Significance

These results suggest that representing electrical conduction in terms of intuitive, agent-level behaviors can enable learners as young as 5th graders to understand and explain (in a scientifically correct manner) the behavior of electric current in linear circuits, which misconceptions researchers have shown to be difficult for even college students to understand (Chi et al., 1994; Reiner et al., 2000; Bagno & Eylon, 1997).

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Seeing Change in the World from Different Levels: Understanding the Mathematics of Complex Systems

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In an increasingly dynamic and data-driven world, it is important for all students to be able to understand and interpret mathematical patterns over time in the context of the real-world phenomena that generate them. While there is a growing body of evidence that students as early as middle school can understand some fundamental and powerful ideas related to the mathematics of change over time in terms of real-world contexts such as motion or banking (Confrey, Maloney, & Castro-Filho, 1997; Nemirovsky, Tierney, & Wright, 1998; Roschelle, Kaput, & Stroup, 2000), as well as the sophisticated dynamics of many complex systems (Wilensky & Reisman, 2006; Wilensky & Resnick, 1999), little is known about how they might think about mathematical change as it relates to complex systems: where a number of *different behaviors and entities all contribute to a single quantity* of interest, rather than a single behavior or phenomenon. But such systems are increasingly important in all aspects of academic and everyday life. From global temperatures that are increasing exponentially due to increased individual consumption to employment patterns that affect individual workers and are influenced by consumer spending, students must be able to understand not only how to interpret rates of change in the world, but also how individual entities and their actions and interactions contribute to and are affected by those rates of change.

Knowing how students think and learn about change over time in such systems is important not only because it can help prepare students as active and informed citizens (Sabelli, 2006), but also because it can serve as a new access point to more formal mathematical topics such as calculus (Nemirovsky, Tierney, & Ogonowski, 1993; Stroup, 2002), and provide a better foundation for students entering the natural and social sciences where such systems are especially common (AAAS, 1991). That traditional calculus-based mathematics and notions of rate of change are so widely applicable and powerful, yet can so easily obfuscate the very mechanisms and elements that are at the core of those systems, presents a fundamental challenge for mathematics and science educators. In this presentation, we will explore the potential for *Agent-Based Modeling (ABM)* to provide students with an alternative, intellectually honest means to construct and analyze the mathematical trends produced by complex systems.

Agent-based modeling shifts the encoding of *quantitative change* from an aggregate-level quantitative trend (for example, the rate of change of a population count) to a collection of agent-level behaviors that produce those trends (the reproduction behaviors of individuals). As a result of this shift, agent-based modeling can represent and reflect quantitative change in a way that includes notions of randomness, sensitivity to local conditions, the role of nonuniform distributions in aggregation, and other powerful aspects characteristic of change in *systems* that are not dealt with in traditional calculus. While considerable research has investigated how computation has expanded *who* can learn the mathematics of change, less is known about how it can expand *what* can be learned about the mathematics of change.

We propose that such a practice can:

- Broaden student access to the mathematics of change and variation, by providing an alternative language with which to "speak" and "build" mathematical models of multi-agent systems that change over time.
- Make more accessible to students the connections between the mathematical model(s) of a system, and the behavior and mechanisms that comprise that system.
- Provide students with an infrastructure within which the ideas of calculus can be applied to a large class of interesting, and personally relevant, phenomena.

We will support these claims with evidence from think-aloud (Ericsson & Simon, 1984), semi-clinical (Clement, 2000) interviews conducted with 12 high school students enrolled in a precalculus course, who engaged in agent-based model building activities related to population growth and analyzed the graphs of mathematical trends produced by those models. These interviews are part of a larger design research agenda that intends to explore the potential for a constructionist, computational agent-based modeling environment to provide students a flexible, meaningful context within which students can explore the mathematics of complex systems. Findings suggest that while students were initially able to connect the behavior of a population plot to the behavior of the agent-based model, they were not able to articulate how the modeled reproductive behavior of that population, or how certain real-world factors such as a catastrophic event or individual preferences on childbearing might be manifested in the notion of "rate of change" or in the featured plots. They were also likely to attribute fluctuations in the plot to real-world events that were not included in the model, or develop incorrect mathematical explanations for real-world factors. In other words, students had great difficulty connecting the behavior of the model to mathematical ideas such as rate of change; when they tried to do so, they struggled and were often unsuccessful. After interacting with, and building their own, agent based models of population growth, students were much more likely to consider explicitly how behaviors that they knew were included in the model might play a role in the mathematical trends generated by that model, and were better able to predict mathematical trends given behaviors that were not included in the model. They were more likely to consider issues of randomness, individual heterogeneity, and the "stripped down" nature of modeling (that is, that not every real-world factor can be included in a given model) when analyzing mathematical trends and discussing mathematical models.

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Theoretical and Methodological Implications of Complexity

Learning as an Emergent Phenomenon: Methodological Implications

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In this paper, we put forth a theoretical cum methodological proposal for a line of inquiry that seeks to understand learning as an emergent phenomenon. Our theoretical and methodological arguments detail how an emergent conception of learning places limits on experimental and descriptive approaches, whether used alone or in combination. These limits are not so much a function of causality or reduction, but the need to deal with the dialectical co-existence of linearity and non-linearity that often characterizes complex phenomenon. To overcome these limits, albeit only partially, we leverage complexity theory to advance computational agent-based models as part of an integrated, iteratively validated phenomenological-ABM inquiry cycle to understand learning as an emergent phenomenon from the "bottom up."

Although there is much excitement about the possibilities that computational methods bring to the table, there remains little theoretical and methodological exposition of why and how computational methods can be integrated with existing quantitative and qualitative methods to potentially expand the research toolkit of educational researchers. How do existing quantitative and qualitative methods fall short and how might computational methods be integrated to provide better understanding of the phenomenon of learning, and vice versa? Our argument is based on the premise that learning is a complex phenomenon, which under certain conditions exhibits emergent properties. Indeed, many contexts of learning—formal or informal, groups or individuals—are in fact complex systems (Jacobson & Wilensky, 2006; Kapur et al., 2007). It is this very complexity that sets up the stage for the emergence of knowledge structures, interactional patterns, values, norms, identity, culture, and so on. Invoking emergence, however, requires that we deal with a fundamental tenet of complexity: an emergent phenomenon is ontologically and methodologically irreducible, i.e., *an emergent phenomenon is its own shortest description* (Kauffman, 1995). This simple yet powerful tenet poses serious methodological challenges. Through a careful analysis of the assumptions underpinning quantitative and qualitative methods, we will build a case that existing methods fail to adequately address the issues of *non-linearity*, *temporality*, *spatiality*, and *phase-space* that are of central to understanding emergent phenomenon. We will discuss how and the extent to which these issues can be addressed by *integrating* computational agent-based methods with existing quantitative and qualitative methods (Abrahamson & Wilensky, 2005; Blikstein, Abrahamson, & Wilensky, 2006; Goldstone & Janssen, 2005). In the final analysis, we propose an iterative process of building from and validating with phenomenological theory and data to seek a better understanding of the complex phenomenon of learning noting very well that, any method, be it quantitative, qualitative, or computational, used alone or in combination, will necessarily remain reductive.

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