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We hope that members will use the proceedings to give feedback to the authors and that through discussion and debate we will develop an energetic and critical research community. We particularly welcome presentations and papers from new researchers.

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Themed Strand:

USING MULTIPLE REPRESENTATIONS TO ASSESS STUDENTS' UNDERSTANDING OF THE DERIVATIVE CONCEPT

Victor Amoah, Alperton Community School, Wembley

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Calculus is highly symbolic in nature and therefore students often try to get through calculus by manipulating the symbols without understanding the meaning of such symbols (i.e. having a procedural but not a conceptual understanding of the topics in calculus). Educators are looking for ways to help students achieve higher levels of conceptual understanding. This study explored Science Foundation Year students' graphical, numerical and algebraic understanding of the derivative concepts after differential calculus course. The course was designed to develop students' conceptual understanding of the derivative concept.

INTRODUCTION

Tall (1996) points out that for students who take an initial calculus course based on elementary procedures, there is evidence that this may have an unforeseen limiting effect on their attitudes when they take a more rigorous course at a later stage. For example, Ferrini-Mundi and Gaudard (1992) found that it is possible that procedural, technique-oriented secondary school courses in calculus may predispose students to attend more to the procedural aspect of the college course. Students can be seen to be developing short-term techniques for survival when experiencing conceptual difficulties in the calculus at university. One of the possible approaches to teaching calculus Tall (1996) suggested involves numerical, symbolic and graphical representation.

A number of influential professional groups have put forth compelling proposals for the reform of mathematics education and calculus in particular. These proposals express a new vision of mathematical achievement, in which conceptual understanding plays a central role. According to Ohlsson (1987), one effect of developing conceptual understanding is that procedures and principles become easy to learn and understand. Conceptual knowledge is equated with connected networks of knowledge. In other words, conceptual knowledge is rich in relationships (Hiebert and Lefevre, 1986). On the other hand, procedural knowledge is defined as a sequence of actions. It is important to emphasize that both kinds of knowledge are required for mathematical expertise. Procedures allow mathematical task to be completed efficiently (Hiebert and Carpenter, 1992).

Assessment of multiple representation of derivative

The idea of connections between representations provides a way of thinking about assessing understanding and provides a general criterion for constructing useful tasks (Hiebert and Carpenter, 1992). Hiebert and Carpenter (1992) point out that errors may imply a lack of understanding, but an absence of errors on the type of items used on most diagnostic instruments does not imply understanding is present. They argue that the assumption that understanding implies well-connected knowledge suggests that we should assess understanding by attempting to determine how knowledge is connected.

Use of multiple representations, particularly when interconnections are formed, is expected to increase students' understanding. Some educators call for more emphasis on multiple representation of calculus concepts (Heid, 1988; Tall, 1996; Ostebee and Zorn, 1997) and this had made traditional skills test items no longer most appropriate for testing understanding. Therefore, a suitable assessment tool that reflects the teaching objectives should be developed. The primary purpose of this study was to assess the students' understanding of the derivative concept. This paper does not provide many details of the course itself but focuses on the aspect of assessing the understanding of the concept of the derivative.

METHOD

Key Questions

1. Does the student understand that, for a point a in the domain of a function f, the value of $f'(a)$ is the slope of the line tangent to the graph f at the point $(a, f(a))$?
2. In the absence of a defining equation for the function, is the student able to think about and work with the derivative using only graphical information?
3. Is the student able to think about and work with the derivative using numerical information?
4. Is the student able to apply the ideas of the derivative to solve a problem?
5. Is the student able to recognize the graph of a function if the graph of its derivative is drawn.

Participants

The participants for this study were 150 Science Foundation Programme (also known as **U**niversity of the **N**orth **F**oundation **Y**ear **P**rogramme – **UNIFY**) students of the University of the North in the Limpopo Province of South Africa. Of the 150 students, 24.5% were females and 75.5% males. The mean age of the participants in the study was 19.7 yrs (S.D. = 1.6). The students were selected from a pool of 802 students who wrote the UNIFY selection test at the beginning of the 2001 academic year. UNIFY is almost exclusively serving students from disadvantaged backgrounds. These are students who received their secondary education under adverse school conditions that could not provide them sufficient opportunities to realise their potential and thus gain immediate entry into mainstream courses in mathematics.

Procedure

This study took place in the year 2001. The data was collected after the UNIFY students had completed a course in differential calculus in the second semester of the 2001 academic year. A paper and pencil test developed by the researcher was administered to 150 students (from five different groups) who made themselves available for the test. The test was administered to students in their classroom. The teaching approach for all the five groups was similar in that the groups were taught with the emphasis on concepts. All five groups used the same worksheets. The worksheets were developed by UNIFY and contain mostly numerical, graphical and elementary applications of the derivative. One group in addition, used the graphing capabilities of computers in order to provide for the visualisation of the derivative concept. Teaching at UNIFY is aimed at mathematics sense-making. The teaching involved (i) continuous identification of student ideas which in this case was through the written work of students and through mathematical discussions that were held in class, (ii) continuous exchange of mathematical ideas between peers and/or with the facilitator/lecturer.

Test Items

The test was designed to obtain information on the students' conceptual understanding of differential calculus. The test items were initially reviewed by six experts in the fields of mathematics and mathematics education at the University of Transkei and the University of the Witwatersrand (in South Africa) for content validity. Appropriate modifications were then made and piloted in 2000 by administering the test to 160 science foundation year and first year students of the University of Transkei at the end of their calculus course. From the data generated by the pilot, further improvements wee made to the instrument.

The test items used in the main study and what they gauged are now discussed.

- The ability of students to calculate average rate of change graphically. (**Item 1a**)
- The ability of the students to find the derivative at a point from a graph. (**Item 1b**)
- The ability of the students to explain what is meant by a derivative at a point. (**Item 1c**)

Item 1

The sketch of the graph of a function was given.

a) what is the average rate of change from point A to point B?
b) Find the derivative of $f(x)$ *at* $x = 3$*?*
c) What do you understand by the derivative of f *at point A?*

The average rate of change over an interval can be obtained from the *graph* by noting the amount of vertical increase (rise) or decrease (drop) in the function values as read from the graph over an interval of the independent variable and dividing this difference by the width of the interval. If the function is represented by a *graph*, students were expected to approximate the instantaneous rate of change of the dependent variable for a particular value of the independent variable by estimating the slope of the tangent to the graph at that particular point.

- The ability of the students to calculate the slope of a curve at a point algebraically. (**Item 2**)

Item 2

Find the slope of the curve at x = 1 for the function $f(x) = 2x^2 - x$.

The slope of the curve $f(x) = 2x^2 - x$ at $x = 1$ can be obtained from the equation by symbolically computing the first derivative and evaluating it for the appropriate value of the independent variable.

- The ability of the students of the students to estimate the derivative at a point numerically. (**Item 3**)

Item 3

The table below shows values of $f(x) = x^3$ *near* $x = 2$. *Use the table of values to estimate the derivative of f(x) at x = 2.*

x	*1.998*	*1.999*	*2.000*	*2.001*	*2.002*
$F(x) = x^3$	*7.976*	*7.988*	*8.000*	*8.012*	*8.024*

The derivative of $f(x) = x^3$ at $x = 2$ can be estimated from a *table of values* by computing the average rate of change in the dependent variable over intervals immediately before and after the value of the independent variable.

- The ability of the students to apply the derivative concept to solve a problem. (**Item 4**)

Item 4

When an antibiotic is introduced into a culture of bacteria, the number of bacteria present after t hours is given by $N(t) = 2000 + 10t - 5t^2$, *where N(t) is the number (in thousands) of bacteria present at the end of t hours.*

Find the rate of change in the number of bacteria present at the end of 2 hours.

Rate of change in the number of bacteria present can be obtained from the given equation by symbolically computing the first derivative and evaluating it for the appropriate value of the independent variable (i.e., at $t = 2$).

- The ability to recognize the graph of a function if its derivative is drawn. (**Item 5**)

Item 5

Students were given a diagram in which a particular curve was indicated as being the derivative of a function. Students then had to identify the graph of the function from a collection of curves in the diagram. . They were also required to explain how they arrived at their choice of curve for the function.

RESULTS AND DISCUSSIONS

Table 1: Results of Item 1a and 1b

Item	Correct answer		Erroneous element		Incorrect answer	
	n	%	n	%	n	%
1a	75	50.0	42	28	33	22
1b (*graphical*)	39	26.0	43	28.7	68	45.3

Item 1a was answered correctly by 50 % of the 150 students who wrote the test. 28 % of the students showed evidence of knowledge of the solution to the problem but minor errors occurred and 22 % of the students did not demonstrate knowledge of relevant procedure to answer this question.

In item 1b, graphical competency is demonstrated if the gradient of the tangent to the curve at the required value of x is determined. Most of the students could not find the derivative at a point from the graph. Only 39 (26 %) students out of the 150 students were able to demonstrate their ability to calculate the derivative at a point graphically. Some of the incorrect answers came about because some of the students confused the derivative at the point with y-value of the point of tangency. Other students endeavoured to find an equation for the function represented graphically. Some students made errors because they had difficulty in computing the gradient of the tangent to the curve although the principle essential to the solution was understood.

Table 2: Results of Item 1c (*explain*)

Correct Explanation		**Erroneous element**		**Incorrect Explanation**	
N	**%**	**n**	**%**	**n**	**%**
90	60.0	23	15.3	37	24.7

Results of item 1c indicate that 60 % of the students were able to demonstrate their understanding of derivative at a point.

Table 3: Results of Item 2 (*algebraically*)

Correct answer		**Erroneous element**		**Incorrect answer**	
N	%	n	%	n	%
81	54.0	14	9.3	55	36.7

About 54 % of the students demonstrated the ability to calculate the slope of a curve at a point algebraically. Some students (9 %) knew how to differentiate but some errors occurred due to incorrect differentiation or manipulation. About 37 % of the 150 students did not demonstrate any conceptual knowledge of the problem, some instead substituted $x = 1$ directly into the equation for the function.

Table 4: Results of Item 3 (*numerical*)

Correct answer		**Erroneous element**		**Incorrect answer**	
N	**%**	**n**	**%**	**n**	**%**
56	37.3	4	2.7	90	60.0

Item 3 was answered poorly. Only 8 (37%) of the students managed to cope with this problem. Most of the students (60 %) were unable to estimate the derivative at the point numerically. Incorrect answers occurred because most of the students responded to this question by differentiating symbolically although symbolic differentiation was not indicated.

Table 5: Results of Item 4 (*application*)

Item	Correct answer		Erroneous element		Incorrect answer	
	n	%	n	%	n	%
4	44	29.3	18	12.0	88	58.7

Item 4 was answered correctly by about 29 % of the students. About 59 % of the students could not apply the derivative concept to solve this problem. Most of the incorrect answers came about because the students attempted to substituted $t = 2$ directly into the equation for the function.

Table 6: Results of Item 5 (*derivative Function*)

Correct answer		Erroneous element		Incorrect answer	
n	%	n	%	n	%
13	8.7	51	34.0	86	57.3

Students were asked to identify the graph of a function given the graph of its derivative. They were then asked to explain how they arrived at their answer. This item was answered poorly. Most of the students (57 %) did not identify the graph of the function correctly from the graph of its derivative. About 9 % of the 150 students managed to get the correct answer and provide an acceptable explanation. However 34 % could not explain their choice although it was correct.

Although calculus reform movement emphasizes the ability to move freely amongst multiple representations as central to building the interconnectedness which indicates understanding, the results indicate that there is not much consistency across the responses to various items. Despite the similarities among Items 1b, 2, 3 and 4 the students lacked the ability to move comfortably among the different representational modes as in symbolic equations, tables of values and graphs.

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STUDENTS' EXPERIENCES OF 'EQUIVALENCE RELATIONS'

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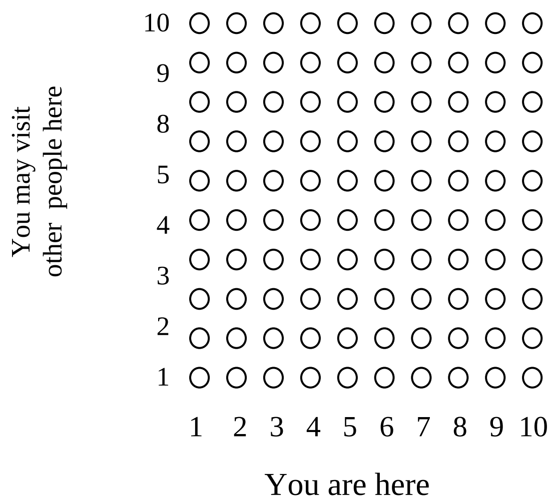
University of Warwick

We engaged a smallish sample of students in a designed situation based on equivalence relations (from an expert point of view). The students were different from each other in age and educational background, and all were unfamiliar with the formal treatment of equivalence relations. The study was conducted by holding individual in-depth task-based interviews, in which we aimed at investigating the ways that students organize the given situation, rather than teaching them any particular ways of organizing that. As result, I will report a certain way of organizing the given situation, from that a 'new' definition of equivalence relations, and consequently a new representation for them, is emerged; a definition that seems to be overlooked by the experts.

INTRODUCTION

Before giving any introduction in a normal way, let us invite you to give an example of a "visiting law" as defined below.

> A country has ten cities. A mad dictator of the country has decided that he wants to introduce a strict law about visiting other people. He calls this 'the visiting law'.
>
> A visiting-city of the city, which you are in, is: A city where you are allowed to visit other people.
>
> A visiting law must obey two conditions to satisfy the mad dictator:
>
> 1. When you are in a particular city, you are allowed to visit other people in that city.
>
> 2. For each pair of cities, either their visiting-cities are identical or they mustn't have any visiting-cities in common.
>
> The dictator asks different officials to come up with valid visiting laws, which obey both of these rules. In order to allow the dictator to compare the different laws, the officials are asked to represent their laws on a grid such as the one below.

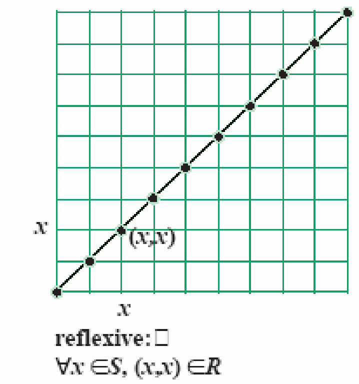


If you have not yet generated your own example, please before reading the next line that is about the original aim of this task try to generate one.

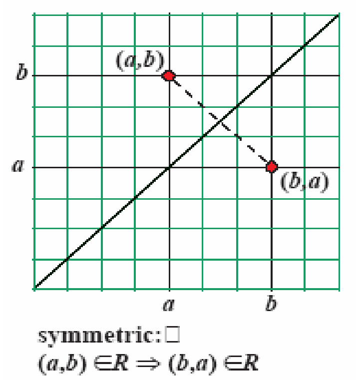
When devising this particular situation, the researcher had the standard formulation of ‘equivalence relation’ and ‘partition’ in mind (see below). And the situation was originally designed with the *intention* of seeing how the students proceed with what was then considered to be *the only way* of organizing the situation in order to come to *the* definitions of ‘equivalence relation' and ‘partition’.

Even though we can find different forms of the standard definition of equivalence relation in any text book about the foundation of mathematics (e.g. Stewart and Tall, 2000), let us choose one of them from a research paper that is highly related to the present study. Chin and Tall (2001) uses the following version of the standard definition of equivalence relation, i.e. a subset of S × S, say R, in which:

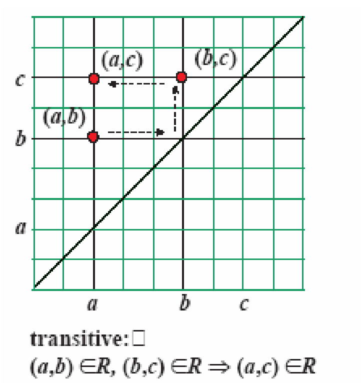
The elements (x, x) are all in R
(R is *reflexive*)



If (a, b) is in R then (b, a) is in R
(R is *symmetric*)



And it is *transitive*, i.e. If (a, b) and (b, c) are in R then (a, c) is in R. Although you could not find a picture for this form of expressing of the transitive property in the text books, Chin and Tall give the following picture for it:



If you generate an example of a visiting law and then try to generate more examples, you will wonder at the original aim of this study that was leading student from the above task to such complex definition of equivalence relation. Thus let us have a close look at the task and what we originally aimed for.

Analysis of the situation

As it can be seen an equivalence relation is first and foremost a relation. Thus let us start from relations in general. Set theoretic treatment of relations gives a unit and plural character to those elements that relate to each other. This aspect can be implicitly seen in the eloquent and still informal introductory paragraph of the chapter on relations in Stewart and Tall (2000, p.62):

> The notion of a relation is one that is found throughout mathematics and applies in many situations outside the subject as well. Examples involving numbers include 'greater than', 'less than', 'divides', 'is not equal to', examples from the realms of set theory include 'is a subset of', 'belongs to'; examples from other areas include 'is the brother of', 'is the son of'. What all these have in common is that they refer to two things and the first is either related to the second in the manner described, or not.

As it can be seen each one of that 'two things' in Stewart and tall examples implicitly belongs to a set; therefore, even though, for example, 1 in $2 > 1$ is treated as an individual, being in the set of integer gives an infinite access to it and illuminates its plurality. In general, those 'two things' are not only single individuals, but also something that can fill one of the two sides of a relationship, or more importantly fill both side of a relationship; they are simultaneously unit and plural.

As a particular relation, equivalence relation inherits above peculiarities in a more remarkable way. When we are looking for a concrete example of equivalence relation, we are apt to define a relation between *two* different things or people, say, *both* have the same colour, *both* live in the same street; we can check the possession of the given relationship between those two things or people by pointing to those two; even we can do that in a more concrete level, or using Dienes words(1976, p.9), in 'first order attributes' realms, say, they are both green, for the first relation, and they both live in Oxford street, for the second. However, as Dienes pointed out, the former way of checking, described by 'second order attributes', is more abstract and more difficult than the latter:

> To have the same colour as something else is a much more sophisticated judgement than to say that they are both green. (ibid, p.9)

Regardless of the difficulty, passing to 'second order attributes' realms seems inextricable for grasping reflexive property. To grasp reflexive property, first we must go one step further of the situation, and look at the situation as '…having the same colour as…', '…living in the same street as…', and so on; that demands, on the one hand, a transfer from unity to plurality in the sense described for relations in general, and on the other hand, a transfer from plurality to unity, i.e. coming from *both* to *each*.

In sum, although bringing plurality and unity together is hardly accessible in the concrete cases, we tried to achieve it, in the designed situation, by giving a "metonymical definition" in which more than often, city is used to refer to people in city. In consequence, "each city is its own visiting-city" metonymically stands for "in each city you can visit other people". And as it can be seen the former is an expression of the reflexive property. Having captured the reflexivity (the points on the diagonal), the situation aimed at leading students to the symmetry and transitivity through creating their own examples demanded in the first task and then giving the minimum amount of information demanded in the following task:

> The mad dictator decides that the officials are using too much ink in drawing up these laws. He decrees that, on each grid, the officials must give the least amount of information possible so that the dictator (who is an intelligent person and who knows the two rules) could deduce the whole of the official's visiting law. Looking at each of the examples you have created, what is the least amount of information you need to give to enable the dictator to deduce the whole of your visiting law.

Participants

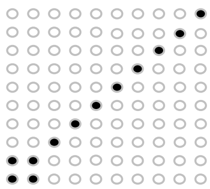
Having considered such details, our study started with a small opportunistic sample of students that their only commonality was that they had not been formally taught equivalence relations and related concepts. The initial data revealed that the students spontaneously create their own way of organizing the given situation which were not necessarily those intended by the situation designer; in other words they had their own concepts to use and their own ways of relating them to each other. Accordingly, the *intention* of the study became an investigation of *the ways* that students organize the given situation including a careful consideration of what *they* use to organize the situation.

Results

To manifest a flavour of the present study, let us present a snapshot of our data coming from interview with Tyler who is an undergraduate computer science student.

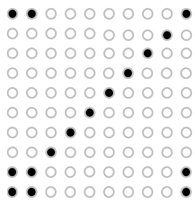
To satisfy the first condition of the given situation, Tyler blacked the diagonal and continued as follows:

Tyler: If I am in city one, and we allow to visit city two, how the other things need to change, to keep the rules consistent and see either they are completely the same or completely different, so aha, so city two now have to be able to visit city one…



Then he considers two things: "mirroring in y equals x"and "box"(square) and then "to see what was happening" he decides to make city one visit city ten:

Tyler: … and I realised first that, city ten has to visit city one… so that the second law …city ten has to visit city two…now I look at the city two, now I realised they are different from city one…so I copy number one on to number two also just to keep them the same…



As a result, Tyler abandons the "block square", keeps the "mirroring" and *proves* it as a "general pattern of these dots" (if (x, y) then (y, x)). In addition, the way that he proves "mirroring", gives him a new insight, i.e. considering the relationship between any two individual cities:

Tyler: If you allow a city to visit any other city, then it's gonna end up with having the same visiting-rules as that city that's allowed to visit and vice versa...

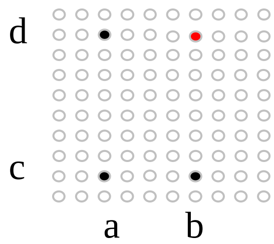
Having passed through many different concepts, he transcends the situation by introducing a new concept with general applicability (the 'box concept'):

Tyler: How do I say that columns must be the same mathematically? (He writes). If (x1, y1) and (x1, y2) and (x2, y1) then (x2, y2)

Interviewer: Could you explain.

Tyler: I think it's a mathematical way of saying …if a column has two dots, and there is another column with a dot in the same row, then that column must also have the second dot in the same row…I take maybe *a box of four dots*…I use the coordinate because that makes it very general, and so if I made that my second law, for a mathematician might be easier to follow.

It is worth saying that the box concept can be easily illustrated by a picture:

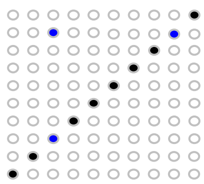


If (a, c) and (a, d) and (b, c) then (b, d)
(Box concept)

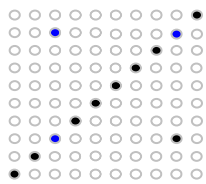
Given this, an equivalence relation can be understood as a relation having the reflexive property and the box property. That is, Tyler has explicitly generated a new (and, for us, unexpected) definition (which happens to be mathematically equivalent to the standard definition of equivalence) in order to organize this situation.

Equivalence relations, revisited

The following diagrams show how having reflexivity and box concept, we can deduce symmetry and transitivity.

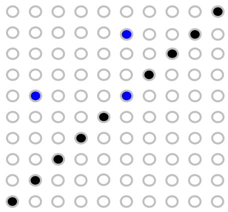


(**a, b**), (a, a), (b, b)
are three corner of the box

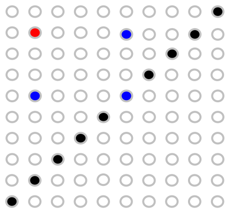


(**b, a**) is the
fourth corner

As it can be seen it is our old friend symmetry; the following diagrams illustrate the other one, transitivity.



(**a, b**), (b, b), (b, c) are three corner of the box



(**a, c**) is the fourth corner

On the other hand, it can be seen that having the normative definition of equivalence relation, based on reflexivity, symmetry and transitivity, we can deduce box concept.

Although the normative way of defining equivalence relations and its definition based on the box concept are logically equivalent, they have dramatically two different representations that could affect students' understanding of the subject. For example, Chin and Tall (ibid, p.5) suggested "*the complexity of the visual representation*" as to the transitive law as a source of a "*complete dichotomy between the notion of relation* (interpreted in terms of Cartesian coordinates) *represented by pictures and the notion of the equivalence relation which is not*". Accordingly, they suspected that that dichotomy inhibits students from grasping the notion of relation encompassing the notion of equivalence relation. However, the above figures show that the stated dichotomy, to a large extent, depends on the standard way of defining equivalence relation, i.e. if we define equivalence relation as a relation having the reflexive property and the box property, that dichotomy would disappear.

CONCLUSION

It is worth saying that the notion of equivalence relation defined by the box concept and its normative definition reveal two different ways of organizing the related concepts. While the former provides us with a simpler visual representation, the latter endows the subject with a seemingly more comprehensive quality in which two important types of relations, equivalence relations and order relations can be seen as particular types of transitive relations. Generally speaking, relinquishing a concept suitable for organizing a local situation in favour of grasping a more global picture appears as a particular aspect of mathematics.

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‘GUESSING’ IN A YEAR 1 MATHEMATICS LESSON WHEN ENGLISH IS AN ADDITIONAL LANGUAGE

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Young bilingual students in the UK face the challenge of learning mathematics and learning English simultaneously. In this paper, I draw on work in bilingual education concerning the role of participation in meaningful interaction in language acquisition. Using an approach to analysis based on ideas in discursive psychology, I present an analysis of a short extract of interaction between a Year 1 learner of English as an additional language (EAL) and his teacher in a mathematics lesson. The student appears to make ‘guesses’ in response to the teacher’s questions. My analysis suggests, however, that this behaviour arises from the socially organized structure of the interaction, as much as from the student’s arithmetic proficiency.

INTRODUCTION

There has been little research into the learning of mathematics in the UK by students who are also learners of English as an additional language (EAL) [1]. In particular, there has been little investigation of the participation of learners of EAL in lower primary school mathematics. In this paper, I analyse a short extract from interaction in a Year 1 classroom in a multicultural classroom in London.

THEORETICAL PERSPECTIVE ON LANGUAGE LEARNING

Research in bilingual education has considered the role of interaction in language acquisition. Cummins (e.g. 2000, p. 68), for example, proposed a 2 dimensional framework relating linguistic context with the cognitive demands of the interaction:

Cognitively undemanding

Context embedded ——————+—————— Context reduced

Cognitively demanding

Context refers to the context available to participants to support their interaction. Face-to-face talk, for example, relies on a high degree of context, in the form of gestures, facial expressions and the presence of many of the objects of discussion. Such context supports sense-making and so tends to reduce the cognitive demands of the interaction. Some interaction involves less context. In telephone conversations, for example, it is not possible to draw on facial expressions or gestures. Reduced context tends to lead to more cognitively demanding interaction.

Cummins ideas, however, are pitched at a rather general level, saying little about the detail of interaction. Such detail has been explored by Swain (e.g. 2000), whose work suggests that participation in interaction can contribute to language acquisition. In particular, she argues that “[linguistic] output pushes learners to process language more deeply – with more mental effort – than does input...Students’ meaningful production of language – output – would thus seem to have a potentially significant role in language development" (Swain, 2000, p. 99). These ideas suggest that 'meaningful production' in a rich linguistic context will support learners of EAL to learn English in and of the mathematics classroom.

THEORETICAL AND METHODOLOGICAL PERSPECTIVE ON INTERACTION

My research has involved the development of an approach to the analysis of interaction in multicultural classrooms which focuses on examining the discursive practices used by participants, rather than on the individual meanings participants have 'inside' their heads. This approach draws on discursive psychology (Edwards, 1997) and conversation analysis (Sacks, 1992). In particular, the social functions of interaction such as arguing, agreeing, negotiating or conducting relationships, are seen as primary in structuring discourse. In effect, the social structures the 'content'. Conversation analysis shows how, for example, talk is structured in turns, with the turn-taking structure both enabling and organising interpretation. A common feature of turn-taking is the occurrence of two-part structures, such as question-answer, greeting-greeting or invitation-acceptance. These two-part exchanges are called adjacency pairs. The second part of an adjacency pair may appear directly after the first, or may appear some turns later, often with other pairs nested in between, as in the following example, used by Sacks (1992, vol. 2, p. 529; see also Silverman, 1998, p. 106):

A:	Can I borrow your car?
B:	When?
A:	This afternoon
B:	For how long?
A:	A couple of hours
B:	Okay.

In this exchange, the first and last turns in the extract form an adjacency pair, with two question-answer pairs inserted in between. An important feature of adjacency pairs is that once the first part has been deployed, it is difficult for the addressee to avoid completing the pair in some way. Indeed any response will be interpreted in the light of the adjacency pair structure, so that even if, for example, B were silent after A's question, that silence would still be heard as a response. These ideas will be used to analyse a short extract of interaction from a Year 1 classroom, following an outline of the research context.

RESEARCH CONTEXT

The lesson featured in this paper took place in a primary school in London. There were 26 students in the class, including EAL learners from Kosovan, Bengali and Anglophone and Francophone Africa. In this particular lesson, another teacher (T2) joined the class for part of the lesson and supported individual students with their work. The lesson, which focused on halving and doubling, began with the students using number fans to respond to teacher’s questions. Later, the teacher moved on to a problem-like scenario about two children who have various items, one child having double or half the amount of the other. The teacher introduced the use of multi-link cubes formed into rods to support thinking about halving.

K is a refugee Kosovan student. He joined the school at the start of Reception. He was assessed by the school as EAL stage 1 (new to English) in November. The teacher estimated that he is probably stage 2 (becoming familiar with English) by the time of this recording. His parents were reported as being supportive, though K’s mother did not speak much English. K had Albanian language books on English and mathematics. The teacher felt he had a good memory, giving spelling as an example, characterising his memory as ‘very visual’. The teacher reported that K relied on guessing, often not listening to instructions before embarking on a course of action. The teacher believed K was working at a relatively high level in mathematics but was concerned that he could not show what he knew. In school tests, he scored more highly in English than in mathematics. I recorded K using a lapel microphone connected to a mini-disc recorder, worn in a pouch attached to his waistband (rather like a small walkman). K was recorded for an entire numeracy hour lesson, apart from a few minutes at the end, after the microphone became disconnected.

GUESSING

The teacher reported that K tended to guess in his responses to questions. During the lesson there were a number of sequences in which K’s participation could be interpreted as guessing. In the following extract, for example, T2 is working with K and Steven, reviewing K’s written responses on part of a worksheet [2]:

K I’m trying my second one//

Ste now you can do your **own** one//

T2 okay **now**/ four cars// d’you know what you’ve done look here// ‘kay it’s eight cars and it should be **double** eight and you’ve **halved** it/ you’ve made half of eight and it must be **double** eight/ what’s double eight?

K umm=

T2 =eight plus eight

K two

T2 eight and eight together

K seven!
T2 what’s eight/ and another eight/
Ste I know
T2 eight plus eight
K two!
T2 [no
Ste [sixteen
T2 sixteen
K oh
T2 so it should be sixteen cars/ /woah now you have to work out/ one and a six/

In this extract, T2 indicates that K has mis-interpreted the question on the worksheet, saying that K has halved a number of cars, when the task is to double the quantity. She formulates this point twice, emphasising the words ‘double’ and ‘halved’. She concludes with the question ‘what’s double eight?’ which is contextualised by the preceding formulations. She has moved from interpreting the task to a direct question. By asking a question, the first part of an adjacency pair, she creates an opening for K to contribute, although the nature of the question also indicates the kind of responses that might be given: a number is expectable. K’s response is ‘umm’, an utterance which allows him to take up his allotted turn, whilst buying some time. His turn is cut off, however, by T2, who reformulates ‘double eight’ as ‘eight plus eight’. Such reformulations can be seen as guiding students, glossing previous utterances to provide a range of interpretations for the student to work with. They might also be seen as supporting the student in engaging with the language of the task, in this case by relating a mathematical term ‘double’ with an operation ‘plus’. As a socially organised exchange, however, T2’s glossing also serves to raise the stakes for K. Having been offered two formulations, ‘double eight’ and ‘eight plus eight’, there is a greater obligation on K to come up with a suitable response to complete the pair. This obligation, I should emphasise, comes from the interaction, rather than any intention on the part of the teacher. It is a feature of talk that the more information that is provided with a question, the harder it is to not respond. K does provide a response: ‘two’. This response is generically suitable: it is a number. K has taken the turn for which T2 has nominated him, and rather than giving a non-committal ‘umm’, a response which was marked as unsuitable by the teacher’s swift intervention, K offers something generically appropriate and which completes the pair. T2 again indicates this response is not suitable, however, by again reformulating, this time saying ‘eight and eight together’. The stakes continue to rise. K offers another generically appropriate but mathematically unsuitable response, this time as an exclamation, ‘seven!’. Again T2 indicates unsuitability by reformulating, ‘what’s eight/ and another eight’. This time Stephen takes the open slot, saying ‘I know'. He indicates that the question is answerable and that, given the opportunity, he would be able to give a suitable response. The effect is to raise the stakes again. Not only is T2 reformulating the question, but Stephen claims to know the solution, implying K should too. T2 returns to an earlier reformulation 'eight plus eight' and K gives the same response he offered on the first occasion it was used: 'two!'. Both T2 and Stephen break the pattern of the preceding turns. T2 now explicitly evaluates K's latest (re)offering, 'no'. Stephen, overlapping, takes up the opportunity created by his previous turn, to give a response of his own, 'sixteen'. This response is accepted by T2 through her repetition, 'sixteen'. K accepts this closure, 'oh'. Finally, the teacher recontextualises Stephen's solution within the problem on the worksheet, by referring to 'sixteen cars'.

DISCUSSION

To summarise this analysis: a number of patterns run through this exchange.

- the interaction is structured by the question-answer format;
- the sequence of reformulations raises the stakes through the exchange;
- the reformulations run through a range of glosses for 'double': '...plus...', '...and...together' and '...and another...'.

How might these patterns interact with K's position as a learner of EAL? My first observation is that K is clearly able to participate in the question-answer pattern common in much classroom talk. He takes up turns when he is nominated. Indeed K's 'guessing' can be seen as arising in response to this pattern. It may be linguistically less demanding to provide a 'guess' than to ask for more information or to find some other way out of the pattern, particularly when the teacher's reformulations raise the stakes. Furthermore, K's responses are generically appropriate, indicating more specific familiarity with the norms of mathematics classroom talk. A second observation is that the range of formulations of 'double' provide potentially valuable linguistic input, offering a range of ways of talking about a particular concept. In this particular sequence, K does not appear to respond to these reformulations, but it may be that over time, he would become familiar with a number of ways of talking about 'double' and relate the concept to other arithmetic structures, including addition. It is noticeable, however, that in this extract, as throughout the lesson, K rarely uses the term 'double' himself. The occasions when he does so are in the form of repetitions. If meaningful production is an important part of the acquisition process (Swain, 2000), however, whilst hearing various glosses for a term like double is an important contribution to K's learning of the language of mathematics, supported opportunities to use such terms himself would also be beneficial.

In conclusion, I have argued that K's 'guessing' can be seen as arising from the interactional patterns found in the mathematics classroom as much as from his arithmetic proficiency. It is possible that K attends more to the interaction than the mathematics, perhaps in a bid to maintain an appropriate social role in the class.

NOTES

1. English additional language (EAL) refers to any learner in an English medium environment for whom English is not the first language and for whom English is not developed to native speaker level.

2. Transcription conventions: Bold indicates emphasis. / is a pause < 2 secs. // is a pause > 2 secs. (...) indicates untranscribable. ? is for question intonation. ( ) for where transcription is uncertain. [for concurrent speech. & for utterances which continue on a later line.

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ACCELERATED LEARNING OF PROBLEM SOLVING SKILLS

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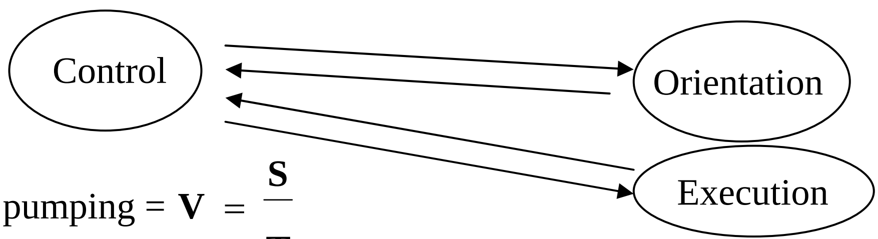
Two year 7 classes in a Manchester school were taught multiplication, division and fractions. An experimental group was taught these numerical skills, but their teaching program included practical problem solving, based upon activity theory principles, as an integral component. A control group practised their number skills in more traditional abstract contexts. As expected, the control group was not able to transfer number fluency to practical problem solving tasks. The experimental group, however, demonstrated a problem solving ability at higher GCSE level and achieved a significant improvement in mean scores over the dynamic assessment that followed the teaching program. The dynamic core of this assessment was computer based and there was a strong negative relationship between hints given by the computer and residual gains. Analyses of the computer records have provided important clues to guide a qualitative analysis of video records of the teaching program.

Engeström (1999) has suggested that a study of mediating artefacts (such as mathematical models) is centrally important in research into practical problem solving. Within this ‘Activity Theory’ perspective, theory is seen to be of greatest importance when it can be used to mediate a process of practical activity. In the teaching program summarised below there is no separation between presentation of theory and practice and the practical creative nature of mathematical tools became directly apparent to the students while they were solving problems. The teaching material was in turn prepared on the basis of detailed analyses of practical problems involving the notion of rate already carried out by the Gal’perin School (See Haenen 1996 for a more detailed account of this teaching method).

The experimental teaching programme focused on teaching basic number skills of multiplication, division and fractions in a meaningful context. I chose to develop these skills in the context of problems on rate of processes, for example the rate of movement, rate of production or rate of flow in water, because the students will encounter these problems regularly in later studies. I chose the notion of rate as a practical (substantive) generalization that would be widely applicable in practical activity. I looked at this learning in terms of developing practical creative abilities rather than simply of acquiring abstract knowledge of formal calculation rules. This diagram shows the key components of actions that were taught.



Rate of pumping = $\mathbf{V} = \dfrac{\mathbf{S}}{\mathbf{T}}$

Fig. 1

Gal'perin's Activity Theory suggests (ibid) that the process of orientation, of knowing what to do next at any point, requires identifying the main operations to carry out and the order in which to do them. An appropriate control model must be taught and developed in the course of problem solving activity. A more or less developed form of this model can then be brought to mind when it is required during practical problem solving. Orienting activity will then appear as 'attention', which directs itself towards the model. This attention is thus an abbreviated and condensed control procedure for practical activity. An example of a problem solving action from the teaching program (fig 1) was: '*A pump produces 100 litres of oil in 5 hours. How much oil would be produced in one hour?*'

In this example, a formal mathematical notion was introduced as a model, which acted as a control for the action. The process of orientation, of knowing what to do next at any point, was also taught, in this case by means of two cards (shown below), which indicate the main operations to carry out in simple calculations, and the order in which to do them. These instructions were operations in verbal form and they were abbreviated during practice to a coded form '1,2,3,4' or '1 to 6'and then, with more practice to a simple awareness of what to do next, or 'attention'.

CARD 1

In each question you must find:

1) **Who is carrying out the action?**
2) **What is the person or thing carrying out the action getting through, producing or using up? (S =?)**
3) **How long do they take? (T=?)**
4) **How much do they get done in one unit of time? (V= ?)**

Card No 2

1) **How many actions are going on?**
2) **Do they begin and finish together?**

Do the actions work: a) together

b) against each other?

4) What is known in the task about overall production values (S_o, T_o, V_o)

5) What is known about component action values (S_1, T_1, and V_1)

6) What do you need to find in the task?

Within this teaching method practical actions were converted to words and then to mental actions. Actions were first presented in materialised form as diagrams. These coded actions changed to a verbal form as they were spoken aloud. During practice, silent speech 'to oneself' was abbreviated and condensed and was eventually no longer accessible to introspection. In this process, the actions changed in their level of generalisation as a deeper understanding of rate formed from notions of speed, wages, flow etc. Abbreviation and fluency of all three aspects of the actions (orientation, execution and control) was developed in order to establish a sound long-term memory of the problem solving skills (see Talyzina 1981). A variety of techniques were employed to develop these skills and the problem solving tasks were gradually increased in difficulty until they eventually reached a level of difficulty presented in higher-level GCSE courses. For example:

'Two bulls charge each other with a combined speed of [40 3/10 mps] and meet after [16 seconds]. The speed of the first bull is [12 1/5 mps]. [How far would the second bull travel] in [this time] if he went [2 1/10 mps slower]?'

Results: The diagram below illustrates the experimental design:

	Initial base line test	Teaching -------- program	First post test	Computer practice	Second post test
Experimental gp.	(IT)	------------------	(T1)	(P1)	(T2)
Control gp.	(IT)	-----------------	(T1)	(P1)	(T2)

Initial baseline measures compared the children from the two classes (the experimental and control groups) in terms of ability to complete questions on multiplication, division and fractions. These were questions taken from the normal school end of unit test on this topic. Average scores on these tests did not vary significantly between the classes. The mean overall scores in these initial tests were 61% and 63% (table 1). These scores indicated that children were generally competent to begin work on the elementary introductory examples in the teaching program. Initial scores on further practical problem solving questions about rates of processes were lower, as would be expected, but again showed no significant differences between the two groups:

	Experimental group		Control group				
Variable	**mean**	**sd**	**mean**	**sd**	**n**	**t**	**p**
Pre-test (number - %)	**61**	**16.5**	**63**	**16.5**	**23**	**0.6**	**0.57**
Pre-test (rate - %)	**18.6**	**18.6**	**22.9**	**21.4**	**26**	**-1**	**0.42**

Table 1

The control group followed the normal school program, which involved practice of the number skills in more abstract contexts. I expected that the control group would not be able to transfer their number skills to practical problem solving tasks. This was in accord with the principles of activity theory, which explicitly propose that mastery depends on the quality of particular orienting bases (cards one and two) employed, and is not an ingredient that is added separately from the material that is taught.

At the end of the program I looked both at what the children could do in formal numerical questions from the school end of unit test and at how easily they could apply this knowledge to problem solving questions on rate, with help from an hour of computer based teaching. In this three-part dynamic assessment, a computer-based assisted practice session separated two isometrically similar formal unassisted tests (see Day 2001 pp. 176-207 for a discussion of this procedure). No significant differences emerged between the groups in their measured abilities at numerical questions on these topics. A two-way Repeated-Measures ANOVA analysis of the number test scores showed no significant differences between classes (see Chart 1).

An analysis of rate test scores showed a significant difference between experimental group post-tests ($p<0.01$) and a significant difference between the two groups overall

($p<0.05$) (see Chart 2). The experimental group scores on rate improved from a mean of 26% to 41% over the dynamic assessment and were more than twice as high as those of the control group in the final post test.

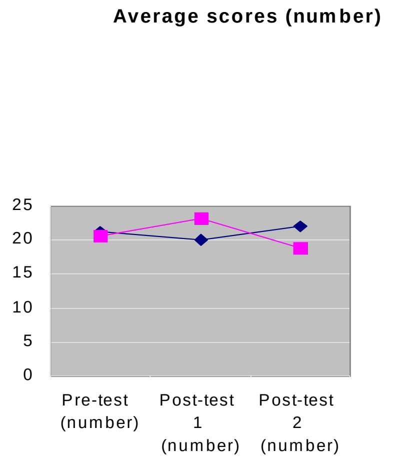


Chart 1

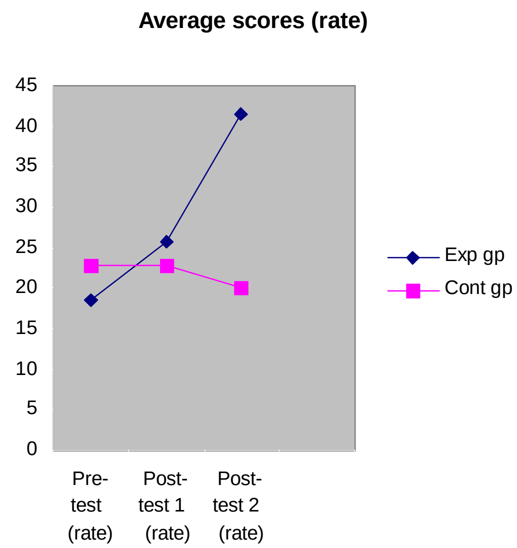


Chart 2

The next table summarises the comparison between mean scores on the pre-test and mean scores on the post-tests (T1, T2) for the two groups.

Descriptive statistics and t values for experimental and control groups

	Experimental group		Control group				
Variable	**mean**	**sd**	**mean**	**sd**	**n**	**t**	**p**
Pre-test (number)	21.3	5.5	20.6	5.2	23	0.6	0.57
Post-test 1 (number)	20.1	6.6	23.3	4.8	26	-2	0.05
Post-test 2 (number)	22	5.9	18.8	7.4	12	1.5	0.16
Pre-test (rate)%	18.6	18.6	22.9	21.4	26	-1	0.42
Post-test 1 (rate)%	25.7	25.7	22.9	25.7	26	0.3	0.8
Post-test 2 (rate)%	41.4	31.4	20.0	24.3	26	2.6	*0.014

table 2

I have already shown (Day 2001) that the rate of adaptation to similar but more complex problems can provide important information about a child's 'Zone of Proximal Development'. In this work I demonstrated that the number of hints given during interaction with a tutor can provide a useful (inverse) index of intellectual maturity and readiness for the more difficult problems. Because of time constraints within the busy school teaching program, theoretically based hints, generated from the teaching program, were given during a computer-based practice until all the problems were solved. Amount of help required was recorded, categorized and compared with the mathematical gains made in unassisted performance over the two tests carried out before and after the computer-assisted session.

[In order to minimise validity problems due to effects of the distribution of results in test T1 on results in test T2, mathematical gains over the practice session were defined by residual gains in test two, above or below what was predicted by the overall trend of results. (See Elliot and Lauchlan (1997), Embretson (1990) for a discussion of the reliability of gain scores measured by test, train, retest procedures). Unreliability due to unequal scaling effects for level of difficulty between test items remains, but scaling defects will be the same in both experimental and control groups and changes of scale that occur because items that become easier in the second test will also be generally replicated over the two groups. Use of the second post-test will therefore not affect the reliability of my comparison between the two classes].

Mean residual gains and hints given for the two groups are shown in the diagrams below. The negative mean residual gains for the control group (fig.2) suggest that the control group children were unable to gain as much from the practice sessions as children from the experimental group. They could not transfer their formal mathematical knowledge to their practice session on the computer. This was, of course, what was expected. These results largely replicated the results of earlier studies and confirm the number of hints needed in practice to be a useful indicator of proximal development zones. Control group children received almost 50% more assistance on average (fig. 3) than children from the experimental group.

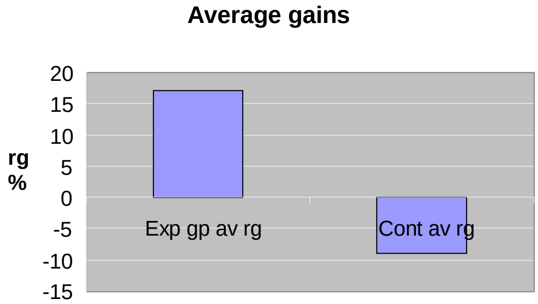


Fig 2

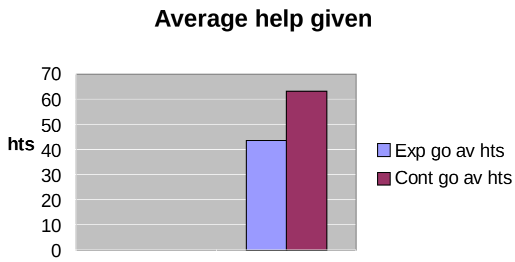


Fig 3

It can be seen from table 3 that the number of hints provided was negatively correlated with gains that were made and with scores on the first post-test. I found, as expected, that lower scores on the post-test meant that, generally, more help would be needed in completing the practice papers and lower gains would be made during practice. The amount of help needed was clearly an important factor in predicting these gains. In a multiple regression (table 4) the number of hints (hts) accounted for 13% of variation in residual gain scores over and above the general mathematics ability measured in the pre-test and a specific test of the work (t1, rate).

	Rg	**t1 num**	**t1 rate**
t1 num	**0.1**		
t1 rate	**-0.030**	***0.55**	
Hts	***-0.31**	***-0.46**	***-0.52**

*P < 0.05

table 3

STEPS TABLE

Model	STEP	df	deviation reduced	df2	F		rsq chg%
0 (no predictors)							
1 (pre)	1(0-1)	1	0.97	27	0.25	ns0.010	1
2 (pre,t1)	2(1-2)	1	0.95	26	0.24	ns0.009	1
3 (pre,t1,hts)	3(2-3)	1	13.18	25	3.64	P<.05	13

table 4

Computer-based hints have thus been shown to have some validity as an index of a students progress and can help to guide a more descriptive account of the program.

Qualitative results: This final diagram shows hints plotted against residual gains for two children who completed the entire program. These children (Louise and Lisa) sat together in class, achieved equal scores in the second post-test and made similar mathematical gains. Lisa, however, needed far more help in completing the computer-based practice than Louise. Video records of these two children confirm that Louise was far more influential in interactions between the two children. She seemed to have a greater mastery of the topic than Lisa and because of this would be expected to progress more quickly in future. The imbalance in their working relationship was only indicated quantitatively by amount of help required during practice. An inductive analysis of the data, beginning with video transcript records of Lisa and Laura and then looking at other related events that were observed has modified my view of the teaching activity. As I review the data looking for events that contradict my original idea, I hope to arrive at set of ideas developed within an integrated and well-defined theory that could describe aspects of the teaching program in a way that will provide suggestions for future improvements. A model that accounts for the qualitative observations will be presented later.

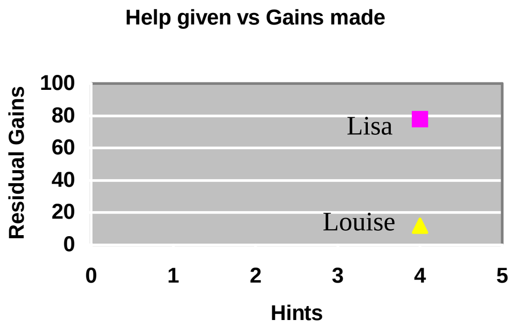


Chart 3

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THE END OF SPOON FED MATHEMATICS? A REPORT OF A YEAR'S BPRS RESEARCH

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An outline of a development project initiated to prevent the continuation of spoon feeding teaching at a grammar school. The report is covers the background information, research process, some examples of student work, and finally gives some tentative conclusions.

BACKGROUND

"I like school, you don't have to think, they tell you what to do." Anonymous

This quotation concluded my findings at the end of 2001 and summarised very succinctly the challenge presenting my development work. In April 2000 I joined the staff of Tonbridge Grammar School for Girls as the second in the mathematics department. Tonbridge Grammar School for Girls is often perceived as a very successful school, it has a selective intake and achieves outstanding performances from its students in many different areas.

However, as I looked around the mathematics department at the staff and students, I found that the apparent success of the school was founded on a very traditional style of mathematics teaching and the girls that we taught were often unsure of themselves and found it hard to make decisions. The girls appeared to work best when given explicit instructions and some would often prefer to do nothing, rather than make the wrong choice. Many of the girls seemed very unhappy at presenting any piece of work to be marked if that work was not 100% correct. Some of the braver students seemed to have adopted a "working in pencil" strategy for when they were unsure. This demonstrated that they did not really trust their answers, but had at least made the attempt demanded of them.

It was also clear that these habits were more obvious with the sixth form students than with the younger years. It has been suggested that students working at such high levels do not tend to find GCSE all that difficult and are, perhaps, only meeting a real challenge when they start their A-level studies.

In recent years this approach has often been called "spoon-feeding". In a teaching situation it is easy to see how such an approach has been developed. In explaining a difficult concept to a group of students the students query each individual step and need each part of the concept explained in great detail. When they are trying to solve a similar problem on their own they seem more successful if the teacher has broken the initial problem down into a number of smaller problems. Without necessarily having had this strategy in their head, the teacher has, deliberately or otherwise, taken the initial problem and split it into smaller pieces. Each piece is far more readily solved by the students, thus they can solve the whole problem, and therefore feel more successful. Initially all seems well but it becomes clear that this approach leaves the students clutching at a long list of rules to apply in every possible situation. Attempts have been made to teach the students how to break the initial problem down into manageable pieces but students vigorously resist such processes in favour of seemingly easier strategies. This problem was raised by many researchers in the early 1980s, with one pair reporting

> Mathematics is the study of relationships, and not the memorisation of predetermined processes and answers.(Dawson & Trivett, 1981, p36)

Although this was written some twenty-two years ago it is clear to me that many of my students think exactly the opposite, that it is possible to be very successful at mathematics by entirely memorising everything they come across.

A VISION OF THE FUTURE

As I considered the nature of the mathematical experience our girls were undergoing I was forced to question whether this was the best experience we could offer. At the beginning of this academic year I asked my sixth form students what they thought mathematics was all about. These are a typical cross-section of their responses.

> Maths is about solving seemingly impossible problems, and for use in everyday life.

> Maths is all about messy books with loads of wrong answers so I get v frustrated and do it all again! I know there is a point to it, but sometimes it's very difficult to see what use its ever going to be.

> Maths is about achieving an A-level in a subject that is respected because everybody sees it as difficult, thereby proving yourself to be a competent, intelligent human being, even if you don't feel like one when in the process of doing this A-level.

These students seem to have their understanding of mathematics largely based on some understanding that mathematics is about solving problems, using the skills that they have learnt. These problems have been based in real-life, though with some fear that these problems are not really of any genuine use. There is some sense also of looking for patterns and trends in numbers. This is coupled with a little of cynicism based on having been forced to learn things that can easily be accompanied by other means, for example learning how to multiply numbers when their calculator can carry out such calculations very easily.

The Technology College Trust produced a report in 1999 entitled “Engaging Mathematics”. This report was written to try to find solutions to the problem of declining numbers of students studying mathematics. In its suggestions for teachers to make mathematics more engaging and enjoyable for students the authors write

> Involve pupils in simple starting-points, then ask how they might vary these, or what questions they could think up to answer next.

> Think of ways in which pupils can be involved in processes such as searching for patterns, making and testing conjectures. (Oldknow & Taylor, 1999, p20.)

These comments guide us to another interesting suggestion – that the students could be more involved with their choice of work. Perhaps we need to move away from the teacher directing all of the students' work, but seek tasks that the students can begin to develop under their own initiative. This has two immediately obvious long-term benefits. Firstly that this skill of individual development of problems is much needed for GCSE coursework in years 10 and 11, and secondly that this is also working towards a more independent approach to the students' learning, which addresses many of the problems that my previous research uncovered. There are many places to turn to uncover the right sort of starting point for such tasks. There may come a time when it is appropriate to give the students a very wide choice of task, perhaps even allowing them to choose their own area to investigate. For our first year we will seek a more modest approach and look for a task that the teacher can introduce, but the student can develop and extend.

RESEARCH PROCESS

As a teacher-researcher I had to grapple with defining the type of research methodology that I would be following. Action research is an inquiry-based process. It allows the researcher to focus their attention on a specific situation. This often results in a highly focused study. This process builds on the professionalism of teachers, encouraging further reflection and study of the specific problem. Hegarty writes

> Teaching is a professional, skilled activity. Expert teachers do not come into the classroom programmed with a set of rules drawn from a manual of good teaching practice…. Excellent teaching is founded on insight, creativity and judgement. (Hegarty, 2003, p30)

Action research builds on the insight of teachers by encouraging them to reflect on their current practice and identify parts of it that could be improved. The improvement process is as free of constraints as possible to allow teachers to use their own creativity as much as they are able to. Action research also allows teachers to begin to judge their own work and to develop their own success criteria. Action research requires a disciplined approach. The action researcher has to rise above the mere tinkering in order to make changes to their practice.

One challenge facing action researchers is that of objectivity. The whole process of action research ties up the researcher with the classroom being observed. The researcher cannot remain aloof and detached from the situation. Traditional scientific research made much of the remote investigator who was able to observe a situation without influencing it. In this sense the action researcher fails. The task of the researcher is not to remain detached, but rather to take account of the connections between the observer and the observed. In the educational field progress is being made far more slowly. Stenhouse (1975) argued for direct teacher involvement in the educational research process;

> All well-founded curriculum research and development, whether the work on an individual teacher, of a school, of a group working in a teacher's centre or of a group working with the co-ordinating framework of a national project, is based on the study of classrooms. It thus rests on the work of teachers. (Stenhouse, 1975, p143)

In his view all educational research has its foundations in the work of teachers. Educational theories should have their basis in the classroom. He believed that teachers were professionals who generated theory based on their classroom practice. Almost thirty years later it is encouraging to see more and more research being carried out in this way.

TWO EXAMPLES OF THE WORK

And so in September 2002 we began the new lessons with year seven. To begin with I had planned a series of lessons looking at the way in which we communicate the mathematics that we know.

The initial statement "two odd numbers always add up to make an even number" provoked a good discussion with the first class that I met. The following dialogue was very interesting.

Student: We know that it is true because 1 + 3 = 4.

Teacher: Are you convinced from one example?

Student: What about 3 + 5 = 8, 1 + 1 = 2?
(many more were suggested)

Teacher: How many examples do you want to give?

Student: All of them.

Now at one level this final answer is a very good one. In order to be convinced about the truth of a statement specifying every possible answer is a perfectly sensible strategy. Perhaps this would best be described as a scientific proof? In the same way that a scientific theory is often tested under all possible conditions, perhaps the validity of the mathematical statement should be tested using all possible numbers. The student quickly realised a small flaw in their argument.

Teacher: How many are there?

Student: (looking quite embarrassed) lots!

So the student quickly realises that there are an infinity of possibilities for each number, the number of possible pairs seems more than infinite, if that were possible. With this group nothing further arose from the discussion. Perhaps sensing a dead end in this line of thinking, no one else managed to create a more satisfactory solution. With another class a different approach was taken quite quickly. One student started the following line of attack.

Student: You could think of odd numbers as being some pairs of numbers and an extra one, and so if you put two odd numbers together you'll have a pair of the extra ones, and this would make another pair.

Some of the rest of the class took a little more convincing of this strategy. It seemed too early to try to write something algebraic, but some students were able to produce a diagrammatic representation of an odd number as being a number of pairs and one “odd one”. When each odd number is represented in this way the sum can be seen as a number of pairs plus two “odd ones”. These two “odd ones” thus make another pair, and so the sum can be said to be an even number.

This sort of thinking was exciting to witness. This was thinking beyond the use of numerical examples, the diagrammatic approach made sense to the student and she was able to utilise it to give a very good proof of the general statement. She was also able to explain her approach, so that others in the class could also be convinced by it.

From my limited experience of my two classes I was very pleased with the students’ initial approach. They seemed to understand the problem, and were willing to try and talk about their answers. With continuing examples many more seemed to grasp the concepts of mathematical explanations.

Much later in the year we spend several weeks working with Pascal’s triangle. Much varied work was produced, but one student in particular discovered some significant mathematics entirely on her own. She had been working on powers of 11 and had noticed that the first few powers of 11 were clearly just the first few rows of Pascal’s triangle. Then she hit 115 and had to explain how 161051 could be produced,

> “I saw if you get a row from Pascal’s triangle you can make it come to an answer from the powers of 11. You don this by adjusting all the boxes with the 2 digits inside. For example, in the line 1, 5, 10, 10, 5, 1 by treating the 10 as 1 thousand rather than 10 hundreds and continuing this procedure as you work to the left then you can produce 161061. This also works for higher numbers in the triangle.”

SOME DIFFICULTIES

An interesting staff issue arose part way through the autumn term. After the initial set of tasks my approach was to stay two or three weeks ahead of things, to give me some means to react to the way in which the tasks were being received. I hoped that two or three weeks notice would give staff enough time to digest the information – it has seemed over the past few years that the other teachers only really plan their lessons about a week ahead at the very most. During this term though a couple of staff asked for “solutions” to the problems being set. For “normal” mathematics this is a perfectly reasonable request. The text books that we use for years seven to nine have a teacher’s volume with answers in, and so it does not appear unreasonable that the staff are wishing to receive a set of solutions to accompany this new material. However, this request does strike at the heart of the objectives of the new lessons, but at a new level. The aim is to encourage the students to work more independently, to need less spoon-feeding and to be able to think for themselves, to plan new areas of work without being quite so teacher-led. But what does this mean for the teacher? Can the teacher have every possible solution previously mapped out? At that time the request for answers from a teacher sounded very much like the request for answers from a student. I was left to ponder this dichotomy. In order to prevent spoon-feeding the students is it necessary for me to spoon-feed the staff? For many of these tasks it is very hard to predict all possible interesting spin-offs.

So to what solution? Two immediate solutions presented themselves. Firstly, to encourage staff away from the reliance on knowing all the answers in advance, for me to work at a staff level, in the same way that I want my staff to work at a student level. If it is possible for me to model the behaviour and approach that I expect from them then perhaps the staff will understand more fully. On a second level it might be appropriate for me to give some outline solutions to the most obvious route and what I expect to be the most common solution. Maybe not for every task, and maybe not in the greatest of detail, but perhaps this gives some level of support to the staff – again perhaps I am able to model the same level of progression with the staff that I am expecting my staff to model with their students.

CONCLUSION

At the end of the year it was clear that the students and staff had coped well with the changes. The students were working more independently and we will continue to develop this ability over the coming years. It will be interesting to see how they progress, especially when they face GCSE coursework. The staff were discussing mathematical problems amongst themselves, which I hadn't witnessed at the school before.

As a department this has begun a discussion concerning our beliefs about mathematics. What experiences do we wish our students to have, and how can we ensure that they all obtain a fair deal.

As a conclusion I would like to end with the words of one of our year seven students. When asked for her suggestions for future improvements she wrote

> "Well, ice-cream and music with a couple of playstation games and A-list celebrities would be cool, *but as far as Maths lessons go, this is pretty good*."

At the beginning of this report I spoke of our students' reluctance to show enthusiasm towards their mathematics. In a wonderfully British understated way this comment goes some way towards giving a glimpse of enthusiasm and enjoyment. As an endorsement of our work so far this is quite enough.

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IDENTITY, MOTIVATION AND TEACHER CHANGE IN PRIMARY MATHEMATICS: A DESIRE TO BE A MATHEMATICS TEACHER

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Teacher change in mathematics education is recognised to be a difficult and at times painful process. This is particularly so for generalist primary teachers, who have often had negative experiences of mathematics. In this paper I explore how one teacher developed a desire to be a mathematics teacher, thus enabling her to engage with change despite its difficulty. Drawing on theories of identity and situated learning, I conceive of motivation in terms of desire and argue that emotion is a potentially powerful element of mathematics teacher education.

INTRODUCTION

Negative mathematical experiences and relationships to mathematics are well-documented problems amongst primary teachers (e.g., Bibby, 1999). Teacher change in mathematics is also known to be a hard and painful process (Clarke, 1994). Hence, as Stocks & Schofield (1996) argue, teachers need a "deep desire" (p. 291) in order to engage and persevere with change. Yet, there is little research within mathematics education that seeks to understand and theorise how such motivation develops (Middleton & Spanias, 1999). In this paper, I explore this issue using the case of one teacher, Ursula.

METHODOLOGY AND CONTEXT

The research reported here is based on a four year longitudinal study into the professional change of the six teachers involved as teacher-researchers in the Primary Cognitive Acceleration in Mathematics Education (CAME) Project research team (Hodgen, 2003). The Primary CAME project research team consisted of four researchers, four teacher-researchers and the Local Education Authority mathematics advisor. Over the first three years of the project, the research team met on an approximately fortnightly basis to develop Thinking Maths lessons specifically for Years 5 and 6 in England (ages 9-11). The teacher-researchers participated in the trialling and development of lessons, in addition to leading professional development sessions and acting as tutors for a further cohort of teachers. (Johnson et al., 2004)

The fieldwork was conducted between November 1997 and July 2001. Data collection was qualitative using multiple methods, including observations of the day-long meetings, lessons and PD sessions, semi-structured interviews with individuals and groups, and structured mathematical interviews. My own role was as a participant observer. Initially, the data was analysed through open coding methods and informed by constructivist grounded theory (Charmaz, 2000). As the research progressed, I developed the analysis through narrative methods drawing on Kvale's

(Kvale, 1996) approach. I used participant validation and comparison between data sources to triangulate and develop my analysis.

Ursula

Ursula (a pseudonym), the focus of this paper, participated as a teacher-researcher throughout the four year period. At the start on the research, she had been teaching for five years. She had previously participated in a 20 days mathematics course. In many respects, Ursula was a somewhat unusual primary teacher. The Primary CAME professional development experience was unusually extended and intense. Moreover, during the course of the project, she moved from being a classroom teacher to being a Numeracy Consultant. Hence, I discuss Ursula as a “telling” rather than as a “typical” case in order to amplify and illuminate the possibilities for change through a process of analytical induction (Mitchell, 1984).

MOTIVATION AND DESIRE

My consideration of desire arose in part because Ursula, along with two of the other teachers, referred to mathematics in strongly affective terms. They all talked about their “love” for doing or teaching mathematics and used emotive stories from their past to illustrate this. Yet, the professional change experience was at times painful for all three.

Ursula, like many primary teachers, had anxieties about aspects of the secondary mathematics curriculum that were related to her own schooling. For example, she associated her “fear of algebra” to her experience of starting and giving up A/S mathematics:

> there was just this enormous algebraic equation going across the board. Absolutely enormous, and I’d walked in late because it was after school … and I couldn’t come to grips with this at all. And that was that. Walked out the classroom and didn’t go back again […] It is just a whole negative thing. I assume that I can’t do anything [to do with equations], I have a complete mental blank. Whenever I see anything like that I just get a mental blank and I just think - I can’t do that (Interview, July, 2000).

In addition, Ursula experienced difficulties throughout her involvement in the project. The experience was difficult: “I just can’t do this”; exasperating: “I’m so annoyed, I’m I feel like smacking him [one of the researchers]”; and confusing, “What is special? I’m doing this already.” Despite these difficulties, Ursula not only persevered with the project but also became a Numeracy Consultant and, at least for a time, identified herself as a “subject specialist […] a maths teacher” (Interview, July, 1999). The resulting changes in her beliefs about mathematics and mathematics education were very significant (Hodgen, 2004).

IDENTITY, LEARNING AND CHANGE

I outline my conception of identity in some detail elsewhere (Hodgen, 2003, 2004). Briefly, I draw on Wenger’s (1998) conception of a fragmented identity located in communities of practices and Holland et al.’s (1998) notion of identity change in terms of authorship and improvisation. Wenger argues that that change and learning are facilitated by a “combination of engagement and imagination” which enables identification “with an enterprise as well as to view it in context, with the eyes of an outsider. Imagination enables us to adopt other perspectives across boundaries and time … and to explore possible futures … [and] trigger new interpretations” (p. 217). Holland et al. emphasise “aspects of identities that have to do with figured worlds - story lines, narrativity, generic characters, and desire” (p. 125). Learning is, as Evans (2000) argues, “facilitated by fantasy” (p. 224).

In order to conceptualise desire, I draw on the Lacanian psychoanalytic theory. For Lacan, imagination, fantasy and desire are fundamental to understanding human action. He conceives of identity in terms of an unattainable completeness:

> “[T]he human subject is always seen as incomplete, where identifications of oneself are captured in an image: as an individual I am forever trying to complete the picture I have of myself in relation to the world around me and the others who also inhabit it (Brown & Jones, 2001, p. 10).

Lacanian theory is particularly appropriate, because of the way in which pleasure is seen as dialectically linked to pain. Thus, it provides a way of locating the motivation to sustain change in relation to the very real difficulty of this for teachers.

LOVE, FANTASY AND MATHEMATICS TEACHING

In this section, I discuss two quotes from interviews some 2 1/2 years apart. These were from key moments in Ursula’s professional change and are typical of her engagement at these times. Ursula described the initial trial of the first TM lesson that she herself developed as follows:

> They were really noisy. I had stand up arguments between children about the maths, shouting at each other. If anyone had come in, they’d have thought it was chaos, but I loved it. (Research team, January 1998)

The image presented here was certainly an exciting one in which children were engaged in mathematical talk. However, the way in which she expressed this message is very significant. Schools and classrooms are generally characterised by order, control and turn-taking. “Chaos” and children “shouting at each other” are the very antithesis of what classrooms are expected to be like. Ursula used these descriptions in order to emphasise that mathematics in this incident was different to ordinary primary mathematics lessons. Her description of the children’s mathematical talk was framed in language that implicitly challenged her own authority as the teacher. She presented the children as arguing about mathematics without apparent teacher intervention. This is in marked contrast to the culture of many mathematics classrooms where authority for what is right or wrong, together with what counts as mathematics, rests with the teacher. Thus, in this brief description Ursula pointed to three inter-related issues in relation to school mathematics: the children’s control of the mathematics; the contrast with other people’s mathematics lessons; and, her own strongly expressed belief in this way of working.

Equally important was the form in which she presented presented the lesson as a deviant case. She emphasised that "anyone," implying, I suggest, anyone who taught in the ordinary way, would have judged the episode as chaotic. This highlights the intuitive and undeveloped nature of Ursula's beliefs in relation to mathematical authority at this stage. Ursula believed that, contrary to her own experiences, authority should be dependent not on the teacher but on mathematical discussion. Whilst she believed this to be the case, she did not *know* it to be the case and would have had difficulty justifying this belief to others.

Two and a half years later, Ursula commented on her earlier discomfort and confusion:

> I've had a big shift actually in the fact that I used to like things like Roofs [a TM lesson] 'cos you had a really exciting answer at the end of it and the kids were pleased, but that was it. And I actually like the lessons now where you ask them a question and they go away still talking about it much more. But I used to feel very uncomfortable with those, they used to feel that there was no conclusion to my lesson and there was nothing going for it. … I used to love to get to the end. … I'm much more comfortable now about just leaving up in the air. (Group interview, June 2000)

Ursula described herself as having working with two competing and contradictory approaches within TM lessons: one, as in Roofs, where she was looking for closure with a "really exciting answer at the end of it"; and, another, where there was no conclusion and she left the mathematics "up in the air." The first approach was comfortable, because it was closer to the norm in school mathematics, and to her existing practices of teaching, which, although investigative, nevertheless sought closure. The latter approach had been *very* uncomfortable in part because it was so different to her existing practices. The "up in the air-ness", the very thing that was attractive, was also painful. This discomfort was increased by the way in which Ursula constructed this new identity as deviant to her own ordinary practices in school mathematics. Despite this pain, it is evident from the earlier comment that she found this new approach attractive. Indeed, I suggest this attraction stemmed in part from the way in which she could only glimpse these new ideas. This glimpsing is itself both painful, because of the uncertainty and unpredictability, and attractive, because the unpredictability is interesting. A key feature here is that the desire is for reconciliation in order to understand and overcome the unpredictability.

The use of the strong emotive term of "love" in these extracts is of further significance. It points beyond Ursula's uncertainty and suggests that she herself held competing beliefs in relation to mathematical authority. In the first description, I suggest that she was making a strong statement about her identity as a mathematics teacher. Within the constraints and affordances of the past and present, an individual can "explore, take risks and create unlikely connections." (Wenger, 1998, p. 185) Indeed, an individual's identity, and ultimately legitimacy, within a community depends not simply on their acceptance by the community, but on the individual's identification with it. In expressing "love" for her image of the chaotic and different practice of CAME, a practice which as a newcomer she could only imagine, Ursula was articulating a desire not only for this different way of teaching but also to be a different teacher herself. Yet, because of the difference to her ordinary practices, this different way of being could only be imagined and partially realised. It is interesting that in the later quote Ursula emphasised her changed beliefs by placing her desire firmly in the past, using the emotive “love” to describe her previous practice of looking for a clear end result. At the time of this interview, towards the end of her involvement in the project and as she was beginning to apply for primary management posts, she appears to have achieved a degree of closure on her mathematical desire.

DISCUSSION

I use the term desire deliberately to emphasise not just a personal and emotional investment in professional change but also a compulsion to change. Ursula found the possibility of change deeply attractive in terms of her teaching, despite the difficulties and confusion she experienced throughout her involvement. She seemed to be driven to engage with CAME, a drive she expressed as love. Thus, I suggest, she experienced what Lacan calls “jouissance … which simultaneously attracts and repels.” (Zizek quoted in Brown, Hardy, & Wilson, 1993, p. 14) Ursula’s motivation to change was not simply that she perceived the need; it was rather that she was compelled to change through this powerful emotive and motivating force of desire. Through this, Ursula was able to develop the more rounded emotional relationship with mathematics evident in the later interview.

The professional development experience described here was unusually intense and extended. However, my analysis suggests that the crucial factor in Ursula’s professional change was the richness, quality and affective nature of her experience. It is noteworthy that Ursula’s mathematical desire appeared to pre-date her participation in Primary CAME. It seemed to originate from her participation on a 20 days mathematics course, on which she was able to reflect on her own negative and painful experiences of school mathematics. In particular, she began to challenge external authority figures in the form of the course tutor and through her engagement with him to construct a more positive image of mathematics.

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YEAR 10 STUDENTS' PROOFS OF A STATEMENT IN NUMBER/ALGEBRA AND THEIR RESPONSES TO RELATED MULTIPLE CHOICE ITEMS: LONGITUDINAL AND CROSS-SECTIONAL COMPARISONS

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We found, in two separate studies (1996 and 2002), that high attaining Year 10 students in English schools tend to produce empirical proofs, though many of them seem able to appreciate some of the qualities of more powerful proofs. Students rate algebraic proofs highly, often for superficial reasons, though we found that in the second, longitudinal, study they were more discriminating in Year 10 than they had been in Year 9.

INTRODUCTION

Proof, where it involves deductive reasoning based on general relationships, distinguishes mathematics from science and from argumentation in daily life, where reasoning is more usually based on experimental evidence or analogy.

Our work (eg, Healy and Hoyles, 2000; Küchemann and Hoyles, 2001) suggests that even when school students are able to appreciate the qualities of a mathematical proof, their own explanations may be low in insight and instead consist mainly of empirical support for the statement they are trying to prove. It is possible to find abundant evidence (eg Bell, 1976; Balacheff, 1988; Coe and Ruthven, 1994) of school students having difficulty in providing mathematical explanations and who seem to adopt proof schemes that are empirical or external (Harel and Sowder, 1998) rather than involving general mathematical relationships - ie who at best use what Bills and Rowland (1999) call ‘empirical’ rather than ‘structural’ generalisations. There are also studies to suggest that some students, having learnt a mathematical procedure, may show little interested in why it works (eg Hiebert and Wearne, 1988). On the other hand, even young children seem able to engage in sophisticated forms of explanation and justification, given a classroom culture with appropriate sociomathematical norms (see eg Yackel, 2001).

THE STUDY

In this paper we look particularly at responses to two questions (A3 and HA4) which were devised by Healy and Hoyles (ibid) and which formed part of a written test that they gave to 2459 high attaining Year 10 students in 1996. The same questions were given to a similar sample (N = 1512) of high attaining Year 10 students in 2002, in research undertaken by the authors for the Longitudinal Proof Project, which ran from 1999 to 2003. The aim was to look for similarities and contrasts in patterns of student response.

Students' proof choices

Question A3 had a multiple choice format (see Figure 1, below). Students were presented with various 'proofs' of the statement "When you add any 2 even numbers, your answer is always even" and were asked to choose the proof which was nearest to their *own approach* and which would get the *best mark* from their teacher. In 2002 students were also asked which proof they *liked best*.

Aysha, Brian, Coby, Deon, Eric and Fiona were trying to prove whether the following statement is true or false:

When you add any 2 even numbers, your answer is always even.

Aysha's answer

a is any whole number.

b is any whole number.

$2a$ and $2b$ are any two even numbers.

$2a + 2b = 2(a + b)$.

So Aysha says it's true

Brian's answer

$2 + 2 = 4$ $\quad$ $4 + 2 = 6$

$2 + 4 = 6$ $\quad$ $4 + 4 = 8$

$2 + 6 = 8$ $\quad$ $4 + 6 = 10$

So Brian says it's true

Coby's answer

Even numbers are numbers that can be divided by 2. When you add numbers with a common factor, 2 in this case, the answer will have the same common factor.

So Coby says it's true

Deon's answer

Even numbers end in 0, 2, 4, 6 or 8. When you add any two of these the answer will still end in 0, 2, 4, 6 or 8.

So Deon says it's true

Eric's answer

Let x = any whole number, y = any whole number.

$x + y = z$

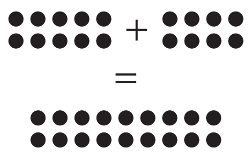
$z - x = y$

$z - y = x$

$z + z - (x + y) = x + y = 2z$

So Eric says it's true

Fiona's answer



So Fiona says it's true

a) Whose answer do you like best? . . . .

b) Whose answer is closest to what you would do? . . . .

c) Whose answer would get the best mark from your teacher? . . . .

Fig 1: Question A3 (2002 version)

In HA4, students were asked to produce a proof for a similar statement to the one in A3 (this time concerning odd numbers). It was placed immediately after A3 on the test, in the belief that the options in A3 might help students devise a proof in HA4.

Option A in A3 is a ‘structural’ proof, expressed in algebraic form. Option B is empirical, based on just 6 examples (albeit fairly systematic ones). C is structural, like A, but expressed in narrative form. D is an exhaustive proof. It says something about the properties of all even numbers (namely, that in our number system they happen to end in 0, 2, 4, etc), but is essentially empirical rather than structural. It describes how even numbers behave, but not why. E is a pseudo or nonsense proof but, like A, is expressed in algebraic form. Option F was intended to be a structural proof, like A and C, but expressed ‘visually’, with sets of dots representing generic examples of even numbers. However, in retrospect the option is perhaps too cryptic, since the sets of dots can easily be interpreted as representing specific even numbers, making it an empirical proof. In the event, F was not a popular choice, perhaps because of this ambiguity, and we do not discuss it further in this paper.

Options A, C and D are all valid proofs of the given statement, in that they verify that the statement is true. However, A and C might be thought to be more satisfying (and educationally more useful) in that they also illuminate the statement, ie explain why it is true. Option B confirms the truth of the statement, but does not prove it, while E is nonsense.

The frequencies of the Year 10 students’ choices in 2002 are shown in Table 1, below. The table lists the six options, in decreasing rank order of popularity, for the three criteria of *like best*, *own approach*, and *best mark*. As is immediately apparent, there are some dramatic changes in order for the different criteria.

	Year 10 choices for A3		LIKE best		OWN approach		BEST mark	
A	ALGEBRA-structure		D	35%	B	41%	E	38%
B	EMPIRICAL-6 examples		B	17%	D	29%	A	24%
C	NARRATIVE-structure		C	17%	A	13%	C	20%
D	EMPIRICAL-exhaustive		A	13%	C	9%	D	9%
E	ALGEBRA-nonsense		F	10%	F	3%	B	3%
F	VISUAL-structure		E	6%	E	3%	F	1%
c9	miscellaneous		c9	2%	c9	3%	c9	5%

Table 1: Y10 students’ choice frequencies for A3 in 2002 (N = 1512)

Looking first at the algebraic proofs, few students seem to *like* them, perhaps because they find them difficult (A, 13%) or impossible (E, 6%) to understand; even fewer claim that they are close to their *own approach*, perhaps for the same reasons; however, they are the two most popular choices for best mark, with option E (38%), which is the more algebraic-looking of the two, even more popular than A (24%). This latter result is perhaps not surprising since in the popular imagination high powered maths is commonly equated with algebra.

As far as the empirical proofs are concerned (B and D), these are the two most popular choices for *like best* and for *own approach*, perhaps in large measure because they are relatively easy to understand. Interestingly, D is the most popular choice for *like best* (35% compared to 17% for B) and B the most popular for *own approach* (41% compared to 29% for D). This suggests that many students can appreciate that D is a powerful proof, but admit that B is closer to their own approach, even though it is more limited. When it comes to *best mark*, these two proofs are ranked very low, perhaps in part because they are not algebraic, but perhaps also because students recognise their limitations (namely that D is not general and B is not illuminating).

Finally, many students seemed able to appreciate the structural quality of proof C: though few chose it for *own approach* (9%), a substantial minority chose it for *like best* (17%) and also for *best mark* (20%) despite it being in narrative rather than algebraic form.

In the Longitudinal Proof Project, the students were given a similar question to A3 in Years 8 and 9. However, there were only 4 options in Year 8, non of which were algebraic, and only 5 options in Year 9. Also, the content, though always involving number/algebra, changed from year to year. Thus it is not possible to make simple longitudinal comparisons, although one can discern some trends. For example, there are some interesting changes in the *best mark* frequencies for each year's narrative-structural proof. In Year 9, this proof has a frequency of just 6% and is swamped by the two algebra proofs (48% and 28%), despite its strong showing in Year 8 (53%). However, its popularity increases again in Year 10 (20%), suggesting that the students are beginning to judge algebraic proofs more critically.

Comparisons with 1996

The version of A3 used with Year 10 students in 2002 was the same as the one used in 1996 with Year 10 students in the predecessor project, except for the addition of the *like best* criterion in the later version. In both cases the sample consisted of students in top sets from randomly selected schools, and though nothing further was undertaken to produce comparable samples, the response frequencies shown in Table 2 below suggest that the samples were in fact remarkably similar.

Criterion for choice	Distribution of choices: algebra (A3)											
	D empirical-exhaustive		B empirical 6 examples		C narrative-structure		F visual-structure		A algebra-structure		E algebra-nonsense	
	1996 %	2002 %	1996 %	2002 %	1996 %	2002 %	1996 %	2002 %	1996 %	2002 %	1996 %	2002 %
like best		35		17		17		10		13		6
own approach	29	29	24	41	17	9	16	3	12	13	2	3
best mark	7	9	3	3	18	20	9	1	22	24	41	38

Note: Underlined frequencies are 'substantially' higher than their other-year counterparts

Table 2: Y10 students' choice frequencies for A3 in 1996 (N = 2459) and 2002 (N = 1512)

The *like best* criterion was added because we had noticed that without it (as for example in our Year 8 version of A3), there seemed to be a tendency, especially amongst boys (Küchemann and Hoyles, ibid), to choose an option that they liked for *own approach*, rather than one that was genuinely similar to what they would have constructed themselves. It is a moot point whether one should 'improve' questions in this way, as it makes comparisons more difficult - and it renders a detailed discussion of the frequencies in Table 2 beyond the scope of this short paper. However, it is interesting to note the large increase in the *own approach* frequency for the empirical proof B, which perhaps indicates a growth in a 'pragmatic', data-generating approach to mathematics.

Students' constructive proofs

Question HA4, which asked for a proof of the statement "When you add any 2 odd numbers, your answer is always even", appeared immediately after A3 on the 1996 and 2002 written tests. Table 3 gives an indication of the *type* (but not the quality) of proof that students constructed in 2002. The 1996 frequencies are broadly similar. What is immediately apparent is the popularity of the purely empirical (31%) and empirical-exhaustive (19%) approaches, which chimes with the *own approach* frequencies for A3. At the same time, a sizeable proportion of students (17%) embarked on narrative proofs in which the structure of odd numbers is described effectively. Very few students, though, attempted an algebraic proof, which again echoes the *own approach* (and *like best)* frequencies for the algebra proofs in A3.

HA4: Proof types

EMPIRICAL	1 example	2%		
	several examples	15%		
	'crucial' example	14%	30%	31%
EMPIRICAL-EXHAUSTIVE	Odds end in 1,3,5,7,9	13%		
	fairly exhaustive	3%		
	very exhaustive	4%	7%	19%
ALGEBRA	NO structure	3%		
	PARTIAL structure: a=even, a+1=odd	2%		
	FULL structure: 2n+1 = odd	3%		
NARRATIVE	NO structure	1%		
	PARTIAL structure: 'up in 2s'	1%		
	FULL structure: odd = even plus 1	17%	18%	
VISUAL	NO structure	0%		
	PARTIAL structure	0%		
	FULL structure: OOOO OOOOO = odd	6%	6%	
Other		16%		

Table 3: Frequency of proof types of Y10 students' constructive proofs in 2002 (N = 1512)

CONCLUSION

Evidence from the *Longitudinal Proof Project*, and its predecessor, suggests that even high attaining students in English schools have a strong propensity to construct empirical rather than structural proofs. At the same time, many students seem able to appreciate some of the qualities of more powerful proofs, even if they cannot, or do not attempt to, construct such proofs themselves. This suggests that carefully designed teaching which helps students evaluate and characterise different kinds of proofs could have a marked impact on the quality of students' explanations, provided it is sustained and built upon over time. We are currently exploring ways of doing this in our new DfES-funded project, *Developing Research-Informed Materials in Mathematical Reasoning for Teachers.*

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A MATHEMATICIAN GOES TO THE MOVIES

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In recent years there have been several films featuring a mathematician as the central character. In this article I focus on four of these: A Beautiful Mind, Enigma, Pi and Good Will Hunting. I offer my own analysis of the films, and make connections to the teaching and learning of mathematics. In particular, I argue that the films create gendered pictures of what being a mathematician and doing mathematics mean, and that these pictures have powerful impacts on the ways in which learners' construct their relationship with the subject.

This exploration has several starting points. My increasingly hybrid identity is one of those as are the pleasures I find in films and other forms of popular culture, and my desires to make explicit some of what I have learnt through my interaction with these media and to explore what these might mean for the teaching and learning of mathematics. The quotes below capture some of these sentiments:

> Trained in algebra and analysis, I identify professionally as a teacher of mathematics. I have applied for a visa for an extended stay in the permeable territories of sociology-as a resident alien or a cross-specific hybrid, naturally. But my real home is the ferociously material and imaginary zones of popular culture, into which I and hundreds of others have been interpellated. (personalised from Donna Haraway, 1997, p. 49)

> We practice culture criticism and feel the fun and excitement of learning in relation to living regular life, of using everything we already know to know more. (bell hooks, 1994, p. 2)

> If media fictions are part and parcel of the living of life in the present, these need to be explored as one aspect in which the fictions and fantasies of the subject are constituted through, or in relation to, the regimes of deeply interdiscursive meaning through which subjects understand themselves and others. (Lisa Blackman & Valerie Walkerdine, 2001, p. 196)

> [My] concern is to provoke others to ponder the role of the school in the 'age of desire', and to consider what all this means for the nature and purposes of contemporary schooling. Indeed, [I] ask readers to contemplate the purposes of schooling if the distinctions between education, advertising and entertainment diminish. (Jane Kenway & Elizabeth Bullen, 2001, p. 7)

Theoretically, sociological work on the media has been torn between top-down approaches that look at media texts independent of how people engage with them and bottom-up approaches based exclusively on how people respond to them. Many people, myself included, are interested in engaging in both of these aspects. I have been very influenced by studies that see young people as active 'readers' of cultural texts, such as work by David Buckingham (1993) and Bronwyn Davies (1989), but which also understand that such texts place important constraints on the kind of work that young 'readers' can do with them.

Yet another starting point for this research was the data I collected during my doctoral research exploring how people come to choose to study mathematics at 16+. The ideas in this paper came out of my attempts to make sense of what the 43 participants said to me in their interviews and how they formed their relationships with mathematics. Key features of their accounts that the analyses that follow help me to understand are:

- The oppositional construction of mathematics: for example, they spoke of the subject as objective, rule-based and ordered as opposed to subjective, creative and emotional. Subjects such as languages, arts and humanities sat on the other side of the divide.
- The oppositional construction of mathematicians: most participants divided the population into maths people and non-maths people. The former were variously depicted as socially incompetent 'nerds' and as active problem-solvers.
- The gendering of identification with mathematics and as mathematicians (6 out of the 24 boys interviewed strongly identified with the subject compared to none of the 19 girls).

DOMINANT REPRESENTATIONS

The dominant discourse around mathematicians in popular culture depicts them as boring, obsessed with the irrelevant, socially incompetent, male, and unsuccessfully heterosexual. Even a 'quality' newspaper described the two mathematicians who solved a puzzle, earning a £1,000,000 prize, as posing "for pictures resplendent in patterned jumpers and sensible haircuts, seem[ing] to typify a certain academic type renowned-to put it diplomatically-more for their fluency with numbers than for their acquaintance with the cutting edge of dance music" (Oliver Burkeman, 2000). Such figures are closely related to computer 'nerds'/hackers who:

> Are invariably male, usually in their late adolescence or early adulthood, …are typically portrayed as social misfits and spectacularly physically unattractive: wearing thick, unflattering spectacles, overweight, pale, pimply skin, poor fashion sense. Their bodies are soft, not hard from too much physical inactivity and junk food…According to the mythology, computer nerds turned to computing as an obsession because of their lack of social graces and physical unattractiveness. Due to their isolation from the 'real' world they have become even more cut off from society. (Lupton, 1995, p. 102)

There is an opposition between the softness of their bodies and the 'hardness' of the mathematics they do. Similarly, Deborah Lupton (1995, p. 103) points out the stark contrast between the body of the 'nerd' and the "rationalized, contained body of the masculine cyborg".

However, in popular culture, in addition to the other-ing of mathematicians as 'nerds' there is the other-ing of mathematicians through their idealisation as adventurers and as geniuses.

HEROES AND GENIUSES

'The mathematical genius' was at the centre of the recent films *A Beautiful Mind* (Goldsman, 2001), *Enigma* (Stoppard, 2001), *Good Will Hunting* (Affleck & Damon, 1997) and *Pi* (Aronofsky, 1998). Celluloid presentations of mathematicians largely avoid the 'fact' of the mathematics and bring the stereotypes discussed above into play in covert and overt ways. The plots of these films interweave conventional storylines, for example, of generational change, finding love and espionage and counter-espionage, with narratives that depend on mathematics. I focus here on two such mathematical narrative strands each of which is central to three of the films: tales telling of quests for rationality and those depicting the costs of that same rationality.

In *A Beautiful Mind* and *Enigma*, the love stories are central. In both films the main characters start with the social unease of the 'nerd' and end as heterosexual heroes. In *A Beautiful Mind*, based loosely on Sylvia Nasar's (2001) biography of the mathematician John Nash, our hero conquers his mental illness and wins the Nobel Prize. In *Enigma*, set in Bletchley during World War II, he uses his mathematical skills to triumph over German codes and his action hero skills to triumph over British spies. In both films our hero gets the girl; *Enigma* includes no reference to Alan Turing the gay real life hero of Bletchley and *A Beautiful Mind* leaves out John Nash's bisexuality, first family and marital problems. The images they present of mathematicians are flattering. Mathematicians are puzzlers/problem solvers, active, independent thinkers; they follow their own road and triumph in the end. These are stories of masculinity, of separated rather than connected ways of relating to the world (Carol Gilligan, 1993), of the love of a good woman, and, above all, of the determined pursuit of a quest.

Quests, with the exception of Jo Boaler's (1997) appropriation of the word to describe girls' mathematical activity as a 'quest for understanding', are usually discursively constructed as masculine enterprises from *Lord of the Rings* to *To the Lighthouse*. In the latter book Virginia Woolf uses an interesting metaphor for Mr. Ramsay's philosophical progress. She describes how he uses his "splendid mind" (Woolf, 1994, p. 57) to range across all the letters from A to Q one by one, but he cannot reach R:

> Qualities that in a desolate expedition across the icy solitudes of the Polar region would have made him the leader, the guide, the counsellor, whose temper, neither sanguine, nor despondent, surveys with equanimity what is to be and faces it, came to his help again. R-…Feelings that would not have disgraced a leader who, now that the snow has begun to fall and the mountain-top is covered in mist, knows that he must lay himself down and die before morning comes, stole upon him, paling the colour of his eyes, giving him, even in the two minutes of his turn on the terrace, the bleached look of withered old age. Yet

> he would not die lying down; he would find some crag of rock, and there, his eyes fixed on the storm, trying to the end to pierce the darkness, he would die standing. He would never reach R. (p. 58)

In this passage Woolf makes explicit the masculinity of Mr. Ramsay's intellectual project in the connection of mental challenges with physical ones. There is a sense here of a boys' own adventure. Woolf's writing also highlights the linearity of the imagined quest and its futility and narrowness. That the rational thought processes demanded by mathematics impose restrictions and that these have consequences is another theme of films about mathematicians.

In all four films the central mathematician has mental health problems. In *Good Will Hunting* and in *Enigma* the suggestion is that these are only indirectly related to mathematics, deriving instead primarily from experiences of childhood abuse and romantic abandonment respectively. However, in *Pi* and *A Beautiful Mind* the character's madness is directly linked to his mathematics. In both films this connection is made in the way that the process of doing mathematics is presented as individual, fevered, mysterious and intuitive. In *A Beautiful Mind*, John Nash is shown scribbling formulas on every available surface, in a state that is indistinguishable from his later insanity. His original work on game theory, the only one of his mathematical results mentioned in the film, is presented as the result of a flash of inspiration brought on in an attempt to maximise his and fellow mathematicians chances of 'success' with a group of women they encounter in a bar.

The connection between mathematics and madness is more marked and more disturbing in *Pi*, being achieved through the use of high contrast black and white film, a fast paced dance soundtrack and the rapid inter-cutting of visual sequences. The main character's unusual and reductive 'philosophy of life' is repeated in voice-over at various points in the film:

> 1: mathematics is the language of nature; 2: everything around us can be represented and understood through numbers; 3: if you graph the numbers of any system patterns emerge. Therefore there are patterns in nature.

He also repeatedly recounts details of how, as a six year-old child, he stared at the sun for a very long time:

> The doctors didn't know whether my eyes would ever heal. I was terrified alone in that darkness. Slowly daylight crept in through the bandages and I could see. But something else had changed inside of me. That day I had my first headache.

He is seen self-medicating, injecting drugs to control his headaches and seizures. The final scene comes immediately after a breakdown and depicts him as now unable to calculate and seemingly having found the inner calm denied him while he was mathematically active.

Good Will Hunting initially looks rather different from *Pi* and more like the other two mainstream films. It too is a love story, telling of a socially awkward young man who overcomes his own background of childhood physical abuse and poverty, to find his true love and true self. However, like *Pi* it is a story of the costs of rationality and of the pain of mathematics that ends with the central character, Will, abandoning the subject. Although this time leaving behind mathematics in order to "go see about a girl" is Will's choice, the internal logic of the film presents this choice as inevitable because of the nature of mathematics. It does this through a series of binaries: mind/body, separation/connection, theory /experience, reading books/living life. Mathematics is attached to the first terms in these oppositions and Will's relationship with his girlfriend Skyla is associated with the second terms. At first, Will, who lives the life of the mind absorbed in books, refuses the emotional connection and experience offered by his relationship with Skyla. He lies to her and, when they begin to get close, denies his love for her and ends the relationship. When, through counselling, he becomes ready for this emotional connection, he abandons mathematics. The idea that mathematics requires such separation is reinforced in the film for example by the actions of the professional mathematician who takes Will under his tutelage. This man relates to women only as conquests and not as partners. Since Skyla is the only female character who gets to say more than one line in the film, femininity too is associated with embodiment, connection, experiencing and living. The film presents these as more valuable ways of being but as ones that exclude mathematics.

In conclusion, these stories of mathematicians work to maintain rationality as masculine and being good at maths as a position that few men and even fewer women can occupy comfortably. Further, although they widen a little the range of behaviours that might be considered to be part of the 'mathematical personality' (to include heroism and madness along with social incompetence), they persist in constructing the mathematician as something you are or are not 'naturally'. Thus they support a key feature of the 'nerd' stereotype. So while literacy is seen as an essential part of being fully human, "in contrast to this framing, arithmetic is not naturalized as genetically human, but as *genetically determined within humans*" (Damarin, 2000, p.76, original emphasis).

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DISTRIBUTION AS EMERGENT PHENOMENON

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In this paper I present the background and intentions that have shaped the design of a microworld to study students' understanding of probability distributions. The outcome of this paper is the microworld design itself.

INTRODUCTION

My research is centered on understanding how people think about the emergent behaviour of distributions. I am fascinated by the often unexpected macrobehaviour that emerges from the interaction of thousands of autonomous agents. As a statistician, I make analogous personal connections with the emergent behaviour of distribution. As a researcher, I want to observe the reasons that lie behind people's struggling to attend to the emergent behaviour and the core notions of probability and trace their thinking. As a teacher and a researcher, I wish to help them to adopt a new way of viewing mathematics through simulation models, and see probability distribution and stochastics more generally through the lenses of emergent binoculars.

EPISTEMOLOGY OF PROBABILITY

The epistemology side of probability is marked by two easily distinguishable approaches, namely frequency-type and belief-type theories. The former place emphasis on how often events occur in order to measure chance. The latter regard chance as subjective.

This separation of probability into distinct epistemologies notoriously leads to a profound confusion and controversy which surrounds the frequency and subjective interpretations of probability. This confusion and controversy, in fact, persists miraculously in the mathematical treatment of probability for the past 300 years.

My study will build upon a new type of epistemology of probability that has emerged from the science of complexity and self-organization and is based on Wilensky's work (1997). Will emergent behaviour add to this catalogue and confuse further, or will it offer some sort of unification?

EMERGENT PHENOMENA

When numerous (micro) agents of a system dynamically interact in multiple ways, following local rules and utterly oblivious to any higher-level instructions, they can form higher - level patterns. These kinds of discernible macrobehaviour are called "emergent phenomena"- that is phenomena that emerge from parallel complex interactions between local agents. For example, prices emerge from the accumulated interactions of buyers and sellers, physical objects emerge from particles, and the transmission of a virus in a human population emerges from the interactions of individual human beings. In fact, as our appreciation of emergence advances, we see our world as dominated by emergent entities.

An understanding of complex systems is increasingly becoming a core part of scientific knowledge, the adoption of this new perspective is essential….so it is time for mathematics educators to make a crucial shift from the traditional paradigms.

Spurred by the necessity of adopting the perspective of emergence, I am seeing probability distribution as a complex personality that self organizes out of many individual decisions (data); a global order emerges out of uncoordinated local interactions over its duration or pattern; distribution emerges out of the anarchy of randomness: that is "probability distribution as emergent phenomenon".

The centralized mindset (Resnick, 1991; Resnick & Wilensky, 1993) is considered as one source of many people's deep-seated misunderstandings about the workings of patterns and emergent phenomena in the world. Resnick termed as "centralized mindset" the existing globalized tendency to have strong attachments to centralized ways of thinking that means that someone (the leader) or something explicitly creates and orchestrates the pattern. For example, people appreciate distributions in the same way and wonder how a patterned behaviour without apparent causal explanation emerges out of low level behaviour that does have explicit causal explanation.

To help people move beyond the centralized mindset, Resnick (1991) and Wilensky (1999) (also, Wilensky & Stroup, 2000) designed StarLogo and NetLogo respectively. Both are modeling complex, dynamic systems evolving over time. They allow modelers to give instructions to thousands of independent computational objects, all operating in parallel, and controlling their actions and the interactions among them as well.

Another major source of confusion lies in the failure to discriminate and move between levels (Resnick & Wilensky, 1998). Levels characterize the micro-behaviour and macrobehaviour of a complex system. According to Wilensky and Resnick(1998) the notion of levels is the a cornerstone to a new framework for developing better causal accounts of the relationships, and interactions among simple elements of various systems in the micro-level and understanding the mechanisms which underlie emergent phenomena and patterns.

One difficulty felt by people is that the relationship between micro and macro levels does not sit comfortably with the conventional view of relationships as being either hierarchical or inclusive. Another difficulty is that people's mind struggle to focus on the appropriate objects. Should they attend to the many individual interacting agents or the singular emergent pattern? Finally, there appear to be a natural or scheduled tendency to hold tightly to the deterministic mindset (Wilensky, 1997).

APPROACH

My research study adopts a Constructionist approach (Papert, 1991), which advocates the construction of knowledge in the context of constructing personally meaningful artifacts. The playful facet of constructionism is more likely to enhance learning because it is based on the fact that learners are more likely to grasp new "formal" knowledge when engaged in experiences (including verbal ones) and creating some kind of external artifact.

The major challenge for educators is to design powerful artificial environments or microworlds that would engage students in active experimentation and personal construction of knowledge. As an educator I respond to this challenge. Thus, I designed a NetLogo model for the purposes of my study. My study falls into the category of design experiments.

The purpose of design experiments is to develop specific theories about both the forms of learning and the means of supporting them, using an iterative design approach in which design, enactment, invention and revision cycle. The insights gained from each iteration feed into the next iteration.

In my research, I will use NetLogo as a platform for supporting student explorations (and studying student thinking) in high schools, in roughly two phases. In the first phase, I will present a "seed" model (a simple starting model) to the students and students play with the model in small groups and explore the parameter space of the model. I will engage them in discussion during their “game” as to what is going on, why they are observing that particular behaviour, how they can change the emergent macrobehaviour. Later I present the first iteration of the seed model. In the second phase, the group proposes an extension to the model and implements that extension in the NetLogo language.

In these activities, insights gained from the participants’ interactions with the model-setting and the “educational” intervention, both redefine my understanding of the learning issues involved.

The methodology is designed to explore whether and how: 1) students will make a link between causation and emergent distributions, and 2) the tools support students’ thinking as it moves from a micro only perspective to one which flexibly connects a macro emergent perspective to that prior micro view.

Due to the fact that we needed to bring the concept of levels into the mainstream of mathematics education, what is needed is more fine-grained research study that probe the conceptions which underlie the ways students understand (and misunderstand) emergent levels. My research will be devoted to: 1) a more fine-grained research study that probes the conceptions which underlie the ways students understand (and misunderstand) emergent levels in the case of distributions, 2) how learners can develop richer understandings of levels, and 3) how this understanding helped them to gain insight into the phenomena they were investigating.

In order to pursue these aims I realized that I needed to turn causality and distribution into something manipulable for students, a process that Pratt (1998) has called phenomenalising. My aspiration is to phenomenalise distributions in such a way that there is support for a Macro view of the features of distributions that I would like students to tune into. At the same time I would like to support the micro view of the messy randomness.

Furthermore, I intend to gain insight on students' tendency to hold tightly to the deterministic way of thinking by empowering students to view the relationship between them and perceive emergence as letting go of determinism.

The students will first experiment with the micro level where a deterministic world lies and get visual feedback for the macro level where an emergent world lies. The first stage of letting go of determinism will involve the introduction of error to the determining variables.

The students will then experiment at the Macro level where they must let go of determinism. Causal relations are now not determinable. Patterns must be explored as emergent phenomena.

BASKETBALL MODEL

I have now established the influential factor and set out the aims that underpin the first design of my microworld. Below, I describe this microworld in more detail.

Micro level project

At the micro level (Figure 1), the student is challenged to throw a ball into the basket. He can alter the basket design or the way he throws the ball.

This task directs attention of the student to causality (speed and angle of throw). Then the student can begin to let go of causality, by introducing an error factor in throws, perhaps perceived as allowing for skill level. The student is no longer completely in control and, consequently, the system is no longer completely determined.

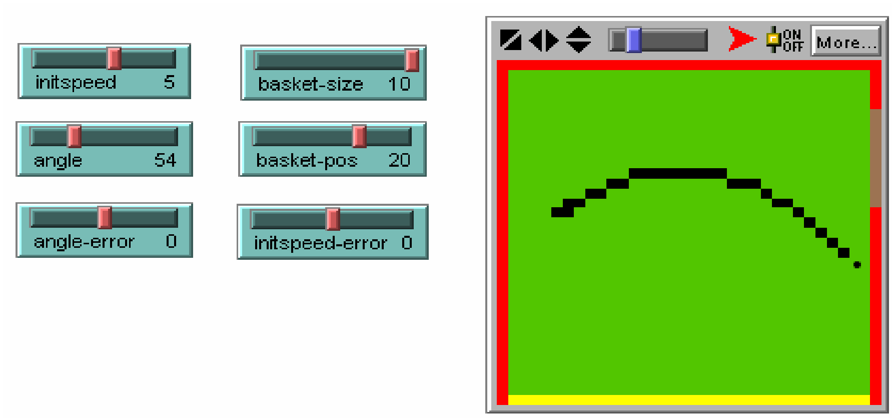

Figure 1: In the graphics window black colored balls move under gravity towards the basket. The top sliders allow the student to change the size or position of the basket. The bottom sliders enable the student to throw the ball at different speeds and angles and introduce error into those throws.

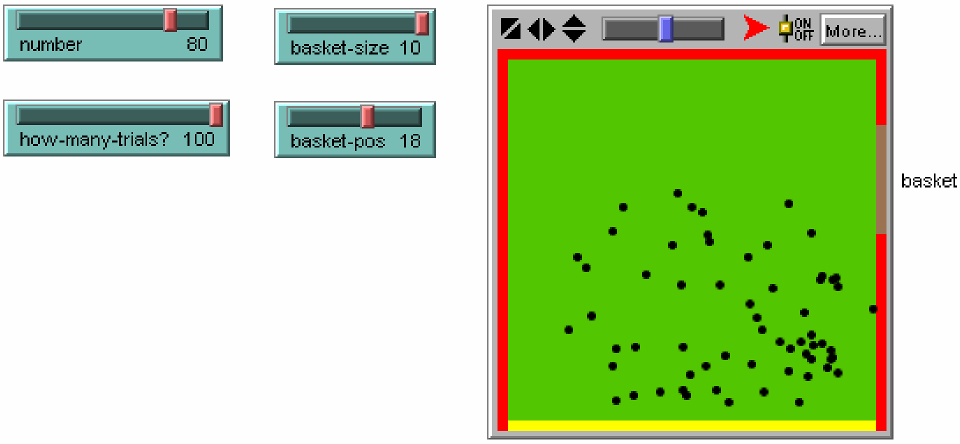

Figure 2: The top sliders again allow the student to change the size or position of the basket. The bottom sliders enable the student to decide how many balls should be thrown and how many repetitions are carried out.

Macro level project

At the Macro level (Figure 2), the student is asked to position the basket in such a way that scoring is neither too easy nor too difficult for a whole class of unknown children.

This task directs the attention of the student towards emergent distribution. The student has to let go of causality. Now, they have no control over the throws and they have to consider the distribution of the throws.

At both levels, feedback is an essential component of a simulation, because it provides the form of an output. At both micro and macro levels, the students can base their decision on various types of feedback, such as counters of goals, and three different types of graphs, namely average rate of goals per trial, a histogram of goals against position of basketball throw each, and successful-shoots against trials.

CONCLUSION

I have described the literature of emergent phenomena and the broadly constructionist approach that has let me to this particular microworld design. I shall be using this software with 16/17 year old students to probe into how students relate to the micro and macro levels and the role of causation and distribution in those two contexts.

In a future iteration I hope to find a way of integrating the two levels, which at present are presented quite separate, onto one project and, in the process, open up the software to enable students to change the model in more fundamental ways. As stated above, this is the first phase of an iterative process, part of a design experiment.

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LINKING MULTIPLE REPRESENTATIONS IN EXPLORING ITERATIONS: DOES CHANGE IN TECHNOLOGY CHANGE STUDENTS' CONJECTURES?

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This study investigates changes in conjectures of four typical students when they are using different kinds of technologies, particularly in relation to their preferences for representations and the way they express their conjectures in understanding the concept and properties of iteration. The first stage of the research was conducted using pen and paper (PP) with graphical calculator (GC) in a classroom while the second stage used PP with graphical software (GS) in a laboratory. The findings suggest, with important caveats, that different technologies significantly influence the students' preferences for representations. Also, this study shows that students' conjectures can be an effective unit of analysis in researching students' understanding of iteration and preferences for representations.

BACKGROUND AND AIMS OF THE STUDY

To be able to understand iteration, students need to successfully link the different representations involved. However, previous studies have revealed that this can be problematic (e.g. Dunham and Osborne, 1991). For example, Sierpinska (1994) has stated that 'students normally conceive that the fixed-point is the intersection of the graph and the iterative formula (p. 91).' But, many have believed that technology can influence students' understanding of mathematics and the way they link multiple representations (e.g. Elliot et al., 2000; Kaput, 1992; Kaput and Thompson, 1994; Hennessy et al., 2002). In fact, Weigand (1991) has found that students' understanding of iterations' properties is influenced by the considered representations generated by technology. Also, Keller and Hirsch (1998) have claimed that students' *preference* for representations is vital to an understanding of how they link multiple representations; that GC users preferred graphical representation; and that the technology lessened the difference in preferences when compared to non-GC users.

Aczel's (1998) study has provided evidence that investigating students' conjectures can be an effective approach to analysing their understanding of algebra concepts. Similarly, Villarreal's (2000) study has confirmed the use of this approach in investigating how students link representations or how students' preferences for representation change. She has categorised students' thinking processes as either preferring algebraic or visual approach which are neither exclusive nor disjoint.

This study aimed to investigate how, when they are using different kinds of technology, students' conjectures change in relation to– a) understanding the concept of iteration b) discovering the properties of iteration and c) their preferences for representation in understanding the concept of iteration. It is hoped that this may help to illuminate the reasons behind the difficulties that students experience in relating multiple representations in learning iterations.

THE METHODOLOGY AND THE ANALYSIS

Two A-level classes (having four and eight students) in a suburban school in England participated in the first stage of the research. The setting was conducted in a classroom environment and involved PP and the GC (TI-83)[1]. This was the students' first formal introduction to iteration at A-level, but they had some experience of rearrangement of equations. The students were asked to work in pairs in order to capture the 'students talk' relevant for analysis.

In the second stage of the research, four students from these classes, along with another student who had no experience of the materials in the first stage, used PP and the GS (*Autograph*)[2]. This second stage was conducted in a laboratory designed to capture the participants' activity by means of video and audio. Four video streams are recorded simultaneously and combined into a single stream: two video streams are of the students working on their worksheet, another video stream is of what they are doing with the mouse and keyboard, and the fourth stream is of the computer display.

Though, there are differences on how representations (i.e. graphical, numeric, and algebraic) can be represented by GC or GS, two parallel worksheets (figure 1) were carefully designed to take account of the differences of the representations that the technologies (i.e. PP, GC, and GS) can present in its interface (figure 2). The worksheets consisted of items 1) requiring procedural skills (required items to make inferences) 2) encouraging the making of inferences and 3) seeking reflection on their experience of using technology and the worksheet in the classroom. The items were also categorised for the purpose of analysis as follows: **I** – in understanding of the concept of iteration; **II** - in discovering the properties of iterations; and, **III** – students' combination of their inferences based on **I** and **II**.

These worksheets, similar in style to that of Weigand (1991) were designed to elicit students' solutions and inferences about their solutions. However, the worksheets from this study are focused more on conceptual skills and on an exploratory approach, where the items are to be connected in order to come up with meaningful conjectures.

The data collected from the worksheets are supported by techniques similar to those of Villarreal (2000) and Weigand (1991): audio transcripts based on think-aloud protocols, video data, interviews and fieldnotes. The data were validated through triangulation of the interview with the teachers and the students, and the researcher's fieldnotes. The main data collected were categorised using a coding scheme based on a number of previous empirical studies relating to how students approach graphing

[1] Texas instrument model TI-83 is graphing tool capable of producing the graphs, equation and coordinates at the same time.

[2] Autograph version 2.10 capable of simultaneously presenting all representations and doing iteration dynamically.

and the linking of different representations (Even, 1998; Ruthven, 1990; Villarreal, 2000). In investigating the students' understanding of the concept and properties of iteration, the data were analysed by sequentially and reflectively looking at the questions, the video data and the data from the two worksheets. In finding the students' preferences for representation the same process was done this time including the coding scheme.

Items	Fieldwork (GC)	Experiment (GS)
Procedural	Find the solution of x2+x-6=0; Sketch the graph using the graphical calculator; Find an iteration formula.	Find the solution of x2+2x-15=0; Sketch the graph using the graphical calculator;Find an iteration formula.
Conjecturing	What can you infer from the graph of x2+x-6=0 in relation to the graphs of y=x and the iteration formula?; What can you conclude based on your inferences?	Write down your inferences for graphing y=x, x2+2x-15, and the iteration formula; What can you conclude based on your inferences?
Difficulty/ Other	Please note down any difficulty encountered in the worksheet or in using the graphical calculator	How does your inference change when you use a computer compared to using a graphing calculator?

Figure 1: Sample items on the two worksheets (extracts)

Type	Fieldwork (GC)			Experiment (GS)		
	Item	paper	GC	Item	Paper	GS
I	A.5	A, N	V	A.3.c.1	A, N	A, N, V
II	D.1.a	A, N, V	A, N, V	A.6.c	A, N, V	A, N, V
III	A.9	A, N, V	A, N, V	A.4	A, N, V	A, N, V
Coding: V – purely visual; A – purely algebraic; N – purely numeric; AV –combined A and V; AN – A and N; VN – V and N; AVN – A, V and N; M – no answer or ambiguous						

Figure 2: Available representations offered by technology (extracts)

General patterns of inferences were considered and revealed that the participants involved in stage two were typical of those involved in the classroom-based stage-one fieldwork. The worksheet data of the selected participants in the two stages were re-analysed to compare the changes in how their preferences for representations and their understanding of iteration in terms of aspects I, II, and III change when using different technologies. Figure 3 shows the general pattern of GC participants' inferences where the four typical participants were determined whilst; figure 4 shows the changes of preferences of the GS participants (See San Diego, 2003 for the description of the coding).

Type	Item	A	V	N	AV	AN	VN	AVN	M
I	A.5	3	1		4		1	1	4
II	D.1.a	2	5	2	2		1		2
III	A.9		4		3			2	5

Figure 3: GC participants' preferences for representation (n = 14) (Extracts)

Fieldwork (using GC)						Experiment (using GS)					
Type	Item	S1	S2	S4	S5	Item	S1	S2	S3	S4	S5
I	A.5	AV	AV	M	AV	A.3.c.1	AVN	AV	V	N	AV
II	D.1.a	A	A	V	AV	A.6.c	VN	VN	N	N	VN
III	A.9	AV	AV	M	AV	A.4	AVN	AVN	AV	AV	AV

Figure 4: Changes of selected participants' preferences

LIMITATION, RESULTS AND DISCUSSION

The difficulty of identifying whether students have failed to notice something or failed to express something is recognised as a limitation of the findings of this study. The subtle differences between the two worksheets meant that comparison was problematic, but these differences could not be avoided. However, the worksheets were repeatedly trialled in order to minimise these effects.

Similarly to Aczel (1998), this has revealed that conjectures can be used as a unit of analysing students' thought processes, particularly in this study, in investigating understanding of the concept and properties of iteration and preferences for representations.

Students in this study have difficulty in explicitly expressing the connection between the fixed-point and the solution of f(x). However, this may be either associated with their difficulty in extracting information from a coordinate (Dunham and Osborne, 1991) or be influenced by the representation considered (Weigand, 1991; Keller and Hirsch, 1998; Elliott et al., 2000). The following are extracts of students' written inferences using GC and GS when asked to relate the graph of f(x) to the graphs of y=x and g(x). The corresponding related interview transcript is also presented.

Stud1: (GC) The intersection of and the iterative formula represent the intersection of the x-axis.

S1: (GS) Where the y=x and the iteration formula meet is where equation B meets the x axis.

Stud2: (GC) The intersection between… and… will give you the value of the roots.

S2: (GS) Where the y=x and the iteration formula intersect, we find the solution of the original equation, by looking at the x values of the intersects.

(Interview after using GS)

Researcher: Did your inference change?

S1: No. It didn't change! It's just faster… I've actually written down the x values.

S2: That's still the same innit?

S1: Yeah we knew that before…

S2: We knew it.

Although the study was limited in various ways, it did provide some tentative results, as in Villarreal's (2000) suggestion on inter-media coordination. Students in this study tend to get confused when solving for the iteration formula expressed in terms of x=g(x) on paper. Instead of recognising the concept of x being a variable that can be changed, students' tendency is to change the set-up in the two technologies where the default setups of the variable are y, Δx, or f(x). The computer is found to be better in this sense, since it can provide immediate feedback for any non-logical operation that students may input (audio transcripts below).

(Using GC)

GroupA,S2: How do you change y?… in the calculator?

Teacher1: You have to change the x into y. What it's asking you to do is to get the iteration formula and graph it but you have to change x to y. Got it?…

GroupB,S4: How do you change y to x?

S9: Basically y equals x… It's y equals x, you change y to x so it's y equals x.

(Using GS)

PairA,S1: y equals x. do it you're faster… x squared plus 2x minus fifteen equals zero. (The computer gave a feedback saying invalid equation entry)

S1: Oh yeah! Equals y innit?

S2: Equals zero.

S1: Are you sure? (typed). . . It's not there!

S2: What is it doing? f equals y innit? No! uhh (laughing a bit, S2 typed in y). Is that right? (The graph appeared.) Cool! (Both laughed)

It appears that in finding the solution of f(x), students prefer to solve it algebraically rather than graphically, supporting Knuth's (2000) findings. The evidence provided by the video data shows that students in this study normally prefer to check their conceived algebraic solution using the graphical calculator or the computer.

Given the limitations of this study, the implications drawn from it are deemed to be tentative. This study does not attempt to find the exact nature of the links between representations; moreover, it attempts to show the value of using conjectures as a way of researching issues concerning the understanding of maths and suggests more empirical studies are needed focusing on students' conjectures or thought processes.

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OBJECTIVES DRIVEN LESSONS IN PRIMARY SCHOOLS: CART BEFORE THE HORSE?

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Advice to share learning outcomes with pupils may be based on sound theoretical and practical principles. However, in order to turn intended outcomes into classroom experiences teachers have to draw on their interpretations of objectives and their mathematical pedagogic content knowledge. In this paper I argue that this may place non-trivial demands on primary school teachers and requires a subtle understanding of the mathematics involved.

INTRODUCTION

The introduction of the National Numeracy Strategy into virtually all primary schools in England during 1999/2000 has impacted on many aspects of teaching mathematics. Not least amongst these has been the importance attached to sharing learning objectives with pupils. The introductory section to the 'Framework for Teaching Mathematics from Reception to Year 6' (Department for Education and Employment (DfEE), 1999) emphasises that within the main teaching part of the daily dedicated mathematics lesson, teachers need to:

- make clear to the class what they will learn (Section 1. p. 14).

Although not specified in such terms, this appears to have been interpreted by many teachers as sharing with the pupils the objective(s) from the framework that lessons address (even though these objectives are couched as teaching objectives rather than learning outcomes: 'Pupils should be taught to …).

Such advice assumes either that objectives have been presented in an unambiguous way or that teachers will have no difficulties in interpreting them. In this paper I question such assumptions by examining two lesson vignettes and the ways in which the teachers' interpreted the teaching objectives and associated classroom tasks. These examples raise questions about the nature of mathematical subject knowledge needed to turn objectives into meaningful learning experiences.

DATA SOURCES

I present data from two lesson observed over the course of five years of lesson observations undertaken as part of the Leverhulme Numeracy Research Programme. (For a more detailed account of the research from which this example is drawn see Brown, 2002). Given the level of detail that the analysis yields, restrictions of space prevent the reporting on the response of more lessons. However the two lessons chosen were not a-typical of the sorts of examples identified in many of the lessons that we observed and provide 'telling cases' of some the issues that objectives driven teaching raises.

The first is a Year 4 lesson on multiplication that the teacher had planned from the ‘Framework’ and after she had attended the five-day training. The second comes from a Year 4 lesson that the teacher was ‘delivering’ from down-loaded lesson plans. Both lessons demonstrate the demands placed on teachers’ subject knowledge in taking pre-specified objectives and turning them into meaningful experiences for the children.

MAXINE’S MULTIPLICATION LESSON

At the beginning of her lesson, Maxine wrote the day’s learning objective on the board and read it out to her class of 8 and 9-year-olds:

> ‘to understand multiplication as repeated addition’.

With the whole class sitting together on a carpetted area at the front of the class, Maxine encouraged them all to count on in tens from zero. In time with the chanting Maxine pointed to divisions on a counting stick (a metre rule marked in ten segments). She then worked with the class on recording calculations such as 10 x 4 as 10 + 10 + 10 + 10 = 40 and then recording 4 + 4 + …+ 4 = 40 and which in turn was expressed as 4 x 10.

The children went off to complete one of two worksheets with multiplication calculations that they had to express in these different ways. Both worksheets were entitled ‘ x as +’ and each had exactly the same structure: a list of multiplication calculations to be re-written in three other ways. For example given a multiplication sentence such as 4 x 5 = they had to write down three other ‘equivalences’:

5 + 5 + 5 + 5 =

4 + 4 + 4 + 4 + 4 =

5 x 4 =

The two sheets were differentiated for two levels of ‘ability’; the ‘harder’ worksheet had larger numbers. For example, the ‘easier’ sheet started with ‘2 x 3 = ‘ while the ‘harder’ started with ‘7 x 8 =‘

Finally Maxine brought the class back together to discuss the work and what they had learned

Commentary

Within the pedagogic parameters of the ‘three part lesson’ that is expected in English schools as part of the National Numeracy Strategy Maxine’s lesson was typical of the majority of lessons that we observed over the latter years of the Leverhulme Numeracy Programme. The lesson had many of the recommended pedagogic elements as set out in the ‘Framework’: a shared, explicit learning objective, a whole class oral and mental ‘starter’, a main teaching part that had differentiated follow-up work for the pupils and a summarising plenary.

However, despite this fit of the lesson structure with the Strategy's prescription for 'effective teaching' our observations during the lesson of children in the class suggested that the fit between teaching and learning was less good. The children's work and their comments when questioned about it suggested that many of them engaged in activities during the course of the lesson that were not always close to leading to what the teacher might have intended as learning outcomes. For example, in the part of the lesson when the children were working individually, errors in their recording indicated that some had adopted a procedural stance towards the task. Typically, given 4 x 5 children were writing down things like:

$4 + 4 + 4 + 4 + 4 + 5 + 5 + 5 + 5 =$

or $4 + 5 + 5 + 5 + 5 =$

The task seemed, for these children, to have become 'a guessing game that is empty of mathematical meaning' (Steinbring, 1998).

Two things to note about this. Firstly, there was no explicit support to help the children make meaning of the connection between, say, 4 + 4 + 4 and 3 + 3 + 3 + 3. The use of an artefact such as an array model for multiplication, setting out tiles or counters in a 4 by 3 arrangement might have been appropriate. As it was, the counting stick as an artefact is not helpful. Trying to establish such equivalences is not easy as the actions of making three jumps of four along the stick and then making four jumps of three do not result in arriving at the same position on the stick. While the pronounced result of 12 is the same in each case, the mismatch of this with the visual image is likely to confuse children.

Secondly, while it may take longer to write out 8 + 8 + 8 + 8 + 8 + 8 + 8 = ('harder' worksheet) than it does to write 2 + 2 + 2 = ('easier' worksheet) the understanding required to do each of these is no different. The worksheets may have been differentiated by the amount of 'work' required to complete them (as measured by the amount of writing involved), but they could hardly be considered to be differentiated in terms of the cognitive demand placed upon the children. This I suggest is linked to the teacher's subject knowledge and lack of explicit awareness of how the meaning making demands of a task may be modified, as opposed to simply altering the action demands. Even amongst those children who were correctly recording what Maxine expected, many replied 'no' or shook their heads when asked if they could explain what they were doing. At best, while children could write down the required strings of symbols, any conceptual understandings of the links between the different forms of representation appeared limited.

Locating Maxine's intentions for the lesson within policy

Maxine's stated objective for the lesson –'understand multiplication as repeated addition' – appears in the 'Framework' examples for 8-year-olds within the strand 'calculations' (understanding multiplication) and under the objective that

> ‘pupils should be taught to: understand the operation of multiplication and the associated vocabulary, and that multiplication can be carried out in any order’. (DfEE, 1999, Y123 example, p 46)

There is only one example later in the Framework that further elaborates this objective.

‘Understand multiplication as:

- repeated addition: for example,

  5 added together 3 times is 5 + 5 + 5, or 3 lots of 5, or 3 times 5,
  or 5 x 3 (or 3 x 5)’ (DfEE, 1999, Y123 example, p 47, original emphasis)

As this example compresses a repeated addition into a multiplication statement, it seems reasonable to interpret the intention behind the objective as helping children come to understand multiplication as a more efficient method of calculation than repeated addition.

I now turn to examine Maxine’s interpretation of this objective. Although we can surmise something of her intent from the lesson itself, here I draw on data from an interview carried out in the afternoon following the lesson observation.

Interview extract: Maxine’s interpretation of the lesson objective

Maxine: This morning was a bit different because it’s kind of a lead up to a culmination. So this week we started looking at different strategies and today we introduced the strategy and they were trying the strategy. Trying the strategy and how to use the strategy and then tomorrow we’ll look at another strategy and doing the same. Then on Wednesday we’ll be looking at another strategy and doing the same and then Thursday and Friday we’ll be exploring problems and deciding which strategy we want to use to work them out. So it’s a bit different, you’ve got different lessons really sometimes, sometimes it is more procedural. ….

MA: And I know you’ve talked about this a moment ago, but just to clarify, talk to me about repeated addition as a strategy.

Maxine: Because a lot of the children feel insecure. As soon as they see a multiplication sign it’s like, I can’t do multiplication. But all children generally are quite secure in what they have to do now. They can count on, especially when it’s quite small stuff so using repeated addition they find it a lot easier because it’s a method, it’s something that they are confident with so they already have their strategy developed to show them how they can use something that they are already secure about to help them work out some method they are less secure about.

Commentary

From this extract, it seems clear that Maxine’s interpretation was somewhat different to the one I argue above: that if a child did not know the answer to a multiplication then they might use repeated addition to calculate the answer. She repeatedly refers to what is being taught as a ‘strategy’, possibly then associating the objective with the part of the framework that deals with ‘mental calculation strategies’. Her clear intent is that children should use addition as a means of calculating multiplications. Interestingly, although in her account of the lesson she emphasises that this means the children have a strategy that they understand, at no point in the lesson itself were the children ever required to actually calculate the multiplication; they simply had to express it in different forms.

SANDRA’S ADDITION AND SUBTRACTION LESSON

After some introductory work on multiplying by nine, Sandra began the main part of her lesson by writing up on the whiteboard the objective for the lesson.

Sandra: Now, your learning intention for today is (writing on board) add or subtract the nearest multiple of ten, then adjust.

Immediately after writing up the lesson objective and reading it with the class, Sandra asked for explication:

Sandra: What do we mean by nearest multiple of ten?

Child: To the nearest ten.

Sandra: Nearest to nine?

Child: Ten

Child: Add or subtract to xxx, then adjust

Sandra: Add or subtract nearest multiple of ten. It does make sense, you’ll see.

Sandra wrote on the board

48 + 20 =

Sandra: Forty-eight plus twenty?

Girl: Sixty eight

Sandra: Good girl. How did you work it out?

Girl: I took off the eight from the forty then added on the 20 and added the eight on again

Sandra: Good, forty plus twenty is sixty and add eight back on.

Sandra then wrote up

58 – 30 =

which also was explained by ‘taking away’ 30 from 50 and then adding back the 8.

Sandra wrote on the board

46 + 17 =

Children’s hands went up more slowly than for the previous two examples. Sandra asked for the answer from a boy whose hand was one of the first to go up.

Boy: Sixty two

Sandra: How did you do it so quickly

Boy: I added ten to forty six, then added seven

Child: Fifty six plus seven

Sandra wrote on the board

56 + 7 =

Sandra: (to boy giving 62 as answer) How did you know it was …?

Children call out that the answer should be sixty-three before he has a chance to answer.

Commentary

To understand Sandra’s examples and approach here, I need to explicate the lesson plan from which she was working.

As part of the support material for the implementation of the National Numeracy Strategy, the policy makers produced a series of ‘down-loadable’ lesson plans that took the Strategies’ objectives and provided tasks through which they might be taught. Sandra was working from one such lesson plan. The section of the plan that she was working from is in table 1

Objectives and Vocabulary	Teaching Activities
Add or subtract the nearest multiple of 10, then adjust	• Write on the board: 46 + 20 58 – 30 Collect answers and discuss methods. Amend to: 46 + 20 – 3 58 – 30 + 5 Collect answers Remind children of work in previous lesson and use of number line. Q. What single addition and subtraction do these statements represent? Establish the single calculations 46 + 17, 58 – 25. Repeat and discuss methods.

TABLE 1

Unless the reader already has a clear sense of what is expected in terms of methods of calculating here, considerable effort is required to make the connection between the teaching examples provided and the lesson objective. Firstly, the task does not start with a number that is near to a multiple of ten and then use a rounding method to carry out the addition or subtraction. It starts with adding an exact multiple of ten and then ‘amends’ this calculation so that, implicitly, a number that is near to the multiple of ten is added.

Secondly, the way in which this ‘amending’ is supposed to be developed is neither clear nor unambiguous. Given ‘46 + 20 – 3 and 58 – 30 + 5’ which the teacher is supposed to have arrived at by amending ‘46 + 20 and 58 – 30’ asking ‘which single addition or subtraction do these statements represent?’ does not produce unique answers. 46 + 20 – 3 could be expressed as 66 – 3, 43 + 20 or 46 + 17. Only in the light of the stated teaching objective of ‘add or subtract the nearest multiple of 10, then adjust’ does the answer of 46 + 17 best fit. The lesson plan then goes on to advise the teacher to ‘repeat and discuss methods’. But what exactly is to be repeated?

In the case of Sandra’s lesson, it would seem that these connections were not available to her. Note that she starts with 48 + 20 rather than 46 + 20 so the potential to link the later calculation of 46 + 17 with the one previously carried out is lost. Sandra’s interpretation of what it means to ‘establish the single calculation’ 46 + 17 is simply to present this to the children as the next one to carry out. The interplay between the tasks and the learning objective did not appear to be clearly established for Sandra

DISCUSSION

How do teachers make sense of the relationship between the specific and the general, between particular lesson tasks and intended learning outcomes? Do examples help clarify the objective or does the objective help you sort out how to interpret the examples? It is not simply a case of understanding the meaning of an objective and then selecting suitable examples. There is an interplay between the two – objectives and examples – and teachers subject matter knowledge for teaching will be central in mediating between these.

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STANDARDISATION AND INDIVIDUALISATION IN ADULT NUMERACY

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In this paper we outline the policies that have created parallels between numeracy work in schools and with adults. A 'one size fits all' pedagogical and curriculum stance has led to an adult numeracy curriculum which is very largely based on the national numeracy strategy; we discuss potentially contradictory issues in the field of adult education.

THE ADULT NUMERACY POLICY CONTEXT IN ENGLAND

Adult numeracy courses in England are now organised through the *Skills for Life* framework. This includes standards from Entry Level 1 to Level 2 (QCA, March 2000) with associated curricular, examination, teacher qualification (to Level 4) and funding arrangements – and targets (http://www.dfes.gov.uk/readwriteplus). The *Skills for Life* survey suggests that 15 million adults (47%) in England have numeracy skills below the government's target level (DfES, 2003). However, it also found that adults' own assessment of their numeracy did not match the test results. 67% of those with Entry 1 or lower level numeracy (that is, assessed as weaker in numeracy than the students whose work is discussed here) felt that they were very or fairly good at number work. The survey writers propose that many people do not realise the negative effect this has on their lives; have found jobs that demand only the appropriate level of skills; or 'have developed coping strategies so their limitations are not exposed'. Those limitations are exposed through testing. As well as increasing provision and advertising provision (for example, through the 'gremlins' TV advertising), 'conditionality' policies have been introduced. Thus the government has trialled, for example, the use of benefit restrictions and probation conditions to support 'motivation'. The government measures achievement of its targets through national tests (http://www.dfes.gov.uk/readwriteplus/learning); funding is linked to students' achievement of targets set in their individual learning plans (ILPs), which may or may not include success in examinations. There have been hints however that the national tests too may be linked to funding, to encourage education and training providers to enter their students for them.

The adult numeracy core curriculum (ANCC) is a slimmed down version of the national numeracy strategy (DfES & Basic Skills Agency, 2001a), focusing on those skills deemed to be appropriate to adults. Thus although Level 2 is equivalent, in the terms of the national qualifications framework, to the top levels of GCSE, colleges suggest at least one and often two years' study for students to progress from L2 to GCSE.

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[1] This paper also draws on work by teacher-researchers Elizabeth Baker, Mark Baxter, Debbie Holder, Eamonn Leddy, Barbara Newmarch, Liz Richards and Topo Wresniwiro.

The three day training programme to introduce the new policies to teachers strongly reflected the curriculum's roots in the national numeracy strategy, promoting lesson plans with mental and oral starters, whole class teaching and a plenary (DfES & Basic Skills Agency, 2001b). This is a new pedagogical culture for most adult numeracy teachers. Until around 1990 teachers had considerable freedom to set the curriculum (consulting students as they wished), and many courses (perhaps the majority) were organised as roll-on, roll-off, individualised programmes. From 1990 funding for the majority of provision was linked to achievement, but much teaching remained individualised.

Meanwhile there may be further changes in adult numeracy policy, with the Smith report arguing that progress in adult numeracy may be undermined by uncertainties about the teaching and assessment of mathematics in general, the limited pool of teachers and the lack of employer engagement (Smith, 2004).

One element of the sea change that has engulfed adult numeracy work is the government's establishment of the National Research and Development Centre for adult literacy and numeracy (http://www.nrdc.org.uk): there has been comparatively little research in adult numeracy but this is now shifting, with the NRDC prioritising numeracy within its programmes. The first numeracy project to be completed is a review of research (Coben & with contributions by Colwell, 2003), and the two teacher-research projects on which we draw in this paper are funded by the NRDC

We have said that the ANCC is based on the national numeracy strategy, but it is also to be geared to adults' contexts:

> [The] adult numeracy core curriculum provides the skills framework, the learner provides the context, and the teacher needs to bring them together in a learning programme using relevant materials at the appropriate level, to support learners in achieving their goals. (DfES & Basic Skills Agency, 2001a)

In the rest of this paper we look at some of the contradictions and difficulties in this view of adult numeracy work. In the terms of the Inspectorate,

> In too many colleges … the initial assessment is not being used effectively to inform the individual learning plan, and the learning targets do not match closely enough the needs, interests and aspirations of individuals. (Adult Learning Inspectorate & Office for Standards in Education, 2003; Ofsted, September 2003)

STUDENTS' CONTEXTS AND MOTIVATIONS

Two adult numeracy teacher-research projects are based at King's College London: *Making numeracy teaching meaningful to adult learners and Teaching and learning common measures, especially at entry levels*. The first is concerned with learners' identities within and outside the classroom, the relationship between learners' numerate practices and their experience of numeracy education, and teachers' ways of relating their teaching to students' contexts. The second aims to investigate the effective learning of measures, identify and trial teaching strategies and produce learning materials. Our data sources include participant observation, teacher reflection, individual and group interviews, critical comment by students on emerging data and materials (in the case of the *Measures* project), student numeracy diaries and photographs taken by students. Here we draw on interview data from both projects.

First, a snapshot of adult numeracy 'education' in a prison. Ade spoke to the researcher as he waited for others to arrive for an examination, his third in six weeks (Entry 1, Entry 2 and now Entry 3). They were getting harder and harder each week. He described the first examination as

> really easy ... I haven't had a maths class, I just started straight on exams. I put in for IT and maths - maybe after you cope with the exams they'll put you into different groups.

An informal 'go-slow' by prison officers meant prisoners were often not escorted to classes. Ade went straight to accreditation and took tests every 2-3 weeks; his 'achievement' contributed to the prison education service's achievement of its targets, and he was paid to enter national tests (£3 for Entry levels or Level 1, £5 for L2). This prison was perhaps *buying* its own achievement - an extreme example of funding for achievement leading the 'education' process, rather than students' interests or engagement. We don't suggest this is typical, and we understand that the situation in that prison has since improved. But the funding regime has an impact on numeracy in quite different contexts, too: the staff of a London FE college were directed to make sure the ILP was written in such a way that assessment of students' work against 'their' targets was straightforward and achievement was guaranteed.

Next we turn to data from the *Common Measures* project . Geraldine, a former seamstress, values new learning in metric measurement as a way to save money and prevent shop-keepers cheating her:

> When I went to buy material to do my work, the man behind the counter … was using a yard stick on metre measurements. I said, 'What are you doing?' He said, 'I'm just using my finger to give you a little bit more'. And I said, 'You are supposed to use the right measurement …You are cheating me.' And so I'm [in the classes] for them to teach me - if they don't, people easily can rob you. If I know, I can shout out and say, 'You are cheating the people in the shop' [In the supermarket] they just slap on a price on the lettuce and the cucumber and I'll stand there and weigh all of them and I take the biggest one, I get more for my money. I apply [what I learn in the class] when I go out, anytime.

Geraldine is the kind of student we believe the government has in mind. However, she is the only student we have interviewed who fits the government model. No-one has told us that measurement should be omitted from the curriculum, but their reasons for working on it are not that it is of direct practical benefit, but rather it is included in both their own examinations and their children's. Most students say they have all the measurement skills they need for everyday life outside the classroom. In some cases that's because they do little formal measurement; in others, it's because they are already highly skilled (beyond 'their level' of the curriculum). Elizabeth is in the first of those groups:

> We went over painting a room, the area of a wall and then working out how many pots of paint you'd need, but in reality you just get some paint and if it runs out you go to Homebase and get some more. …. So although the lessons are valuable I don't think I'd ever use it in my life. But I can see how a painter and decorator would, to cut down his costs, because he'd have less wastage.

Simon is a carpet layer, and has some numeracy skills well beyond his supposed level (Entry 3). He told the interviewer how to find the area of a circle:

> You square it off from the widest point. It's about seven eighths of the total area. No, four fifths, round about 80%.

He said a carpet layer needs to know 'how to charge up the biggest area you can'. The ANCC does not include work on area at Simon's level (or indeed fractions and percentages at this level); and it assumes throughout that students are buyers rather than sellers. Simon has developed the skills he needs without (until prison) going to adult numeracy courses. He has a 'spiky profile' (DfES & Basic Skills Agency, 2001b) – but we haven't met anyone with a 'flat' profile.

We turn now to the *Making Numeracy Teaching Meaningful* project. Students' motivations may be more subtle and complex than is envisaged in government discourses. They want to prove to themselves that they have the intellectual capacity; almost without exception, they want to understand the mathematical system, its principles and underlying relationships. They also want to show that they have the durability to succeed:

> I'm not really sure that I can use maths but I just want to learn it for me, it's just something that I want to achieve for myself, that I can do things. … I want to be able to have some sort of qualification that shows me that I've done that because in my life I don't think that I've done anything, apart from growing up and having two babies. (Selena)

A few students say that the mathematics they have learnt in their numeracy classes has really helped them in their lives outside the classroom. Some used to be embarrassed at their lack of mathematical knowledge and skills:

> Beryl: I tell you the most embarrassing thing is when I had to send my children to the shop, or they came with me, and I used to say to them (whispering)… Yeah, how much have I got to give them? I had to ask them and that's embarrassing for a mother, let alone an adult, asking a 7 to 8 year old how much money do I give them, how much change do I get back? I'm not so bad now, I can near enough do it but it was very embarrassing

But improving numeracy does not only have 'practical' outcomes:

> Nisha: Because maths has had the label of being hard and complicated, if a person feels like - oh I'm stupid - or anything like that, and you sit them down and get them to do an algebra problem and they realise - 'Oh wow, I can do it'. It will make a person feel really good about themselves.

Nisha shares with other students, in both projects, an interest in exploring mathematics beyond the limitations of the curriculum. Manman argues for algebra from a different perspective:

> They should teach us what we need to survive on a day to day basis. Algebra, decimals, fractions – if you leave here [prison] and you want business or you have an interview, they give you a maths test.

But … the focus of the ANCC on what are declared to be 'practical' and 'everyday' skills means it doesn't include algebra.

CONCLUSION

The 'one size fits all' curriculum fits few of the students whom we have interviewed. The notion of a 'spiky profile' suggests that somewhere there is a flat-profiled 'normal' student, but we haven't met her. The ANCC shapes accreditation, teacher training and records of work, both in the classroom and in terms of meeting government targets, and there is little doubt that the teachers and students whose work is reflected here will contribute to the success of the *Skills for Life* strategy. But students' ambitions, both for success in mathematics itself and for other goals to which numeracy is subsidiary, reach far beyond those ILPs on which their programmes of work are based, and the development of adults' numeracy skills is not easily categorised and measured in the terms adopted in government policy. We return, then, to the *Skills for Life* survey, based on the ANCC standards, and the gap between the survey's findings and self assessment. It seems that students may have both skills and aspirations which are different from those expected in the survey, the national tests or the curriculum. We noted above the Inspectorate view that too often students' aspirations are ignored. Bringing together students' aspirations and the ANCC in a learning programme is indeed a difficult task.

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NATIONAL POLICY, DEPARTMENTAL RESPONSES: THE IMPLEMENTATION OF THE MATHEMATICS STRAND OF THE KEY STAGE 3 STRATEGY

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In this article we use data from two mathematics departments within one local education authority implementing a national reform policy – the mathematics strand of the Key Stage 3 (KS3) Strategy – to explore the contrasts in the interrelationships between the views of, and goals for, mathematics teaching and learning that teachers see within the policy compared to their own views and priorities. The ways in which these contrasting interrelationships in views and goals impact upon the profile of the department in the context of policy implementation are considered.

INTRODUCTION

Evidence from a broad swathe of previous reform efforts points to the interpretation of policy into practice, rather than Evidence from a broad swathe of previous reform efforts points to the interpretation of policy into practice, rather than a more direct correspondence between the two (Bowe et al., 1992; Pollard et al., 1994; Askew, 1996). This evidence suggests that local contexts are of importance, and led to a shift in policy implementation research away from measures of 'fidelity' of implementation, towards a move to understanding the variations in response. In larger-scale studies, this shift resulted in typologies of school responses to external reform efforts (e.g.Corbett and Wilson, 1991). Smaller scale ethnographic studies pointed to the ways in which these interpretations were filtered through teachers' values and goals (Broadfoot and Osborn, 1988), and the local cultures of work that they were based in (Mac an Ghaill, 1992). Ethnographic studies focused too, on the conflicts faced by teachers as they negotiated the implementation of policies with underlying philosophies of teaching and learning that they felt stood at some distance from their own values (Hammersley, 1999).

THE MATHEMATICS STRAND OF THE KEY STAGE 3 STRATEGY

The reform policy under consideration in this article is the mathematics strand of the Key Stage 3 Strategy (referred to henceforth as the 'mathematics strand'). This policy was launched nationally in English secondary schools in September 2001. The policy was modelled closely on its primary level predecessor, the National Numeracy Strategy (NNS), which had been introduced in September 1999. The mathematics strand carried through the stress within the NNS on improving pedagogic practice and securing progression through the curriculum for students. Amongst the key features of the mathematics strand were:

- Structured 3-part lessons: starter, main activity, plenary, and a call for 'pace' in lessons.

- A curriculum written in the form of numerous specific learning objectives, set in ‘Yearly teaching programmes’, each pitched at a narrow range of National Curriculum levels, set out within the KS3 ‘Framework’ (Department for Education and Employment, 2001)
- Pedagogy – predominant use of interactive whole class teaching
- Training/support programme for KS3 maths teachers, provided by local KS3 Strategy mathematics consultants

Data from an earlier small-scale study that we were involved in tracing teachers’ views on the implementation of the pilot projects of the mathematics and English strands (Barnes *et al.*, 2003) showed some areas of common ground in implementation of the mathematics strand, but in comparison to the early stages of implementation of the NNS where a much greater degree of ‘fidelity’ was apparent, there were widespread indications that policy implementation in different schools varied in content and degree. Variations were found in the following key areas:

- Teachers’ views of the policy varied from highly positive to highly negative, although the majority of teachers stated their support for its overall aims.
- The degree to which interactive teaching was used was highly variable – in many departments, a more interactive style was restricted to the starter activity with little change in pedagogical style in the rest of the lesson.
- About half the sample within the study had changed their schemes of work to align with the curricular format and timeframes given in the draft Year 7 Framework (Department for Education and Employment, 2000); the others had either checked their own schemes for coverage of the ‘key objectives’ given in the Framework, or retained their existing schemes with no reference to the policy’s curriculum.

We used this evidence of partial and varied interpretations of the policy to examine in more detail the ways in which two departments using contrasting practices and structures for organising teaching and learning mathematics at KS3 decided to implement the mathematics strand. These two departments, located within one local education authority, came into implementing the policy through their participation in one of the fifteen KS3 Numeracy pilot projects that began in 1999 alongside the introduction of the NNS following the recommendations of the Numeracy Task Force in their Final Report (Department for Education and Employment, 1998).

In this article, we focus on the contrasting decisions taken by the heads of mathematics in the two schools on how to incorporate the policy into their departmental working – decisions that were taken at the end of their first year of participation in the Numeracy Pilot (Summer 2000), as they prepared to meet the first cohorts of students in Year 7 that would have experienced the NNS in their primary schooling.

We collected data on departments' views of the Strategy through attending the half-termly meetings of the Pilot project, taking notes of the proceedings, speaking informally with the representatives from participating departments, and collecting documentation. We also carried out interviews with the Numeracy Coordinators and Heads of department from the focal schools, and the KS3 Numeracy Consultant leading the project.

THE TWO SCHOOLS

The two schools that we focused on were 11-16 co-educational comprehensives with intakes that were negatively skewed in terms of attainment profiles at KS3 and GCSE in relation to national figures. Additionally both schools had rolls in which a little over 50% of students were eligible for free school meals.

The schools were chosen because of the contrasts in their organisation of classroom practice at KS3. In the first school, Evenscroft, teaching was based around the use of a differentiated textbook scheme – Key Maths. Apart from an initial assessment period in September of Year 7, setted grouping was in place across KS3. Whole-class teaching using the textbook scheme formed the predominant model of pedagogy. The second school, Bradstone, used SMILE – an individualised learning card scheme - across KS3, in which students were set individual programmes of work on different topics and levels. Mixed-ability grouping was in place throughout KS3. Teaching at Bradstone in KS3 consisted of a split between choosing appropriate tasks, supporting individual students with their learning, and monitoring progress. It was important to note that in both schools these respective models of organising learning were the results of decisions taken by their heads of department, both of whom were proactive about making changes in structures if they perceived these to be necessary.

VIEWS OF THE MATHEMATICS STRAND/LOCAL PRIORITIES

Different views of the mathematics strand were expressed over the course of the first year by the heads of department at the two schools in the Numeracy pilot meetings. Bradstone's use of individualised learning clearly conflicted with the mathematics strand's advocacy of whole class teaching within the *'recommended approach to teaching'* (KS3 Framework, p.26). Many of the video exemplars of pedagogic practice and curricular frameworks which were used to focus discussion within these meetings were based on a whole-class teaching model, and therefore restricted Bradstone's opportunities to participate whilst also being of limited relevance to the school in terms of helping them to improve existing practices.

There were also clear differences in the priorities of the two heads of departments in terms of what they felt needed changing. Beena Charan, the head of department at Evenscroft repeatedly expressed her dissatisfaction with inactive teaching:

> "I think you know, the kind of teacher who says 'Right, page whatever, questions 1-20' and then just sits at the desk for the rest of the lesson."

In relation to this priority for change, she viewed the mathematics strand in very positive terms, seeing within the policy some levers on pedagogy that would help to effect moves to a more active teaching style:

> "I think staff have to plan their lessons a lot more as well. You can't just go into a lesson and say 'Right, Page 53 of the textbook, Questions 1-20' because it doesn't work any more. You've got to plan your lessons and you've got to think of an oral and mental starter and a plenary and what you are going to do in the middle. So it breaks up the lesson a lot more as well."

There was therefore, a high degree of match between personal priorities and levers within the reform policy that could help to achieve these objectives, levers that were applicable to her local context of practice because of their use of whole class teaching, to effect changes that she viewed as beneficial.

Beena's formal incorporation of the policy in Summer 2000 consisted of writing sets of mental and oral starters into each unit of work in their Year 7 scheme, buying resources that supported the move to more interactive teaching styles such as pupil white boards and loop cards, inviting the Consultant in to talk to her department about the use of three-part lessons, and then supporting and monitoring this use across the department. She retained their existing schemes of work at KS3 in an unchanged format:

> "We've kept the order, but we've fitted the National Numeracy Strategy objectives, you know the key objectives to **our** scheme of work rather than the other way around. We cover all the objectives but not in the order they say because I don't think that's important." (original emphasis)

The policy model of differentiation at three levels was also not viewed as an important priority:

> "I think differentiation – that has been on board for years, hasn't it, and I think people have worked out their own strategies for dealing with it."

Her enthusiasm for the policy therefore, was quite selective, and restricted to the aspects that served her local priorities for improvement.

Diana Norton, the head of department at Bradstone did not see any such congruence of goals within the mathematics strand. She commented that on the fact that whilst the numeracy focus and the notion of building through from primary school practices were useful, the extension of the degree of prescription given within the NNS was inappropriate for secondary teachers:

> "We **are** specialists, and it [the maths strand] doesn't particularly treat us that way. It attempts to tell us **what** to teach, **how** to teach, **when** to teach it. Some of those things will have benefits, but some of them are just far too restrictive." (original emphases)

Her departmental priorities at that stage were focused on KS4 and the changes that had been made to the GCSE syllabus, but she acknowledged that given the announcement of the national rollout of the KS3 Strategy in the following year, changes to KS3 were inevitable. Her aims focused on finding a response that acknowledged their participation in the Numeracy pilot whilst retaining the aspects of their organisation that she felt were important – the use of SMILE and mixed ability grouping - and simultaneously addressed her local priority of wanting to improve the conditions for learning in classrooms. The problem that she focused on as needing improvement was the level of movement around classrooms by students looking for tasks:

> "It's in terms of cutting down the interaction between the students, and the necessity to find cards and find equipment."

Her solution was to modularize their use of SMILE, with the modules created on the basis of the topics and timeframes given within the sample termly plans detailed within the draft Year 7 Framework version available at that time. This structure meant that unlike before, students within classes would now be working on the same set of topics. Information and supporting resources for mental starter activities had been disseminated earlier in the year, but Diana did not provide any guidelines or engage in discussion on how to teach within the modified SMILE structure.

Diana's incorporation too therefore, whilst driven initially by a desire to comply with policy visibly in some way, was still focused firmly on local priorities for improvement, and selective about the aspects of policy that she chose to accommodate.

Interviews with the teachers in both departments towards the end of the first year of implementation (Summer 2001) indicated that they were positive about the ways in which their respective models of policy incorporation had impacted on students.

THE CONSULTANT'S VIEW

Keely Horsham, the local KS3 Numeracy Consultant, had considerable experience of working within mathematics education in teaching, management and advisory roles. Her view towards the end of the first year of implementation (Easter 2001) reflected the views that I had seen developing in the Pilot meetings over the previous two years. She praised Beena's clarity of vision and her ability to channel the resources available into securing the improvements that she wanted, and commented too on the raised profile that had been secured as a result of her willingness to instigate changes in departmental practice:

> "They [Evenscroft] are always at the forefront of everything"

Whilst the two schools had begun their participation in the Numeracy Pilot at similar positions in relation to student attainment at KS3, Evenscroft's enthusiastic response to the pedagogical aspects of the policy and the relevance of these aspects to their existing model of teaching appeared to have conferred higher status and profile than Bradstone's more low-key incorporation of the policy's curricular format.

DISCUSSION

It was important to observe in this study and in our earlier findings that positive and negative views of the policy were both associated with selective implementation. Beena's positive view of the policy did not lead to a more wide-ranging implementation. The high profile she achieved appeared to link to the acceptance of the backcloth of whole-class teaching and a congruence of goals relating to improving pedagogic practice. This allowed for the constructive use of the tools offered within the policy – three part lessons, interactive resources, use of objectives. Bradstone's incorporation of mental starters and Diana's use of the Framework's curriculum as the basis for their Year 7 scheme did not confer this kind of status – the retention of SMILE and the department's unwillingness to embrace whole-class teaching within the modified structure (although they used whole class teaching in KS4), continued to hamper the degree to which teachers could contribute or gain from discussions related to the policy.

Hargreaves (2003) and Fullan (2003) have criticized the narrow agenda associated with many reform policies. The mathematics strand in its textual form, and in the range of aspects covered in the meetings of the Numeracy Pilot in this study, appeared to have considerable breadth, with discussions ranging across curricular, pedagogical, assessment and management issues. Many of these discussions though, were predicated on the use of whole class teaching, and from Bradstone's perspective, Fullan's concerns about policy agendas were very real:

> "a form of performance training that provides intensive support but only in relation to highly prescriptive interventions" (p.7)

The data presented in this article suggests further that the implementation of the mathematics strand relates more to prescribing pedagogy than other aspects of teaching and learning. Beena's attention was directed almost exclusively at improving teaching; Diana referred almost exclusively to ways to improve learning. Beena's views pointed to an underlying sense that learning could and should be directed, at teacher and student levels – a view reflected in the policy texts; Diana viewed learning as a much more autonomous and individual process, again at teacher and student levels – a view at odds with the directions of the policy. Philosophies and goals also then, appeared to contribute to the differences in profile that ensued.

Recent reviews of the implementation of the mathematics strand (Ofsted, 2004), whilst stressing that implementation of the policy has led to improvements in mathematics teaching, have underlined the dangers of an over-emphasis on teaching without an adequate emphasis on learning:

> "However, in some schools, pupils are over-dependent on teachers and there is insufficient emphasis on using independent, collaborative and oral work to encourage pupils to grapple with ideas." (p.23)

The data presented here leaves questions as to whether the legacy of the mathematics strand will be a shift to more teacher-directed pedagogic styles, with less room for the more independent kinds of learning that Diana wishes to encourage.

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