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Who Constructs Constraints? An Ecological-Dynamics Comparison of Two Dynamic Mathematics Environments' Design Rationales

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ABSTRACT: Dynamic mathematics environments (DME) are interactive technological modules for learning curricular content through engaging in inquiry-based activities oriented on manipulating virtual objects. In DMEs, mathematical ontologies, e.g., "parallelogram," are coded into the activity space as a set of to-be-discovered functional interdependencies among figural elements and feedback cues. This conceptual paper draws on the systemic theory of ecological dynamics to characterize student activity in two DME designs as shaped by constraints that are either a priori inherent in the environment (GeoGebra) or ad hoc emergent in the task (Mathematics Imagery Trainer). In turn, drawing on enactivist philosophy, we conjecture the differential cognitive effects of these distinct constraint regimes. We outline empirical research to compare the pedagogical entailments of these two rationales.

1. Objectives

This conceptual paper lays out the motivation and theoretical grounds for an empirical study that would compare two Dynamic Mathematics Environments (DME) with respect to their anticipated effect in fostering students' understanding of curricular concepts.

Imagine a computer screen featuring an interactive quadrilateral. When one drags any of its vertices or edges, it remains quadrilateral. Now further imagine two activities. In Activity A, the quadrilateral is initially a parallelogram, and no matter how one reconfigures it, it always remains a parallelogram; you are tasked to figure out properties of this shape. In Activity B, the quadrilateral is initially not a parallelogram, yet, as one reconfigures it, it turns green whenever it happens to be a parallelogram; you are tasked to keep the shape green as you reconfigure it. Assuming you have never before studied parallelograms, would these activities bear different effects on your conceptual learning? As researchers, how would we approach this comparison?

DME are inquiry-based, technologically enabled learning environments, in which students develop understanding of new mathematical concepts through moving virtual objects on an interface and reflecting on patterns or principles they discern in so doing (Leung, Baccaglini-Frank, & Mariotti, 2013; Schansker & Bikner-Ahsbahs, 2016; Schwartz, Yerushalmy, & Wilson, 1993; Sinclair, 2014). DMEs now constitute a broad spectrum of software products. These vary in activity architecture, apparently due to variation in the software developers' underlying design rationales, theory of learning, and pedagogical philosophy. For educational design researchers, this variation in activity architecture creates context to surface, articulate, and compare theories of cognition, learning, and instruction. These critical evaluations of design products may, in turn, lead to working hypotheses framing new empirical studies, notably studies that look to juxtapose DME variants with respect to their theoretical warrants and cognitive implications.

2. Theoretical frameworks

The conceptual analysis developed in this paper builds on a host of insights from a 10-year design-based international collaborative research project that is investigating implications of the embodied turn in the cognitive sciences for mathematics teaching and learning (e.g., Duijzer, Shayan, Bakker, van der Schaaf, & Abrahamson, 2017). In particular, we have been drawing on theoretical models from a range of scientific disciplines as well as philosophical treatises all concerned with teaching and learning to enact movements. Our resources include literature on biomechanics (Bernstein, 1996), ecological psychology (Gibson, 1966; Heft, 1989; Turvey, 2019), dynamic systems theory (Thelen & Smith, 1994; van Eck, 2018), enactivism (Varela, Thompson, & Rosch, 1991) coordination dynamics (Kostrubiec, Zanone, Fuchs, & Kelso, 2012), ecological dynamics (Chow, Davids, Button, & Renshaw, 2016), systemic kinesiology (Newell & Ranganathan, 2010), and dance scholarship (Sheets-Johnstone, 2015). Our own empirical work includes microgenetic examinations of tutor–student multimodal behaviors, as the student learns to enact movement forms believed to ground targeted mathematical concepts. Integrating study participants' audio–videotaped clinical utterance and eye-tracking data, as they attempt to solve motor-control problems, we have been able to document the emergence of imaginary sensorimotor perceptual structures—*attentional anchors*—that facilitate the coordinated enactment of movement forms satisfying task objectives. Participants are able to describe and inscribe these ways of looking at the screen, and they adopt mathematical symbolic artifacts as frames of reference to enhance their enactment and discourse.

Project work to date has enabled us to offer the field of educational research pioneering empirical evidence supporting historical claims by both Piaget (Abrahamson, Shayan, Bakker, & van der Schaaf, 2016) and Vygotsky (Shvarts, & Abrahamson, 2019) on the centrality of embodied interaction in conceptual learning. In turn, these findings could enable us to evaluate interactive educational activities with respect to the opportunities they occasion for students to develop curricular concepts. Toward doing so, this paper integrates our source literatures to offer task analyses that characterize learning processes with each of two DMEs under comparison and hypothesize their educational efficacy. We thus apply theoretical frameworks from the learning sciences to the educational practices of design and teaching.

3. Mode of Inquiry

We sought to establish an intellectually rigorous, coherent, and consensual set of theoretical constructs that both draw on historical paradigms in the cognitive and learning sciences and lend themselves for direct operationalization in empirical studies that would develop and evaluate educational materials for mathematics teaching and learning. By laying bare our theoretical assumptions and their pragmatic implications, we strive to encourage dialogue across the field.

Our mode of inquiry consisted of reflectively interacting with the sample software packages selected for the study, critically reading publications concerning the design and implementation of these products, and collaboratively deliberating whether and, if so, how, each of these designs embodies the educational implications of the theories that we selected as a basis for this comparison, namely theories from the movement sciences. Informed in particular by the systemic theory of ecological dynamics as it bears on embodied interaction for mathematics learning (Abrahamson & Sánchez-García, 2016), we conducted a comparative analysis of the design rationales, tasks, and learning processes in two specific DMEs that differ in their activity architecture. This analysis surfaced differences along dimensions of sensation, perception, motor action, and insight that, per the theory, should predict different learning outcomes. As such, our inquiry mobilized the research program from a particular theoretical paradigm and through to the conceptualization of an empirical study that could evaluate the pedagogical traction and validity of the paradigm's practical entailments for designing educational technologies.

4. Materials

The two DME selected for the comparison are *GeoGebra* (Hohenwarter, Hohenwarter, & Lavicza, 2009) and *Mathematics Imagery Trainer* (Abrahamson & Trninic, 2011).

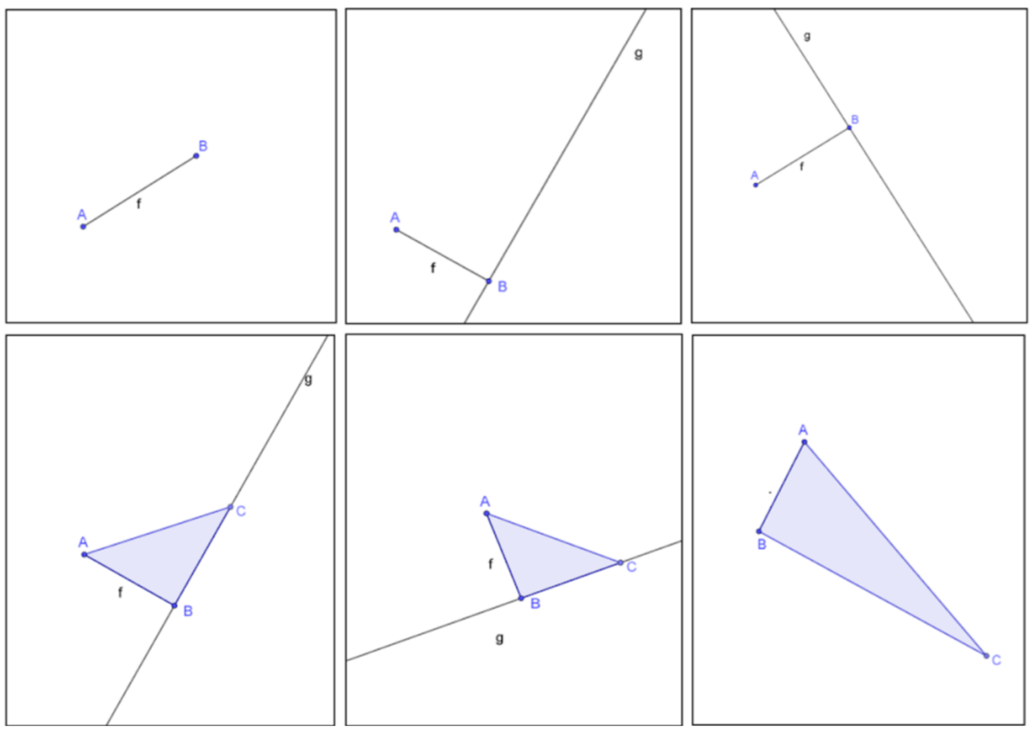


Figure 1. Constructing and manipulating a right triangle in GeoGebra. When dragging Vertices A, B, or C or Edges AB, BC, or CA, Triangle ABC remains right angled (AB ⊥ BC), automatically adjusting lengths of edges and/or measures of Angles BAC and/or BCA.

In the first type of DME (see Figure 1), students work on educational problems involving interactions with ontologically stable figural structures, whose consistent properties students are to discern and identify. We will call this type of DME "*xDME*"—"x" is for "closed." This design approach harks back to early reform-oriented pedagogical regimens of offering children prefabricated concrete materials. The child is to develop new concepts through inquiry-oriented manipulation, where the focal phenomenon is the *object* (Froebel, 1885/2005; Montessori, 1949/1967).

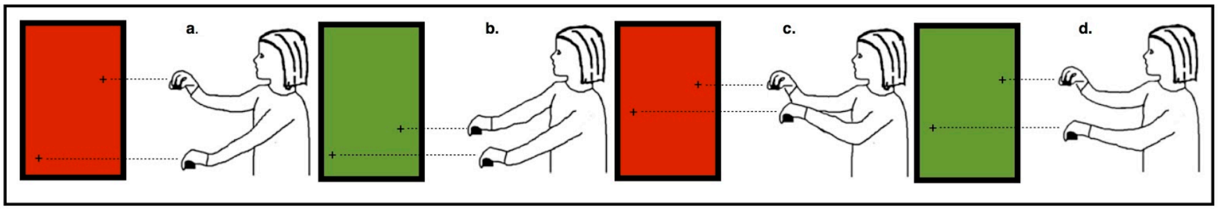
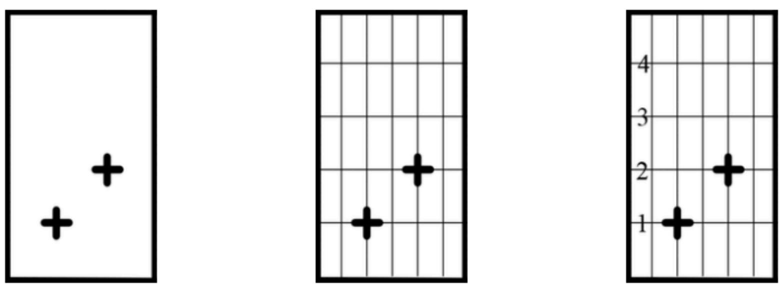


Figure 2. The Mathematics Imagery Trainer for Proportion: schematic activity sequence. Trainer set at a 1:2 ratio, so that the favorable sensory feedback (green background) activates only when the right hand is twice as high along the monitor as the left hand. Glossing over idiosyncratic variability, this figure sketches out Grade 4–6 study participants' paradigmatic interaction sequence toward discovering one effective operatory scheme: (a) while exploring, the student first positions the hands incorrectly (red feedback); (b) stumbles upon a correct position (green); (c) raises hands maintaining a fixed interval between them (red); and (d) corrects position (green). Compare 1b and 1d, the two green configurations, to note the different vertical intervals between the cursors. The child might conclude that, "The higher my hands go, the bigger the interval." She learns to move in a new way centered on a new object (Abrahamson, Lee, Negrete, & Gutiérrez, 2014).



a. b. c.

Figure 3. Symbolic artifacts overlaid onto the Mathematics Imagery Trainer activity space: (a) the two cursors; (b) a grid; (c) numerals. Students learn mathematical concepts by adapting symbolic artifacts as frames of reference for enhancing their enactment, explanation, and evaluation of their effective bimanual manipulation strategies.

a. b. c.

Figure 4. The Mathematics Imagery Trainer for Proportion in action: (a) a child working on her own; (b) a student working with a tutor; and (c) two students collaborating on a Trainer iPad app.

In the second type of DME (see Figures 2–4), students work on educational problems involving interactions with ontologically malleable figural structures whose quiddity can be interrupted. We will call this type of DME "*oDME*"—"o" is for "open." This latter design approach is inspired by cognitive science theories of embodied cognition, which posit the conceptually constitutive role of purposeful, situated, dynamic, and corporeal phenomenology in perceptually organized action (Kiverstein & Clark, 2009; Newen, Bruin, & Gallagher, 2018; Shapiro, 2014; Varela, Thompson, & Rosch, 1991). The child is to learn new concepts through inquiry-oriented manipulation, where the focal phenomenon is not the object per se but the child's dynamic *interaction* with the object, and in particular emergent sensorimotor coordinations—eyes–hands, in this case—enabling the enactment of task-effective adaptive movement forms.

5. Results of Conceptual Analysis

Table 1 (below) compares GeoGebra and the Trainer along multiple dimensions of their design rationale, including their respective interaction mechanics, user experience, and the role of movement in task performance. The comparison implies that these xDME and oDME design rationales differ in terms of their consequences for the user's agency, movement, reasoning, and, potentially, learning through manipulating the mathematical structures on display.

xDME designs bear what we might call *hard constraints*: a priori environmental constraints on movement that the designer encodes and the user is to discern. The user need not exercise agency in keeping the mathematical ontology intact: No matter how the user moves (drags) elements in the structure, the conceptual organization of the structure remains intact. The user cannot work around these constraints, unless they access the source software code. In xDME, the designer constructs constraints on possible movement.

oDME designs bear what we might call, by way of contrast, *soft constraints*: ad hoc task constraints on movement that the user discovers and self-imposes. Moving under these self-imposed constraints, the user self-organizes to receive continuous positive feedback in performing the assigned motor-control task of transitioning between permissible configurations of the display. In oDME, the student constructs constraints on possible movement.

A designed activity's locus of constraints on movement will shape students' attention either to objects (xDME) or action (oDME). Objects and actions are intrinsically different ontologies with far-reaching implications for cognition and epistemology. What students attend to, and how they attend, is of moment for learning processes. A reading into developmental theories of genetic epistemology (Piaget, 1968) and enactivism (Maturana & Varela, 1987/1992; Varela et al., 1991) would suggest that not objects but actions constitute naturalistic foci for the emergence of subjectively meaningful cognitive structures mobilizing mathematical reasoning.

Working with an xDME, students begin with a pre-existing object presented on the screen. It is a compelling perceptual gestalt composed of a coherent assembly of structurally conjoined virtual elements, such as points, lines, and arcs. Students are invited to reconfigure this object. However, given a set of computationally encoded propositions hidden from the students, they will necessarily be operating within this object's constrained morphological degrees of freedom, whereby certain types of reconfiguration are enabled, while others are not. The task is to describe the object's invariant properties, that is, to characterize what it is about this object that remains constant (consistent, conserved, "the same") across all its states one generates. In a sense, one is asked to compose a declarative idea of what this object is, where "is" denotes its haecceity or uniqueness in contradistinction with other familiar (geometrical) objects.

Table 1
Comparing the Design Rationale of Two DME: "Closed" DME and "Open" DME

Dimension		
DME Type	closed DME (xDME)	open DME (oDME)
Example Modules	*GeoGebra*	*Mathematics Imagery Trainer*
Fundamental Design Rationale	Students learn new curricular content by manipulating an interactive display. They notice and articulate invariant properties of an emergent conceptual class	
Input–Output Regimen Mechanics	The designer hard-codes a mathematical ontology as an inherent and immutable relation structure among elements of the display. Consequently, the software automatically maintains these structural properties under any reconfiguration. For example, the ontology "right triangle" remains consistent across permissible modification to the properties of local elements, such as line lengths or orientations. The user's task is not to maintain structure, and so the system does not signal "success" or "failure."	The designer hard-codes a mathematical ontology as a feedback regimen privileging a subset of possible display configurations receiving a differentiated feedback. The display, thus, exhibits figural properties of the goal category dependent on the users' actions. For example, the ontology "right triangle" is instantiated in the display contingent on the user's reconfiguration actions. The user receives positive (e.g., green) feedback on generating category exemplars and negative (e.g., red) feedback on all other configurations.
Functional Relation Between Manipulable Elements	Manipulable elements of a virtual display may or may not be interactionally dependent (moving one element may or may cause automatic movement of the other one).	Manipulable elements of the virtual assembly are interactionally independent (moving one element never causes automatic movement of another one).
Task	Users are tasked to manipulate figural elements of the display in an attempt to discern and articulate what properties of the display remain invariant across manipulation (i.e., what there *is*).	Users are tasked to manipulate figural elements of the display in an attempt to receive "success" feedback, maintain this feedback over continuous manipulation, and then articulate their manipulation strategy (i.e., what I *do*).
User Task Experience	Users drag figural elements of the interactive display, hypothesize invariant properties of an emerging structure, verify or refute the hypotheses, and compose conjectures about the structure's properties.	Users drag figural elements of the interactive display, experiment with manipulation strategies to satisfy the task objective, and use mathematical frames of reference to articulate effective strategies quantitatively. Initially, the user *can* and probably *will* violate the encoded mathematical ontology, because they have not yet constructed it and the technology does not keep it intact. The user responds to the feedback regimen by searching for, and gradually coming to anticipate, the set of privileged configurations. The user develops strategies for transitioning dynamically between these configurations. These strategies emerge in the form of new sensorimotor schemes oriented on new perceptual

		structures (attentional anchors). The to-be-learned ontology materializes through reflecting on the strategies.
Attentional Anchors	No attentional anchors are anticipated to emerge in this context, because there is no explicit movement task.	Attentional anchors are expected to emerge as the user's means of facilitating the coordinated sensorimotor enactment of movements that satisfy the task specification.
The Psychological Role of Movement	Movement serves a pragmatic function by mobilizing the search for information common across a succession of static images. The user moves objects on the screen so as to generate multiple configurations of the intact form in an attempt to discern across them invariant properties of the emergent ontology.	Movement serves a conceptually formative epistemic function in searching for a dynamic form that would enable performing the assigned task effectively. The user attempts to discover and describe a task-effective strategy for moving objects on a screen. The sense of invariance, and thus the new ontology, sprouts from experiencing and then articulating consistency in this dynamically, task-effective stable form.

Working with an oDME, students begin not from prefabricated objects but from virtual utensils (handles, appliances) for operating on a particular property of the background realm (e.g., its color). The students are tasked to handle these utensils such that a particular state of this background realm be achieved and maintained, namely, the state of a particular value of the color parameter (viz. green). In the course of this work, an imaginary perceptual structure, the *attentional anchor* (Hutto & Sánchez-García, 2015), may emerge on the students' sensorimotor interface with the environment. The students, who are further invited to describe their strategy, may refer directly to this new object they are perceiving and explain how they are handling it. Here, the process of articulating the strategy unfolds through specifying what one is keeping constant to operate the apparatus. That is, the stable mathematical attributes of the system come forth as ontologies (what is) through the discursive delineation of one's operatory regimen (what I am doing). There is no a priori object. Rather, the object takes shape as an entity that materializes discovered patterns in effective performance of an assigned task, as the student gropes to enact a new movement form (Varela et al., 1991). A thing is born.

Our hope is that essential issues discussed herein will give rise to empirical research that rigorously evaluates for the impact of constraint source on learning outcome. Where the research design might require much creativity is in building assessment tasks that enable participants in both study conditions to demonstrate their new skills. We predict different learning outcomes for the two designs, with oDME bearing cognitive advantages over xDME.

6. Significance

This conceptual paper has offered a contribution to the field of educational technology design by identifying and articulating a theory-based lens for analyzing, predicting, and explaining differential learning outcomes from interacting with modules that instantiate different design rationales. In the Learning Sciences, it often occurs that research groups who champion similar-enough pedagogies nevertheless operate with little to no scholarly exchange. To our judgment, these missed opportunities for productive dialogue result from an implicit sense that it is normative for various educational "guilds" to coexist, each with their distinct software packages, instructional practices, jargon, and go-to theory. This laissez-faire practice would be most anomalous in the exact sciences, where researchers impel each other to rigorously hone and evaluate competing theories and privilege those whose predictions are better supported by empirical data. By setting the stage for empirically comparing two design rationales, we thus strive to scientificate the learning sciences. The AERA 2020 theme is "The Power and Possibilities for the Public Good When Researchers and Organizational Stakeholders Collaborate." Educational researchers designing for the public good might begin by collaborating *with each other* to resolve theoretical debates.

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