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Dynamic Geometry Meets Participatory Simulations: The Design of PANDA BEAR

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Introduction

Perimeters **and** **A**reas **by** **E**mbodied **A**gent **R**easoning, or PANDA BEAR, is a microworld for mathematics learning that lies at the intersection of dynamic geometry environments (Goldenberg & Cuoco, 1998; Jackiw, 1997; Laborde, 1990; Schwartz & Yerushalmy 1985) and participatory simulation activities (Wilensky & Stroup, 1999b). Building on the restructuration (Wilensky & Papert, 2006; Wilensky et al., 2005) of content in both geometry and mathematical proof developed in dynamic geometry environments and utilizing the agent-based perspective of participatory simulation activities, PANDA BEAR has been designed to develop in its users both geometry understanding as well as skills in mathematical argument. In a PANDA BEAR activity, utilizing the HubNet system (Wilensky & Stroup, 1999a), individual students identify with and control a single vertex of a shared, group-polygon. The measures of perimeter and area of the group-polygon are displayed in the shared environment. Group-level challenges are issued to the students that cannot be solved by one individual. Evidence from a pilot study suggests that by having direct control of only a part of the system for which the challenges are issued, students can acquire an agent-based perspective that enables understanding by way of construction of solutions to the group-level problem. On the basis of very preliminary data, we further speculate that through the communication of ideas and strategies to each other in the context of the shared task, PANDA BEAR activities can foster the development of mathematical argument skills.

Prior Research

It has been proposed that mathematics education be taught as a domain of problem solving (Schoenfeld, 1985). Thus students of mathematics, in addition to the specific content relevant in a given course, must also learn strategies or heuristics for employing that content towards solving a given problem. The student needs to step back further to see when various strategies are to be used and even further to know how to approach the learning of mathematics. Because it is the first course in which many students encounter "proof", geometry lends itself particularly

[1] This version of the paper was updated to include two footnotes that reference published versions of then as yet unpublished papers.

well to the framing of mathematics education as a problem solving domain. As students learn about points, lines, planes, polygons, and how they relate, proofs form the backbone of the domain in traditional geometry classrooms.

Taking advantage of the visual aspects of geometric content, several dynamic geometry environments have been developed (Goldenberg & Cuoco, 1998; Jackiw, 1997; Laborde, 1990; Schwartz & Yerushalmy 1985). This development of dynamic geometry can be viewed as a restructuration (Wilensky & Papert, 2006[2]; Wilensky et al., 2005) of geometry as a domain. Wilensky, Papert and colleagues define a restructuration as a reformulation of the content and practices of a knowledge domain resulting from a change in representational infrastructure. Dynamic geometry environments are examples of reformulating the content of a domain using new technological tools. Instead of using a compass and protractor to create static images from which deductive proofs can be based, dynamic constructions are used to explore large numbers of examples in order to inductively construct propositions. For example, consider the following steps that can be taken in a dynamic geometry environment towards verifying that a triangle's angles always sum to 180 degrees. An initial triangle can be constructed by specifying three vertices. Next, the three angles of that initial triangle can be measured, summed, and stored in a variable on the screen. Then, making use of one of the more powerful elements of dynamic geometry environments, the user is allowed to "drag" the vertices of the triangle throughout the environment. As the vertices are dragged and many example triangles are investigated, the value associated with the variable remains a constant 180.

PANDA BEAR is a different type of dynamic geometry environment, one built with HubNet (Wilensky & Stroup, 1999a), an architecture for constructing participatory simulations (Wilensky & Stroup, 1999b). In a participatory simulation, several participants play the role of individual agents in a complex system. As the individuals interact, patterns emerge at the group level (Stroup & Wilensky, 2003; Wilensky & Resnick, 1999). Several HubNet simulations, in a wide range of sciences and mathematics have been developed and tested in middle and high schools (Abrahamson & Wilensky, 2004a,b; Levy & Wilensky, 2004). Blending dynamic geometry environments with participatory simulations, PANDA BEAR employs HubNet in the domain of geometry.

HubNet is an extension of the NetLogo (Wilensky 1999) multi-agent modeling environment. NetLogo has its roots in Logo (Papert, 1980; Abelson & DiSessa, 1980), and employs the same turtle metaphor as well as much of basic syntax of Logo. The turtle metaphor is one that encourages reasoning at the local level of an individual "agent". NetLogo extends that metaphor to include multiple agents and their interactions. Thus, on many levels, the platform on which PANDA BEAR is based encourages students to adopt an agent-based perspective. This agent-based perspective afforded by agent-based modeling and participatory simulations, as part of the larger complex systems perspective, has proven to be a powerful tool in many areas of science (Wilensky, 2001; Wilensky & Stroup, 2000; Goldstone & Wilensky, 2005[3]; Wilensky &

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[2] Later published as: Wilensky, U., & Papert, S. (2010). *Restructurations: Reformulations of knowledge disciplines through new representational forms*. Proceedings of the Constructionism 2010 Conference, Paris, France.

[3] Later published as: Goldstone, R. L., & Wilensky, U. (2008). Promoting transfer through complex systems principles. *Journal of the Learning Sciences, 26*(1), 465-516.

Abrahamson, 2006). PANDA BEAR has been designed to utilize this agent-based perspective to promote learning geometry content and to engage students in mathematical argument about this content.

Design

All HubNet participatory simulation activities consist of a server model and a client model. The interface of the server model (Figure 1, below) provides a global view of the polygon, as well as administrative tools for setting up and running the activity. When students move their vertex around, the perimeter and area monitors automatically update. Plots of these measures over time show a record of the group's actions as they work towards a goal.

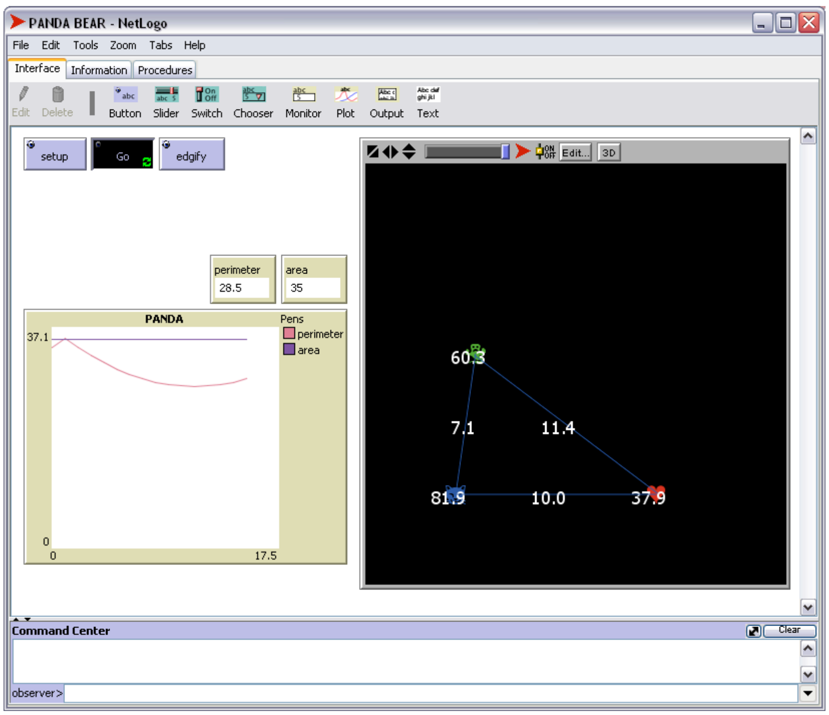


Figure 1. Server interface

The client model (Figure 2, below) displays for the users the group polygon along with the current perimeter and area of the polygon. Also on the client interface are buttons for the students to move their individual vertex around the screen, thereby affecting the shape and size of the encompassing polygon.

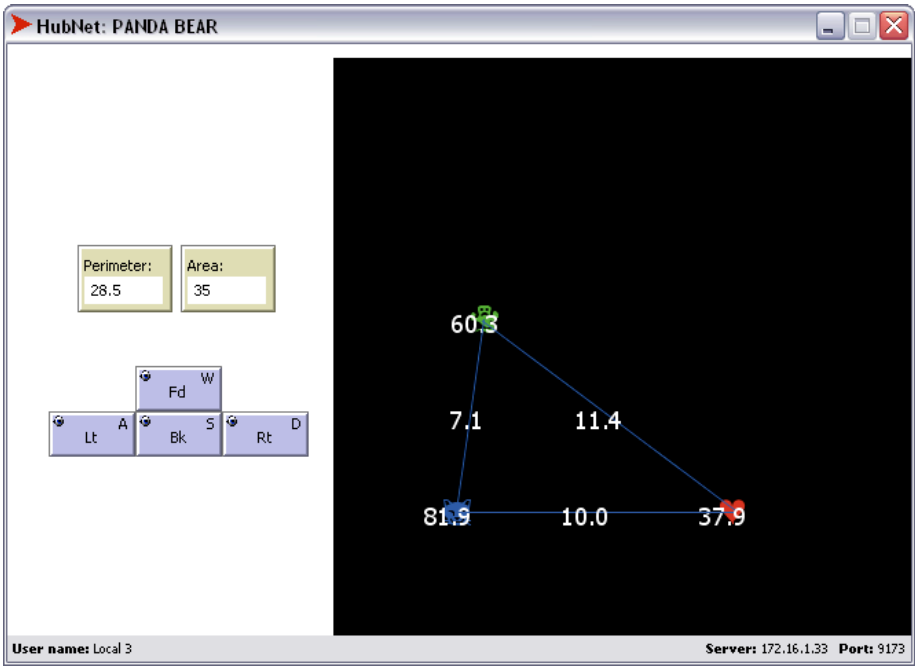


Figure 2. Client interface

Activity Sequence

In our pilot runs of the PANDA BEAR activities we have refined a sequence of activities. After the students initially "login" to the server through their individual clients, some time is spent acquainting them to the environment: learning to move their agents and to observe the displays dynamically changing. Once the students are more comfortable with the basics of the software, they break up into groups of three. As each person is represented by an agent on the screen, the agents of each group form the vertices of a triangle. By moving their three vertices, the students are offered a challenge: Make the area as big as possible while keeping the perimeter below 50. The correct solution to this challenge is an equilateral triangle. As the students try out different ideas, they are encouraged to discuss their strategies with each other. What strategy are they using? Is the strategy working? Why are they using that strategy? As shown below in the pilot study section, interesting strategies can be developed in the triangle case that can carry through in the subsequent activities. Additional variant challenges, such as making the perimeter as small as possible while keeping the area above 50, are also used in this stage to help students develop robust strategies that work across multiple examples.

After the students have begun to master the triangle case, the number of vertices is increased one at a time, with the motivation to encourage students to see the limit of such increases as a circle, while generalizing that each solution is a regular polygon.

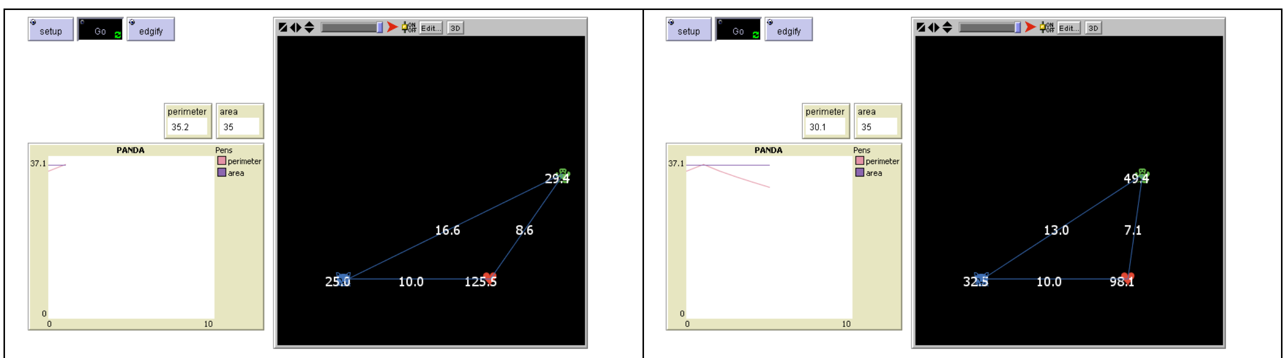
Testing

A small focus group composed of university students provided rich data and feedback. After further rounds of improvements following the focus group, we conducted a pilot study of

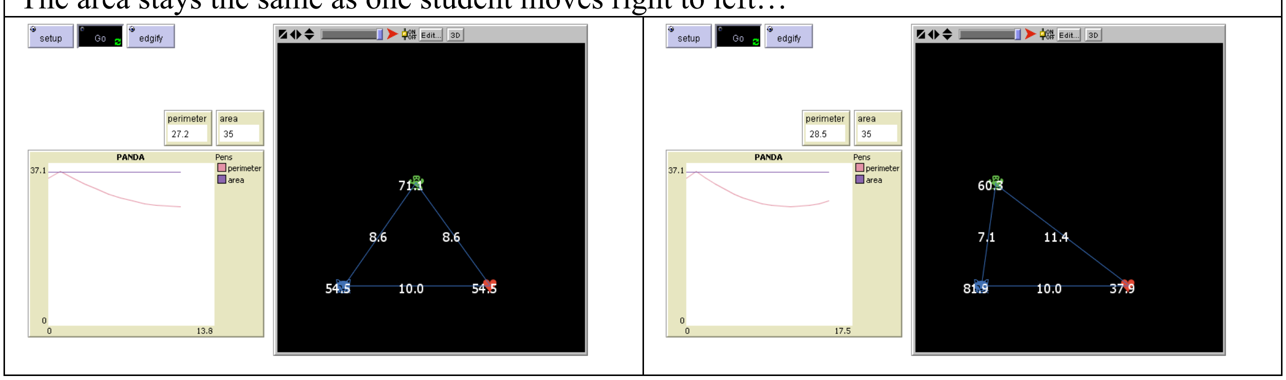
PANDA BEAR with a group of five 7th grade girls. The students filled out pre- and post-tests and went through post-interviews following the 4 hour intervention. In this study, we paid close attention to the strategies the students were employing, as well as how they were communicating about them. In addition, we were watching for changes in what the students counted as geometric justification while they were reasoning through the challenges imposed by the activity leader.

From the pre-test data, several patterns of responses have been noticed. Most of the participants answered the first item (see Figure 4a, below) responding that the first figure is bigger. Several of them related the figure on the right to the figure on the left by saying that it was the same except for having a part “cut out”. Several of them were very sure of their answers as measured by the exclamation points they included in their answers such as “this one!!!!!!” However, one student said that while the first figure has “several units missing”, the question doesn’t specify what is meant by the term “bigger”. As an example, this student put forth “units missing” could be what’s being measured, in which case the second figure would be bigger. This student recognized that the answer to the problem depended very much on an interpretation of the question, but they nonetheless still showed an obvious preference towards area and away from perimeter.

During the activities, students showed great enthusiasm and engagement. Also, the activities seemed to provide a social context for them to engage in geometric argument and justification. As an example, one group decided to make two of their triangle’s vertices stationary while the third student found the right spot for her vertex (see Figure 3, below). A useful strategy was formulated: to minimize the perimeter of a triangle (given a fixed area) she needed to move “between” the other two vertices.



The area stays the same as one student moves right to left…



… and the perimeter is minimized when the student reaches the middle

Figure 3. A local strategy for minimizing the perimeter for a given area

In the post-interviews, mixed results were observed. For the first set of figures (see Figure 4b, below), every student compared correctly both the area and the perimeter of the two figures. One student added a new measure: outside-space, which appears to be the complement of the area of the figure as it sits on the page. Results for the second, similar, set of figures (see Figure 4c, below) in the post-interview are similar. Only one student thought that the perimeter of the second figure is bigger than that of the first. In addition to getting the correct answers, many of the students provided a transformational explanation for their answers. For instance, they talked of “pushing the sides out” to see that the second figure in the first set has a bigger perimeter than the first figure.

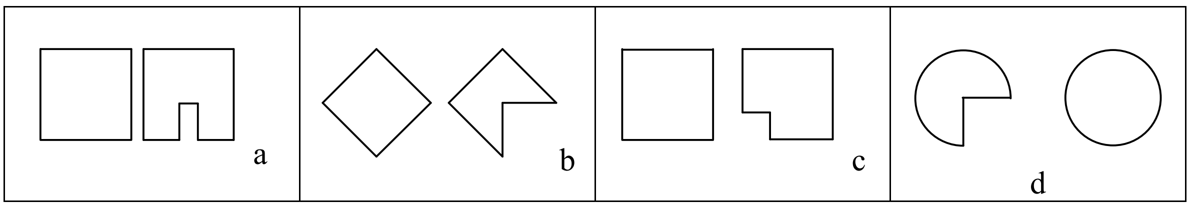


Figure 4. Pre-test (a) and post-test (b, c, d) items

The third set of figures (see Figure 4d, above) involved circles and “partial” circles. This problem presented many troubles to the students. Several students had no difficulty with a comparison of areas. However, at least one student had difficulty with that despite being able to solve the polygon version. She fidgeted in her chair, spoke softly, and finally put forward that the two figures were the same “size”, but the one on the right was missing “space”. When directly asked how the perimeters compared, she asked if the question was about “circumference”, showing some prior knowledge in the domain. Instead of making comparisons by searching for transformations from one figure to the other, this student stated that she did not know how to find the circumference, so she could not answer as to which figure had a bigger perimeter. Not all the students approached the problem in this way though. Two students in particular continued with their transformational approach and decided that the first figure has a bigger perimeter.
We had hoped that the same approach they applied successfully with polygons would extend to circles. However, circles clearly presented difficulties for the students in both the pre- and post-tests. In a subsequent design, this will be addressed by better facilitating the shift to larger polygons. Tools to better manage the logistics of changing the number of vertices will be built into the activity. In addition, more time will be allotted to encouraging discussion surrounding this idea.

Summary and Future Directions

As evidenced by the mixed results in the post-interviews of the pilot study, there is room for improvement in the design of PANDA BEAR and the accompanying curriculum. This is not surprising as we are in early phases of the iterative design process. We are encouraged that the students progressed from the pre-interviews – paying attention to both perimeter and area and often using transformational arguments to justify their answers. However, circles were seen as a different kind of object, and were more difficult to comprehend. In addition the independence of a figure's perimeter and its area was not well understood. That is, as a figure's perimeter increases, the area can stay the same, increase, or decrease and vice versa. One potential reason for this particular confusion might stem from the rapidity with which the activities toggled between maximizing area given a maximum perimeter and minimizing perimeter given a minimum area. In addition to the rapid changing of directions of tasks, the multitude of tasks made it difficult for the students to remember what their current task was. To address these concerns, our next iteration would have students focusing first on either maximizing area given a maximum perimeter or minimizing perimeter given a minimum area rather than on switching between the tasks. Also, we plan to build in structure to support this distinction into the environment itself. For example, if the perimeter must be below a certain number, say 50, PANDA BEAR can be modified to not allow students to move if that movement would cause the perimeter to surpass 50. With this revised activity sequence and structure in place, it will be interesting to see what kinds of differences are observable post-intervention. In addition, formations of fixed points not under the students' control could be introduced, leading students to find solutions such as semi-circles. Clicking in the client interface as an alternative to using buttons would provide a different type of interaction with the PANDA BEAR activity that might prove to be beneficial. Finally, extending the activity into 3D would allow similar activities involving surface area and volume.

We are still in early stages of development of PANDA BEAR. The pilot data suggests that students advanced in their understanding of perimeter and area and the relationship between these geometric concepts. The activities were very motivating for the students and provided a social context for sustained engagement with geometric ideas. We believe the pilot study supports that iterative refinement of PANDA BEAR will result in a set of activities deployable in secondary classrooms that will help students with learning geometry and making geometric arguments.

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