TAKING MEASURES TO COORDINATE MOVEMENTS: UNITIZING EMERGES AS A MEANS OF BUILDING EVENT STRUCTURES FOR ENACTING PROPORTION

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Rhythm is a means of production—a scheme for coordinating the enactment of real or imagined physical movements over time, space, material resources, and concerting participants. In activities requiring the coordination of two or more continuous *motor actions, rhythmic re-assembly of the actions creates a goal event structure mediating the enactment. Yet building that structure requires first unitizing continuity. Unitizing could thus be conceptualized as a cultural–historical strategy for supporting mundane routines by parsing, distributing, and codifying activity as a sequence of iterated actions of equivalent magnitude. Ipso facto, unitizing shifts us from naive to disciplinary activity: articulated rhythm is an ontogenetic achievement driving cognitive growth. We present empirical data of a student spontaneously measuring continuous actions as her means of organizing the enactment of a bimanual task designed for proportions.*

Keywords: Cognition, Measurement

Introduction

Mathematics learning activities designed in accordance with embodiment theories of cognition create opportunities for students to engage in the solution of sensorimotor problems prior to interpreting and representing their solutions formally in normative symbolic register (Lee, 2015). This heuristic design principle is grounded in a constructivist (Boom, 2009) and enactivist (Reid, 2014) consensus that concepts emerge through noticing repeated patterns in perceptual dynamics guiding motor action. Empirical work has corroborated this historical conjecture through combining eye-tracking and clinical data analysis (Abrahamson et al., 2016).

The objective of this paper is to contribute to scholarship on students' passage from sensorimotor action to mathematical reasoning. Our study's empirical context is an action-based embodied design, the Mathematical Imagery Trainer (Abrahamson, 2014), wherein participant students enact a challenging bimanual motor action related to the development of proportional reasoning. By micro-analyzing students' behaviors, we demonstrate spontaneous utilization of measurement units. We argue that emergent rhythmic enactment facilitated discretization that led to evoking measurement units and in turn building an event structure mediating the enactment.

Theoretical Background

Cultural–historical positions view the practice of measuring, along with its artifacts, routines, and discourse, as evolved to serve an essential means of mathematizing human action and thought (Malafouris, 2013). In particular, measure units enhance one's ability to estimate, compare, and calculate continuous quantities (Stavy & Babai, 2016). Cognitive-developmental psychology defines measuring as follows: "To measure is to take out of a whole one element, taken as a unit, and to transpose this unit on the remainder of a whole: measurement is therefore a synthesis of sub-division and change of position" (Piaget, Inhelder, & Szerninska, 1960, p. 3). Measuring competently thus requires: (a) conserving the size of the unit; (b) iterating the unit; and (c) transitively, inferring the relative length of two objects by comparing them to a unit. When we imbue these measurement routines with the temporal dimension, we can discern the enactment of rhythmic actions. Indeed Radford (2015) found structured temporal qualities in analyzing students' performance in algebraic pattern-generating activity: meter, rhythmic grouping, prolongation, and theme. Sinclair, Chorney,

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and Rodney (2016) used rhythm as their focal analytic construct in investigating the mathematical activity of children interacting with a tablet application designed for learning number. They implicate rhythmic actions as the embodied origin of cognitive structure, preceding planning and reflection. In like vein, Bautista and Roth (2012) documented the role of rhythmical hand movements in students' haptic engagement with geometrical regularities in material solids (cf. Bamberger & diSessa, 2003).

In summary, embodiment perspectives on mathematical cognition conceptualize dynamical sensorimotor problem solving as constitutive of conceptual growth. Positioned within the embodiment paradigm, we present a case of a student who spontaneously evoked measurement operations as her means of regulating the enactment of a challenging bimanual motor task designed to support the development of proportional reasoning. Our objective is to enrich scholarship on the rhythmic qualities of mathematics learning by way of interpreting the circumstances and process leading to her rhythmic mathematization of continuous quantities.

Methods

The empirical context for this study was the Mathematics Imagery Trainer for Proportion (MIT-P; see Figure 1). Unlike earlier studies in this empirical context (Abrahamson & Trninic, 2011), in the current study no mathematical tools were offered, such as a grid or numerals.

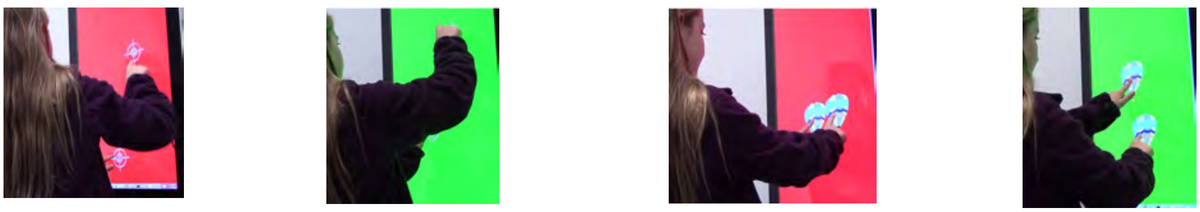

Figure 1. The Mathematical Imagery Trainer for Proportion (MIT-P). The student manipulates two cursors along vertical axes, one by each hand. The task is to make the screen green and then keep it green while moving your hands. The screen will be green only when the heights of the two cursors above the screen base relate by a particular ratio unknown to the user (e.g., here 1:2). Otherwise it will be red. Cursors may be either "stark" (e.g., generic targets; see on left) or "iconic" (e.g., hot air balloons; see on right).

K was an 11-year-old female student, one of 25 students participating voluntarily in a task-based semi-structured clinical interview (for details, see Rosen, Palatnik, & Abrahamson, 2016). The interview lasted in total 19 minutes: a general introduction (1 min.); and the problem-solving phase (18 min.), where she manipulated: (a) hot-air balloons (7 min.); (b) cars (4 min.); and (c) crosshair targets (7 min.). The interview took place in our lab and was audio–video recorded.

We located all the events where the student expressed new insight pertaining to her manipulation strategy. The interview was then parsed into episodes, running from each insight to the next. Episodes were further coded as: (a) either researcher- or self-initiated; and (b) discrete ("finding green" static co-locations) or continuous ("keeping green" while sliding the cursors).

Applying grounded micro-genetic analysis, we focused on the students' range of physical actions and multimodal utterance pertaining to the available media (Ferrara, 2014) as well as on the task-effectiveness of their actions. First, we attended to student actions that preceded their articulation of a new rule for "making green," searching in particular for patterns in the timing and sequencing of student hand movements through space (Sinclair et al., 2016). A notation system emerged for the most frequently used movements. For example, vertical bimanual movement with the right hand going up and the left going down was denoted as "↓,↑ ," and placing both fingers statically on the screen as "●,●." Second, we analyzed K's responses to our recurring question, "How would you explain your strategy for finding green to another person?"

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Results: Spontaneous Evocation of Units of Measurement

In our analysis we will attempt to implicate the specific event of K evoking measure units as a formative moment in her progress from unreflective continuous movement to unit-based discrete movement and, through this, in her learning of proportion.

Before evoking measuring units, K had developed two different rhythmical patterns as her task solutions: continuous movement, where the right hand moves twice faster than left (↑ ↑x2): and discrete, where she placed her fingers together (●,●) then vertically apart (↑,↓), traversing along recurring screen locations: bottom, middle, top. In articulating each of her strategies, K implicitly evoked particular qualities of the situation, such as the distance between her hands, as things she noticed and aimed to control even as she was moving her hands. For instance: "Down here my hands were really close, and then up here they were a little apart, and then up here they were really apart." Hoping for greater specificity of her movement rules, the interviewer asked K, "Ok, do you have any sense of… kind of…how this [the distance between her hands] is changing? How much it is changing, how much faster it is moving?" Beginning at 05:29, where she was working with hot-air balloon cursors, K responded by performing a particular action pattern repeatedly, at a constant pace, at three screen locations: bottom, middle, and top, stating that the balloons were: (bottom) "touching each other"; (middle) "There's about a balloon between them….the length of the balloon"; and (top) "Two balloons [apart], maybe." At 6:45 K repeated: "Ammm… [Quick succession of demonstrations: bottom ●,●, ↓,↑; middle ●,●, ↓,↑; top ●,●, ↓,↑] kind of at the bottom, there… it goes *zero* balloons between them, in the middle there is *one* balloon between them, and at the top, *two* balloons between them. So *it grows by one* at a time." K thus spontaneously utilized an available virtual object as a measure unit.

When the interviewer asked her to show "how to keep the screen green," K first gestured and then moved her hands continuously, with one hand moving twice as fast [↑ , ↑ x2]. It is of note that she tried to use her insights from the previous enactment as landmarks, that is, to connect the bottom, middle, and top discrete solutions into a single continuous enactment, as follows:

> (7:15) K: I would say, like, start at the bottom, and put them close together. And then, move one hand up faster… Wait, actually, [inaudible] …and as I said, in the middle, they are separated like one balloon [inaudible], and at the top two balloons.

Thus, a qualitative scheme for finding and keeping green, "one hand moves faster than the other as it goes up," assimilated a quantitative scheme, "it grows by one at a time," to better serve K's goals. Our claim is that the rhythmic qualities of K's actions—iteration, grouping, stability—as well as the interviewer's prompt to quantify ("how much") catalyzed this process. We observed feedback loops, where movements were coordinated into action patterns, and those patterns in turn were iteratively repeated, both spatially (bottom, middle, and top of the screen) and temporally. The linear extents of the hands' respective displacements came to attention *as a result of experiencing/enacting the emergent rhythm*. Namely, the rapid, cyclic repetition of implicitly measured actions gave rise to rhythmic enactment. The unit of measure emerged as a spontaneous combination of a stable pattern of movements and relatively stable perceptual elements, driven by a task demand to reflect on her own actions. Later in the interview, K quickly reenacted the new quantitative scheme in the case of car icons as well as the stark icons.

Conclusion

Rhythm is a means for coordinating physical operations over time, space, and material (or virtual) resources into new sensorimotor schemes. In the absence of any explicit frame of reference, rhythmic enactment bootstraps discretization, thus leading to further evocation of measurement units, which in turn improve performance and are thus adopted and codified. K's actions evolved from

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independent, explorative, seemingly uncoordinated movements into a stable temporal–spatial choreography comprising a succession of coordinated, measured clusters of movements preserving a relational invariant (see also Sinclair, Chorney, & Rodney, 2016).

K succeeded in coordinating her actions to produce green effectively well before she was able to articulate quantitative properties or her actions. When she first constructed a quantitative scheme for these actions, K was conscious not of a static structure. Rather, she responded to epiphenomenal features in the rhythmic cadence of enacting these coordinated actions. As such, rhythmic enactment mediated a transitioning from naïve to scientific reasoning. The temporal qualities of K's rhythmic enactment across the continuous display assimilated spatial qualities of available objects (hot-air balloons) to deploy motor-action execution over imaginary discrete units of measure. K thus extracted a measure unit from the situation as her means of extending insights from discrete to continuous actions. Unitizing is thus an evolved strategy for enhancing the coordination of continuous action by distributing it over regulated cycles of iterated enactment over projected spatial extensions. Further research is needed to understand the interplay of rhythm, action, and discourse in the elicitation of unitizing operations.

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