sum_{i=0}^{N-1} f_i \Delta x = \sum_{i=0}^{N-1} m_i/M = M/M = 1.$$

to confirm that f represents a probability measure, eqn. (6).

The cumulative distribution, F, is the integral of f according to eqn. (7), or, letting x_n denote the left endpoint of the $n - th$ interval,

$$F_n \equiv F(x_n) = \sum_{i=0}^{n-1} f_i \Delta x = \Delta x \sum_{i=0}^{n-1} f_i = (1/M) \sum_{i=0}^{n-1} m_i \tag{11}$$

and clearly, $F(x_0) = 0$ and $F(x_{N-1}) = 1$.

Fixing the distribution, F, give ϕ the value ϕ_i on the $i - th$ interval of the strategy space. Again, let x_i denote the left endpoint of the $i - th$ interval. Then we have, from eqn. (8), the average payoff on the $n - th$ patch is,

$$\phi_n = c \ln(x_n) - \sum_{i=0}^{n-1} F_i \Delta x = c \ln(x_n) - \Delta x \sum_{i=0}^{n-1} F_i = c \ln(x_n) - (1/N) \sum_{i=0}^{n-1} F_i \tag{12}$$

7

We display the payoff function in the simulation, but use only its gradient. Note that the payoff function defined thusly is piecewise constant. It is constant on each patch, so increasing the number, N, of patches on a row results in a better approximation of a smooth function. In the illustrations below, in which the graph of ϕ looks smooth, $N = 35$.

Using the notations above, throughout the $n - th$ interval of the strategy space, the average gradient is,

$$(\phi_x)_n = c/x_n - F_n, \tag{13}$$

from eqn. (9).

4.2 Our NetLogo model, Veblen

NetLogo displays an interface including control widgets, command center, and a two-dimensional graphics window called the screen. The screen comprises a rectangular grid of patches, which are squares of pixels. One may deploy agents, called turtles, in the screen, with floating point coordinates. We have used one row of patches to model the strategy space, and a number of turtles to model consumers.

Our NetLogo implementation closely follows the numerical equations above. We do not use the Master Equation for the evolution of F, but instead use a primitive integration in which each consumer changes her strategy according to the gradient rule, as follows. Each consumer has a numerical ID, and a position, a floating point number. For each discrete time step of size *stepsize*:

- Each consumer (in numerical order) moves $stepsize * (\phi_x)_n$ within the strategy space, where n is the index of the patch containing her current strategy, x.

- After all consumers have adjusted in this way, the density, f, distribution, F, payoff, ϕ, and gradient, ϕ_x, are recomputed.

- The time is incremented by *stepsize*.

- After each tenth step, the two plots are updated.

Given an initial distribution of consumers, upon clicking the "go" button, our program proceeds step-by-step, until stopped with another click on the "go" button.

The interface of Veblen is shown in Fig. 3. On the upper left are a number of controls that allow the operator to approximate an arbitrary initial distribution of consumers, as follows.

1. There are five parallel rows, all of which are overlays of the strategy space. Using the pop-down menu labeled "puff-row", choose a row.

2. Choose a number of consumers to add to the initial distribution on the chosen row, using the slider labeled "population".

3. Choose a subinterval of the strategy space in which to randomly locate them, using the sliders labeled "center" and "width". Both are calibrated in percent of the unit interval.

4. Push the button labeled "setup".

5. Repeat 1-2-3, then push "puff" to add another square wave of consumers.

Mathematically, all consumers are on the same strategy space, the gray row. But the consumers are shown, as colored triangles, in five puff-rows on the screen. The two plots show the density and the landscape for the initial distribution. The color bar at the bottom of the screen shows where the slope is positive(magenta), zero (yellow), and negative (cyan).

On the lower left are controls for the step-by-step integration. Set "amp" (the parameter 'c' in the model) and "stepsize" (proportion of slope for a consumer to move). Then push "step" for one step of integration, or "go" for a sequence of steps. These continue until "go" is pushed again. The current step number and time are shown in the boxes labeled "totalsteps" and "totaltime". A run may be continued by again pushing "go". (The command center is for NetLogo experts.)

4.3 The hump initial state

The hump is a heap, under an inverted parabola, as shown in the screen (upper right) of Fig. 3. This is a piecewise constant approximation of the density $f(x, 0) = 6x(1 - x)$ featured in the previous section. After a long run it settles down to a static attractor, as shown in Fig. 4. The transient behavior of the herd is interesting, as discussed above. A movie of this transient may be found at our website:

http://www.vismath.org/research/landscapedyn/models/veblen/hump01.gif

This movie, lasting only a few seconds, compresses an integration run that took about half an hour on our desktop computer. The number of consumers in this run was about 400. In future, we plan to accelerate these integrations by parallelizing the model with a cluster of computers. This will be particularly important when the number of agents is much larger, or if different initial distributions must be studied.

In case another model of landscape dynamics is to be studied numerically, and Theorem 1 does not apply, then there may in general be several attractors with complicated basins, and it will be necessary to experiment with many initial distributions. Super computational speed may be needed.

More details concerning landscape dynamics, the Veblen model, our NetLogo implementation applet, and the program in full, may be found at our website,

http://www.vismath.org/research/landscapedyn/

4.4 Why NetLogo?

For the computer simulation of a landscape dynamical model, agent based modeling (ABM) is a very natural choice, in fact, a perfect fit. For ABM work there are a dozen or more programming environments, of which NetLogo is but one. We have found this a good choice for our work, and here are some of the reasons.

- Our model requires only five pages (about 240 lines) of code.

- The programming is as simple as BASIC.

- NetLogo is freeware, has excellent support, and a large and active user community.

- The programming environment is highly evolved, with many features useful for teaching as well as for experiments.

- A model may be saved effortlessly as an applet for WWW browsers.

- The programming environment is cross platform.

- It is easy to read and execute a text file of commands that setup, run, and record the results of an experiment with a model.

- NetLogo has a companion system, HubNet, that allows human subjects to interact with a model.
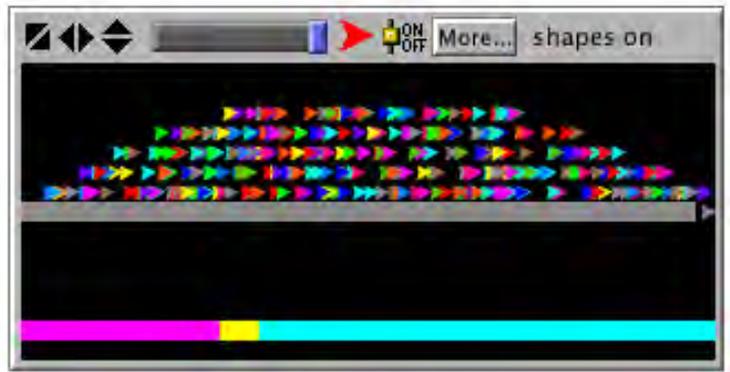
Figure 3: Interface of Veblen 5.2, showing the hump distribution.
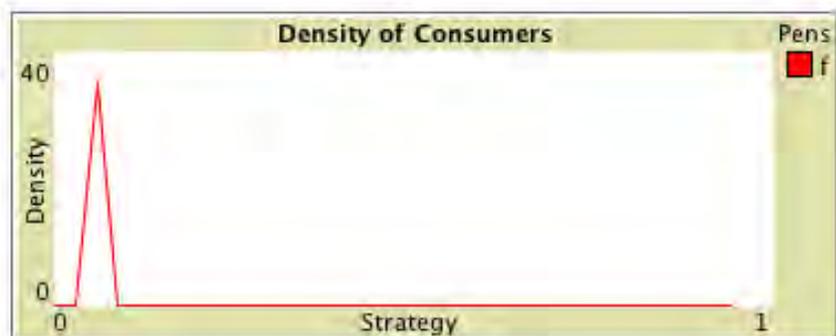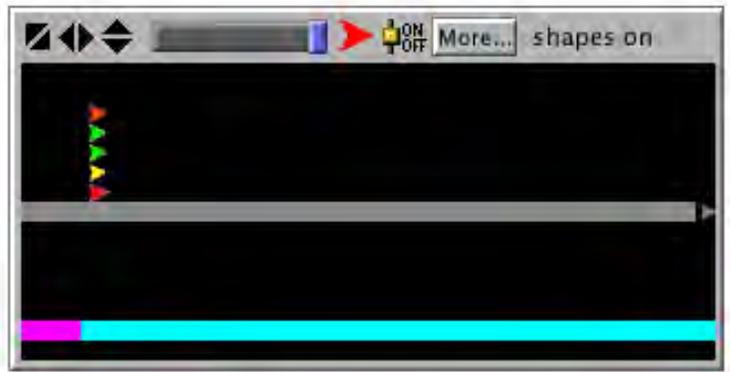
Figure 4: End of the hump run, after 14,000 steps.

5 Discussion

The consumption model can be generalized in many respects. First of all, it is reasonable to include a little behavioral or perceptual noise in the form of a diffusion term in (2), as is assumed in quantal response equilibrium models; see Anderson, Goeree and Holt (1998) for example. Low amplitude noise will have no perceptible effect on our simulations, but does alter the Propositions slightly. The long run equilibrium described in Proposition 1 is slightly smoothed, approximately Gaussian with small variance. Likewise the shock waves described in Proposition 2 are also smoothed. Instead of travelling jump discontinuities in the distribution function, we get travelling steep segments of a continuous distribution, locally approximately Gaussian.

The current version of the model suggests that conspicuous consumption is ultimately fruitless: pride is frustrated in the long run because everyone ends up with the same consumption pattern $\widetilde{x}$. A more realistic model would include individual differences in the personal tradeoff parameter c for conspicuous and ordinary consumption, as well as income differences. The state then would have to include the income distribution as well as the choice distribution, and one would have to keep track of several player populations, one for each value of c. Such complications seem analytically challenging, but are not difficult to simulate in NetLogo.

Our priority in future work is not to extend the conspicuous consumption model but rather to encourage a host of applications for landscape dynamics. We think that many social science applications (and some biological applications) are appropriately modeled by assuming that adjustment takes place at the individual level and is continuous. It is worth repeating that, although the assumption might seem innocuous, it is violated by generalized replicator dynamics and other dynamics that don't respect the ordering of the action set A.

Here is a sketch of an application to financial markets. Consider a single population of portfolio managers whose utility depends on performance. As a practical matter, relative performance, or rank, matters because higher rank brings bonuses and competing job offers, and also indirectly increases managers' compensation by attracting more customers. Each manager in the model would continuously adjust x, the portfolio risk or leverage. The manager's payoff function would depend on absolute and as well as relative payoff. Using orthodox specifications of asset pricing, one would obtain (possibly noisy) specifications of how the payoff depends on the manager's own choice and the distribution of choices across financial market participants. The payoff apparently would be non-linear in the distribution F and rather different than the the simple local dependence we obtained in the conspicuous consumption model. On the other hand, the case for gradient dynamics is especially strong for financial markets and, given Proposition 2 above, it is reasonable to expect compressive shock waves. A natural interpretation would be a financial crash or panic, as managers scramble to unload risky assets as their price tumbles and margin calls must be met.

Other natural economic applications of landscape dynamics include Hotelling-style models of spatial competition in industry; Sonnenschein (1982) is a good beginning. Political as well as economic questions might fruitfully be attacked using the same tools. In particular, political models in the tradition of Downs (1957) are natural for landscape dynamics. One could begin with one dimensional position spaces A and then move to higher dimensions. Likewise, one could begin with a single polity and later consider two separate polities that interact with each other, e.g., members of different ethnic groups such as Protestants and Catholics in Belfast. The goal would be to characterize the conditions that lead to polarization and those that lead to convergence.

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