Body of Knowledge: Rethinking Mathematical Concepts as Signified Embodied Procedures[1]

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We present a novel theoretical framework for articulating the relation between performance and understanding across the disciplines. The framework, which intersects embodiment and sociocultural theory, hinges on juxtaposing epistemological assumptions and pedagogical practices in *explicitly* embodied disciplines (e.g., dance) with those in *implicitly* embodied disciplines (e.g., mathematics). Researchers of mathematics learning have viewed physical performance as an unrefined precursor to understanding en route to its abstraction as conceptual structures. In contrast, the explicitly embodied domains view understanding as emerging *in*, *through*, *about*, and *for* performance. We develop the framework in the context of an empirical design-based research project investigating the emergence of conceptual mathematical understanding from perceptuomotor interaction strategy. As such, we propose two complementary constructs. An *embodied artifact* is a rehearsed physical performance serving as a resource for prospective coping with a particular class of situations through coupling with the world. A *conceptual performance* is a disciplinarily signified embodied artifact that serves as a notion's grounding referent. Such an approach may dissolve the barrier between procedures and concepts, performance and understanding: instead of emphasizing procedural fluency and conceptual understanding as separate aspects of disciplinary competence, we hone the constructs' distinctions along semiotic lines and outline somatic-to-semiotic learning trajectories.

The constructs "procedure" and "concept" play curious roles in educational research and practice. On the one hand, these constructs often appear as a duality both in academic journal articles and national policy documents. For example, the influential (US) National Council of Teachers of Mathematics Standards argues for the developmental importance of fostering both "procedural proficiency" and "conceptual understanding." On the other hand, there is little scholarly agreement over the actual nature of these constructs let alone their relations. At best, it is implied that procedures are something you "do" and concepts are what you "know." In only slight caricature, procedural knowledge is rote performance of pen on paper, whereas conceptual knowledge is packets of transcendent understandings in the head.

Notwithstanding their theoretical fuzziness, procedures and concepts loom large in the "math wars"—polemical educational policy debates over perceived advantages of procedural "basics" versus conceptual "learning for understanding" (Schoenfeld, 2004). Indeed, whenever thought and action have been proposed as separate facets of human phenomenology, educational reform has drawn political interrogation. From a pragmaticist perspective, Dewey (1916) asked, "How shall we secure breadth of outlook without sacrificing efficiency of execution?" (p. 248). From a Marxist perspective, Freire (1973)

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"replied" that a community's outlook and concerns should determine what they learn to execute efficiently. From a utilitarian perspective, Freudenthal (1968) believed that utility for realistic situations should inform learners' selection of solution procedures. Arendt (1968), however, opined that in their misguided interest of breadth, educational reformers have capitulated their charge to foster efficiency... Conceivably, then, one's tacit epistemological position on relations between procedures and concepts orients one's pedagogical perspectives.

And yet, what remains undertheorized in the debate is its *a priori* epistemological grounds, namely the implicit framing assumption that procedures are actively embodied whereas concepts are statically disembodied. Cogently, what *could* abstract mathematical concepts be grounded in? (see Harnad, 1990; Wilensky, 1991; see Vergnaud, 1983, on "theorem-in-action"; see Sfard, 1991, and Grey & Tall, 1994, on the ontological status of math expressions as processes or objects).

Beth and Piaget (1966) view concepts as subjective "encapsulation" of procedures—through reflective abstraction, procedural performance is "reorganized on a higher plane of thought and so comes to be understood" (p. 247; see Karmiloff-Smith, 1992). As such, the constructivist view suggests that learning invovles a category shift in the epistemic status of 'doing' into 'knowing.' Yet this enticing model does not appear to obtain in the case of socio-historical practices invovling situated skills, such as dance, wherein the body is the explicitly manifest medium of enactment. Therein, vocabulary, principles, and understandings are perceived not in terms of "higher planes of thought" but as emerging *in*, *through*, *about*, and *for* performance.

Accordingly, we embarked from the conjecture that the discipline of mathematics is not an ideal domain of research for *initiating* an investigation into the theoretical nature and epistemological status of concepts. We have thus been seeking to rethink conceptual development from the vantage point provided by studies of explicitly embodied disciplines, wherein the core practices are directly visible to interested observers (e.g., compare Degas' dancers to Rodin's thinker). These include movement arts, such as traditional dance, as well as crafts, such as herbalism, which literally "embody the knowledge artisans have had with the materials of nature and the circumstances of their communities" (Borgmann, 1992, p. 121). In these domains there is no strong tradition of distinguishing between procedural performance and conceptual understanding. Dancers, for example, "use their body to think with" (Kirsh, 2011). As such, we maintain that explicitly embodied disciplines provide a unique laboratory for investigating the relation between procedure and concept, particularly in relief to how this relation transpires in mathematical domains.

Emerging from our design-based research in this area is an articulation of the following constructs. An *embodied artifact* is a rehearsed physical performance serving as a resource for coping with particular situations in the world. A *conceptual performance* is a disciplinarily signified embodied artifact that serves as a concept's grounding referent. Such an approach may dissolve the barrier between procedures and concepts, performance and understanding: instead of emphasizing procedural fluency and conceptual understanding as separate aspects of disciplinary competence, we hone the constructs' distinctions along semiotic lines (cf. Radford, 2003). One implication for educational practice is that conceptual performances may be targeted for effective instruction of even seemingly "abstract" mathematical concepts.

Accordingly, our educational design and, more generally, our emerging design framework, build on the constructivist tradition of promoting passages from non-symbolic interaction to symbolic representation, that is, from somatics to semiotics (Bamberger, 1991; Bruner, Oliver, & Greenfield, 1966; diSessa, 1983; Edwards, 1995; Forman, 1988; Kamii & DeClark, 1985; Papert, 1980). Yet recent technological affordances of embodied-interaction media enable us to push constructivist methodology in directions that make for microworlds better tailored to how people learn "in the wild." In particular, we have engineered qualitative-to-quantitative trajectories toward targeted mathematical content that provide students meaningful struggle with core learning challenges *even before a single number has been evoked*.

In practice, our design for supporting learning processes instantiates target mathematical content in the form physically immersive interaction (see Nemirovsky, 2003). In this technological system, learners remote-control virtual objects on a computer display in an attempt to effect a specified target feedback, such as causing an initially red display to turn green. Only very particular body *postures*, such as holding the hands at specific heights, cause this target feedback. Moreover, only a very specific way of *moving* in the interaction space sustains the target feedback dynamically. By discovering and rehearsing a strategy for "moving in the green zone," users learn a technologically mediated choreographed sequence of actions (a *"math kata"*)—this is the embodied artifact. Once they discover the target perceptuomotor strategy, we equip users with mathematical frames of reference, such as a Cartesian grid and numerals overlain on the interaction space, by which to evaluate, adapt, and re-signify this strategy in accord with disciplinary norms of discourse and practice—this is the conceptual performance. We are thus witnessing *concepts emerge as descriptions of actions*.

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